MULTIPLE TARGET DETECTION AND ESTIMATION BY EXPLOITING THE AMPLITUDE MODULATION INDUCED BY ANTENNA SCANNING. PART I: PARAMETER ESTIMATION*

Maria Greco[1], Fulvio Gini[1], and Alfonso Farina[2]

[1] Dip. di Ingegneria dell'Informazione, Università di Pisa, via Diotisalvi, 2 - 56126 Pisa, Italia
Tel: +39-050-568550, Fax: +39-050-568522, e-mail: {f.gini,m.greco}@ing.unipi.it

[2] Radar & Technology Division, Alenia-Marconi Systems, via Tiburtina Km 12.4, 00131 Roma, Italia
Tel: +39-06-41502279, Fax: +39-06-41502665, e-mail: afarina@amsjv.it

ABSTRACT

This work deals with the problem of estimating the parameters of multiple targets present in the same range-azimuth resolution cell of a surveillance radar. The maximum likelihood (ML) and the asymptotic ML (AML) estimators are derived. However, they may be too computationally heavy for surveillance applications. To maximize the nonlinear two-dimensional criterion of the AML estimator we propose a computationally efficient algorithm based on the RELAXation method. The proposed method overcomes the resolution limitation of the classical monopulse technique and allows to resolve multiple targets exhibiting an arbitrarily small difference in azimuth as long as their Doppler frequencies differ by a fraction of the intrinsic resolution of the system. The performance of the proposed AML-RELAX estimator is numerically investigated through Monte Carlo simulation and Cramér-Rao lower bound calculation.

1. INTRODUCTION

In most of modern radar systems, the target direction of arrival (DOA) is estimated by the monopulse technique [8], which in principle can work with just a single pulse. The price to pay is the need for two tightly matched receiving channels: the sum (Σ) and the difference (Δ) channels. The estimate of the target DOA is a function of the ratio of the Δ and Σ channel outputs. When multiple targets are present in the range-azimuth cell under test, the monopulse system provides an erroneous DOA measure [5]. This work describes a method to jointly estimate complex amplitudes, Doppler frequencies, and DOAs of multiple targets present in the same range-azimuth cell using only one receiving channel, exploiting knowledge of the antenna beam pattern and the fact that the mechanical scanning impresses an amplitude modulation on the received signal [2].

The rest of this paper is organized as follows. The data model and the problem statement are introduced in Sect. 2. The ML and the asymptotic (large sample size) ML (AML) estimators are derived in Sect. 3. We also propose an efficient implementation of the AML based on the RELAX method. It decouples the AML problem into simpler problems, where the DOA and Doppler frequency of each target signal are estimated separately and sequentially, starting from the strongest target signal and ending with the weakest one. In Sect. 4, the performance of the AML-RELAX algorithm are investigated and compared to the Cramér-Rao lower bound.

2. DATA MODEL AND PROBLEM STATEMENT

Assume that M point-like targets are present in the range-azimuth cell under test with direction of arrivals $\{\theta_{TGi}\}_{i=1}^{M}$ and Doppler frequencies $\{f_{Di}\}_{i=1}^{M}$. The data vector $\mathbf{z}$ is composed by the collection of the N echoes received during the ToT. The nth element of $\mathbf{z}$ is given by:

$$z(n) = \sum_{i=1}^{M} b_i G(n, \theta_{TGi})\, e^{j2\pi f_{Di} n} + d(n), \quad n = 0, \cdots, N-1, \tag{1}$$

where b_i is the unknown complex amplitude of the ith target signal, $G(n,\theta_{TGi})$ is the two-way antenna gain for the nth pulse from the ith DOA, and $f_{Di} \in [-0.5, 0.5)$ is the Doppler frequency of the ith target normalized to the pulse repetition frequency (PRF). The term $d(n)$ models the disturbance, which is composed by clutter and thermal noise. Assuming that the radar antenna rotates mechanically with constant angular velocity ω_R rad/s and that the one-way antenna beam pattern has a Gaussian shape [10], the amplitude of the nth pulse of the signal backscattered by the ith target is proportional to:

$$G(n, \theta_{TGi}) = G_0 \exp\left[-4 \ln 2 \left(\frac{\theta_{TGi} - n\omega_R T}{\theta_B} \right)^2 \right] \tag{2}$$

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where $0 \le \theta_{TGi} < \theta_B$ for $i = 1, \cdots, M$, G_0 is the maximum gain, θ_B is the –3 dB antenna beam width, and T=1/PRF is the radar pulse repetition time. The number N of pulses collected during the *time-on-target* (*ToT*) by the radar within the –3 dB points is given by $N = \theta_B / (\omega_R T)$. In vector notation, the data model is given by $\mathbf{z} = \mathbf{A}(\boldsymbol{\theta})\mathbf{b} + \mathbf{d}$, where $\mathbf{z}$ is the $N \times 1$ complex data vector, $\mathbf{b} = [b_1 \cdots b_M]^T$ is the $M \times 1$ vector of the unknown complex amplitudes, ^{T} denotes the transpose operation, $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_{TG1}, f_{D1}) \cdots \mathbf{a}(\theta_{TGM}, f_{DM})]$ is the $N \times M$ steering matrix, $\boldsymbol{\theta} = [\theta_{TG1} \cdots \theta_{TGM}\ f_{D1} \cdots f_{DM}]^T$ is the $2M \times 1$ vector of the unknown DOAs and Doppler frequencies, $\mathbf{a}(\theta_{TGi}, f_{Di})$ is an $N \times 1$ vector which can be factored as the product of the spatial and temporal steering vectors: $\mathbf{a}(\theta_{TGi}, f_{Di}) = \mathbf{g}(\theta_{TGi}) \odot \mathbf{p}(f_{Di})$, where $\odot$ represents the Hadamard product [9], $[\mathbf{g}(\theta_{TGi})]_n = G(n-1, \theta_{TGi})$, and $[\mathbf{p}(f_{Di})]_n = \exp[j2\pi f_{Di}(n-1)]$, for $n = 1, \cdots, N$. The elements of vectors $\mathbf{b}$ and $\boldsymbol{\theta}$ are modeled as unknown deterministic parameters. The $N \times 1$ disturbance vector $\mathbf{d}$ is composed by the sum of thermal noise and clutter. It is modeled as a complex zero-mean Gaussian random vector with covariance matrix $\mathbf{M}_d = E\{\mathbf{d}\mathbf{d}^H\} = \sigma_d^2 \mathbf{M}$, where σ_d^2 is the unknown total disturbance power and $\mathbf{M}$ is the normalized covariance matrix, i.e. $[\mathbf{M}]_{i,i} = 1$ for $i = 1, \cdots, N$. In this first part of the work we assume that the number M of targets is known and we tackle only the "estimation problem." The "detection problem," which consists of determining the number M of targets, is the subject of the second part [4]. In summary, the goal here is to estimate $\mathbf{b}$ and $\boldsymbol{\theta}$ based on the observation of N consecutive samples $\{z(n)\}_{n=0}^{N-1}$. We exploit knowledge of the antenna main beam pattern and the consequent amplitude modulation impressed on the signal backscattered by each target. This basic idea was originally described in [10] for a single target scenario. In [1] a linear algorithm for the estimation of a single target DOA was proposed. In this paper, we extend on these works to consider the presence of multiple targets in the same range-azimuth resolution cell. In [2] the same estimation problem was solved assuming that the Doppler frequencies were a priori known. Here, we remove this unrealistic assumption.

3. ML AND ASYMPTOTIC ML ESTIMATION

Conditioned to a given $\boldsymbol{\theta}$ and $\mathbf{b}$, the data vector $\mathbf{z}$ is complex Gaussian distributed with probability density function (pdf) given by:

$$p_{\mathbf{z}}(\mathbf{z}; \mathbf{b}, \boldsymbol{\theta}) = \frac{1}{(\pi\sigma_d^2)^N |\mathbf{M}|} \exp\left[-\frac{(\mathbf{z} - \mathbf{A}\mathbf{b})^H \mathbf{M}^{-1} (\mathbf{z} - \mathbf{A}\mathbf{b})}{\sigma_d^2} \right], \quad (3)$$

where, for ease of notation, we omitted the dependence of $\mathbf{A}(\boldsymbol{\theta})$ on $\boldsymbol{\theta}$. Derivation of the ML estimator and of the Cramér-Rao lower bound (CRLB) is formally identical to that presented in [2] for known Doppler frequencies, with the only difference that now $\boldsymbol{\theta}$ is the $2M \times 1$ vector defined in Sect. 2, instead of the $M \times 1$ vector containing only the unknown DOAs. The ML derived for the deterministic target amplitude model is often termed *conditional* ML (CML) [6,9]. In [2], performance analysis of the DOA CML estimator was carried out assuming the stochastic model for $\mathbf{b}$. The CML estimate is obtained by maximizing the likelihood function (LF) $p_{\mathbf{z}}(\mathbf{z}; \mathbf{b}, \boldsymbol{\theta})$ in (3) with respect to $\boldsymbol{\theta}$ and $\mathbf{b}$. After some manipulations we find:

$$\hat{\boldsymbol{\theta}}_{CML} = \arg\max_{\boldsymbol{\theta}} \mathbf{z}^H \mathbf{M}^{-1} \mathbf{A} (\mathbf{A}^H \mathbf{M}^{-1} \mathbf{A})^{-1} \mathbf{A}^H \mathbf{M}^{-1} \mathbf{z}, \quad (4)$$

$$\hat{\mathbf{b}}_{CML} = (\mathbf{A}^H \mathbf{M}^{-1} \mathbf{A})^{-1} \mathbf{A}^H \mathbf{M}^{-1} \mathbf{z}. \quad (5)$$

Calculation of $\hat{\boldsymbol{\theta}}_{CML}$ requires the 2M-dimensional (2M-D) nonlinear maximization of the functional:

$$F(\boldsymbol{\theta}) = \mathbf{z}^H \mathbf{M}^{-1} \mathbf{A} (\mathbf{A}^H \mathbf{M}^{-1} \mathbf{A})^{-1} \mathbf{A}^H \mathbf{M}^{-1} \mathbf{z}, \quad (6)$$

where $\boldsymbol{\theta} = [\theta_1 \ \cdots \ \theta_M \ f_1 \ \cdots \ f_M]^T$ here denotes the generic parameter vector. When only one target is present in the range-azimuth cell under test, $\boldsymbol{\theta}$ is a 2×1 vector and $\mathbf{b}$ is a scalar. In particular, estimator (4) becomes:

$$(\hat{\theta}_{CML}, \hat{f}_{CML}) = \arg\max_{\theta, f} \frac{|\mathbf{z}^H \mathbf{M}^{-1} \mathbf{a}(\theta, f)|^2}{\mathbf{a}^H(\theta, f) \mathbf{M}^{-1} \mathbf{a}(\theta, f)}, \quad (7)$$

where $\mathbf{a}(\theta, f) = \mathbf{g}(\theta) \odot \mathbf{p}(f)$ is the overall target steering vector introduced in Sect. 2. It is useful now, for future developments, to define the following quantity:

$$\Gamma(\theta, f) = F(\boldsymbol{\theta})\big|_{M=1} = \frac{|\mathbf{z}^H \mathbf{M}^{-1} \mathbf{a}(\theta, f)|^2}{\mathbf{a}^H(\theta, f) \mathbf{M}^{-1} \mathbf{a}(\theta, f)}, \quad (8)$$

which is the LF for the single target scenario. Therefore, $(\hat{\theta}_{CML}, \hat{f}_{CML}) = \arg\max_{\theta, f} \Gamma(\theta, f)$ is the CML estimator for the single target scenario. The CML estimator (4) requires a nonlinear 2M-D maximization. Generally this maximization is computationally cumbersome and may be not feasible in real-time. Therefore, it would be useful to find a suboptimum algorithm which trades off good performance with computational complexity. We use the CML estimator as a starting point to derive an algorithm based on M 2-D maximizations instead of one 2M-D maximization of $F(\boldsymbol{\theta})$. Under the hypothesis that the Doppler frequencies are separated, that is $|f_{Di} - f_{Dj}| > 1/N$ when $i \ne j$, in [3] we prove that

$$F(\boldsymbol{\theta}) = \sum_{i=1}^{M} \frac{|\mathbf{z}^H \mathbf{M}^{-1} (\mathbf{g}(\theta_i) \odot \mathbf{p}(f_i))|^2}{(\mathbf{g}(\theta_i) \odot \mathbf{p}(f_i))^H \mathbf{M}^{-1} (\mathbf{g}(\theta_i) \odot \mathbf{p}(f_i))} + o(N) \quad (9)$$

where we used Landau's notation:

$$f(N) = o(N^k) \overset{def}{\Leftrightarrow} \lim_{N \to \infty} N^{-k} f(N) = 0, \quad (10)$$

so that $f(N) = o(N)$ means that $\lim_{N \to \infty} N^{-1} f(N) = 0$, i.e. the term $o(N)$ in (9) is negligible for large N with respect to the sum of the M terms. Therefore, $F(\boldsymbol{\theta})$ is asymptotically formed by the superposition of the M terms $\{\Gamma(\theta_i, f_i); i = 1, \cdots, M\}$. Result (9) implies that:

$$\lim_{N\to\infty} \max_{(\theta_1,\cdots,\theta_M, f_1,\cdots,f_M)} N^{-1}F(\boldsymbol{\theta}) = \lim_{N\to\infty} \sum_{i=1}^{M} \max_{(\theta_i, f_i)} N^{-1}\Gamma(\theta_i, f_i). \quad (11)$$

Based on this observation, we assume large N and we neglect the term $o(1)$ in (9) to obtain the *asymptotic* (large N) maximum likelihood (AML) estimator:

$$\hat{\boldsymbol{\theta}}_{AML} = \arg\max_{(\theta_1,\theta_2,\cdots,\theta_M, f_1, f_2,\cdots,f_M)} \sum_{i=1}^{M} \Gamma(\theta_i, f_i). \quad (12)$$

In words, the AML estimates $\boldsymbol{\theta}$ from the locations of the M highest peaks of $\Gamma(\theta, f)$. It is less computationally heavy than the CML, in fact it replaces the *2M*-D nonlinear search required by the CML of (8) with the search of the locations of the M highest peaks of a 2-D functional. We expect that if N is large enough, the performance loss of the AML over the CML will be negligible. In Fig. 1 we plot $\Gamma(\theta, f)$ in the presence of two targets (M=2); we defined $SDR_i = |b_i|^2 / \sigma_d^2$, for $i = 1, \cdots, M$.

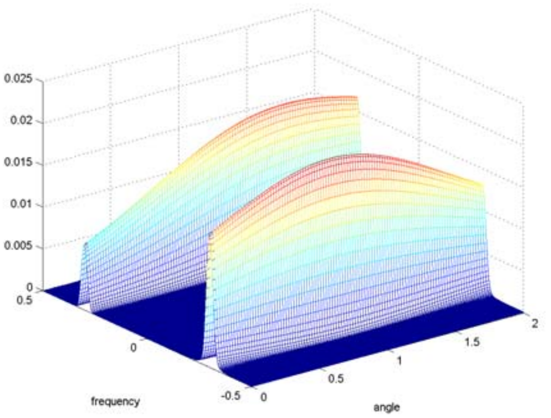

Fig. 1. *Plot of* $\Gamma(\theta, f)$*, for M=2, N=32,* $\theta_B = 2^\circ$*,* $\theta_{TG1} = 0.9^\circ$*,* $\theta_{TG2} = 1.5^\circ$*,* $f_{D1} = -f_{D2} = -0.3$*,* $SDR_1 = SDR_2 = 20\ dB$*, and* $CNR \to -\infty$*, i.e.* **M=I***, where* **I** *is the identity matrix.*

The AML problem is reminiscent of the problem of jointly estimating the frequencies of a signal composed by multiple sinusoids. In that case, the function to be maximized is the 1-D Periodogram and the problem is reverted into a sequence of 1-D maximization problems through a RELAXation approach [7]. In our case for each component we have to estimate two parameters, therefore the 1-D Periodogram is replaced by the 2-D functional $\Gamma(\theta, f)$. Based on this observation, to reduce the computational complexity of the AML algorithm, we used an algorithm based on the RELAX method. The most important feature of RELAX is that it decouples the multidimensional maximization problem into a series of simpler 1-D problems. In general, maximizations with respect to $(\theta_1, f_1), \cdots, (\theta_M, f_M)$ are coupled. It can be shown (see [3]) that when N goes to infinity, $N^{-1}\Gamma(\theta, f)$ approaches to non-zero values only for $f = \{f_{Di}\}_{i=1}^{M}$, whatever the values of θ and $\{\theta_{TGi}\}_{i=1}^{M}$ are. This suggests that it is possible to estimate $\{\theta_{TGi}, f_{Di}\}$ estimating first all the $\{f_{Di}\}$ from the locations of the highest peaks of the FFT of the (whitened) data; then the DOAs can be estimated by plugging $\{\hat{f}_{Di}\}$ in the AML, which estimates sequentially the M DOAs performing M one-dimensional nonlinear maximizations of $\Gamma(\theta, f)$ in (8), calculated sequentially for $f = \hat{f}_{D1}, \cdots, \hat{f}_{DM}$, or by plugging $\{\hat{f}_{Di}\}$ in the CML estimator for known Doppler frequencies [2,3]. These ideas are exploited here to derive an algorithm based on RELAX. The most important feature of RELAX is that it allows us to decouple the search of the locations of the M highest peaks in the 2-D functional $\Gamma(\theta, f)$ into a sequence of 1-D nonlinear maximization problems. Roughly speaking, RELAX estimates first the parameters of the strongest component; then removes the contribution of the strongest component from the data and proceeds with the second strongest component; and so on up to the Mth component. Then, it iteratively refines the estimates of each pair (θ_{TGi}, f_{Di}) working again on a component-by-component basis. The details on the RELAX algorithm are not reported here for lack of space, the details can be found in [3].

4. NUMERICAL PERFORMANCE ANALYSIS

We now investigate the performance of the AML estimator implemented using the RELAX approach. The root mean square error (RMSE) was derived by running 10^3 Monte Carlo simulations and compared with the square root of the Cramér-Rao lower bound (RCRLB). We assumed that two targets are present in the resolution cell under test with deterministic unknown complex amplitudes. The behavior of RMSE and RCRLB was investigated as a function of N, SDR, $\Delta f = f_{D1} - f_{D2}$, $\Delta\theta = \theta_{TG1} - \theta_{TG2}$, and CNR. However, only a small subset of results is shown here. If not otherwise stated, the signal parameters are N=*16*, $\theta_B = 2^\circ$, $\theta_{TG1} = 0.9^\circ$, $\theta_{TG2} = 1.5^\circ$, $f_{D1} = -f_{D2} = -0.3$, $SDR_1 = SDR_2 = 20\ dB$ and $CNR \to -\infty$. $RMSE(\hat{\theta}_{TGi})$ and $RCRLB(\theta_{TGi})$ are measured in degrees. In Figs. 2 and 3, we plot the performance of Doppler frequency and DOA estimators, respectively, as a function of the number N of integrated pulses. For $N \geq 16$ the performance of the estimators are very close to the RCRLB. In Figs. 4 and 5, the same curves are plotted as a function of SDR_2, while b_1 is kept constant, so that $SDR_1 = 20\ dB$. The threshold effect, typical of non-linear estimators, is quite evident for $SDR_2 < 5\ dB$. The numerical results described here were derived assuming **M=I**, i.e. $CNR \to -\infty$. When the clutter is also present **M** is non-diagonal and, in a realistic radar scenario, it must be estimated from secondary data.

5. CONCLUDING REMARKS

In summary, our analyses corroborate the following results: (*i*) efficient estimators of DOAs and Doppler frequencies are asymptotically decoupled; (*ii*) multiple targets can be resolved even if they have the same DOA, provided they have well separated Doppler frequencies, i.e. $|f_{Di} - f_{Dj}| > 1/N$ when $i \neq j$; (*iii*) the DOA estimator is almost insensitive to the relative angular position of the targets and it exhibits good performance even for low values of *N* and *SDR*; (*iv*) for low values of *SDR*, the Doppler frequency estimator is affected by the threshold effect, but it is almost efficient above the threshold; (*v*) the worst case scenario is when the disturbance is composed by only thermal noise.

6. REFERENCES

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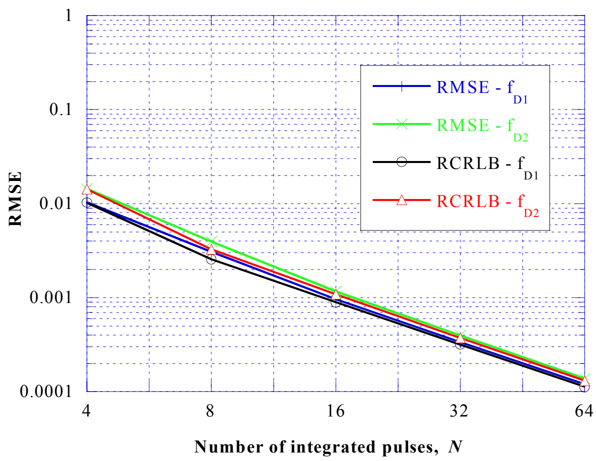

Fig. 2. *Plot of* $RMSE(\hat{f}_{Di})$ *and* $\sqrt{CRLB(f_{Di})}$, *i*=1,2.

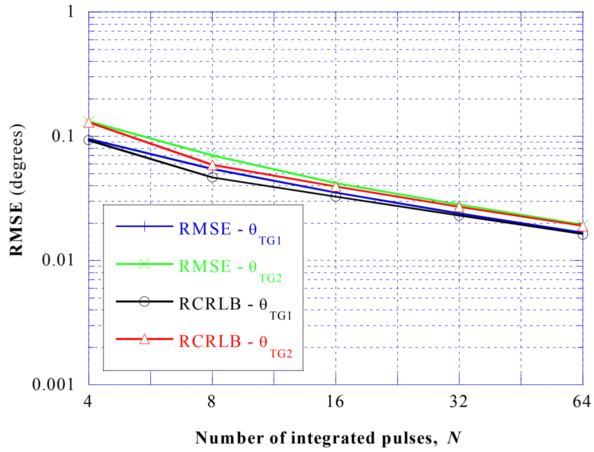

Fig. 3. *Plot of* $RMSE(\hat{\theta}_{TGi})$ *and* $\sqrt{CRLB(\theta_{TGi})}$, *i*=1,2.

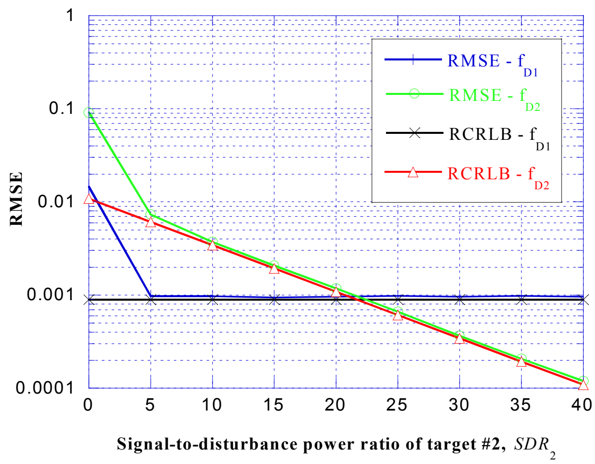

Fig. 4. *Plot of* $RMSE(\hat{f}_{Di})$ *and* $\sqrt{CRLB(f_{Di})}$, *i*=1,2.

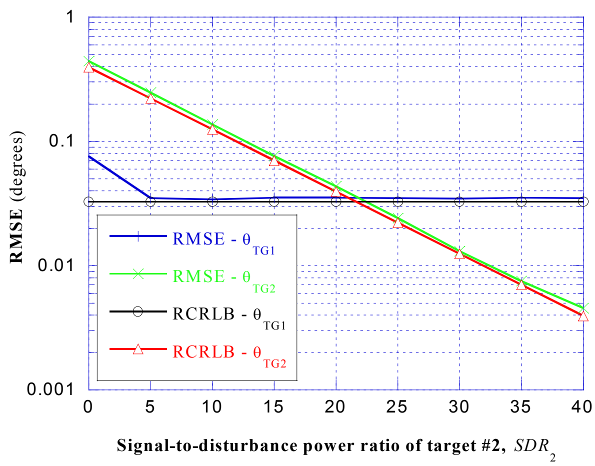

Fig. 5. *Plot of* $RMSE(\hat{\theta}_{TGi})$ *and* $\sqrt{CRLB(\theta_{TGi})}$, *i*=1,2.