

ON EFFICIENT IMPLEMENTATION OF OVERSAMPLED LINEAR PHASE PERFECT RECONSTRUCTION FILTER BANKS

Jie Liang¹, Lu Gan², Chengjie Tu¹, Trac D. Tran¹, and Kai-Kuang Ma²

¹ ECE Department, The Johns Hopkins University
Baltimore, MD, USA 21211. Email: {jliang, cjtu, trac}@jhu.edu

² School of EEE, Nanyang Technological University
Singapore 639798. Email: {pg00137928, ekkma}@ntu.edu.sg

ABSTRACT

In this paper, we first present an alternative way of generating oversampled linear phase perfect reconstruction filter banks (OSLPPRFB). We show that this method provides the minimal factorization of a subset of existing OSLPPRFB. The combination of the new structure and the conventional one leads to efficient implementations of a general class of OSLPPRFB. Possible application of the new scheme is discussed.

1. INTRODUCTION

Oversampled filter bank (OSFB) has received growing attentions recently due to its design flexibility and improved noise resistance and noise shaping capabilities. For applications in image and video processing, linear phase is always a desired property. The necessary conditions on the number of symmetric filters N_s and anti-symmetric filters N_a for the existence of oversampled linear-phase paraunitary filter banks were first studied in [1]. The lattice structure factorizations for systems with either $N_s = N_a$ or $N_s = N_a + 1$ were also developed, based on the result of critically sampled LPPUFB [2]. In [3, 4, 5], the lattice structure in [1] was generalized to perfect reconstruction case. In particular, a thorough study of the necessary conditions, lattice structures, and parameterizations of OSLPPRFB was presented in [4, 5]. The necessary conditions derived in them are tighter than those in [1], and lattice structures for arbitrary feasible N_s and N_a were proposed. Completeness of the structure for certain subsets is also proved.

Minimality is another important issue to be considered in lattice factorization. A structure is said to be *minimal* if it uses the fewest number of delay units to implement a given filter bank [6]. A minimal structure is always preferred in practice since it has lower complexity than a non-minimal one. Although the minimal factorization for critically sampled linear phase perfect reconstruction filter banks has been well known [2], it is still an open problem for oversampled systems. The lattice factorization family in [1, 3, 4, 5] is not minimal for certain filter banks, as observed in [5].

Existing lattice structure generates oversampled systems by first expanding the input data block, and then applying various stages of postprocessing. In this paper, we present an alternative method by applying a postprocessing to a critically sampled LP-PRFB. We show that this method leads to the minimal factorization of a subset of the lattice structure in [5]. We then generalize

the idea and present an efficient lattice structure for the family of OSLPPRFB given in [5]. The new structure contains both pre- and postprocessing of a rectangular core transform, and can be viewed as a natural generalization of the pre- and postprocessing-based lattice structure for critically sampled linear phase FB in [7]. Possible application of the new structure in the robust transmission of image and video over OFDM system is discussed.

Notations: Throughout the paper, vectors and matrices are indicated by bold-faced letters. \mathbf{I}_n , \mathbf{J}_n , \mathbf{O}_n denote $n \times n$ identity, reversal identity and null matrices, respectively.

2. EXISTING LATTICE STRUCTURE

Consider a P -channel, KM -tap oversampled linear-phase perfect reconstruction filter bank with N_s symmetric filters, N_a anti-symmetric filters, and a decimation factor of M ($P > M$). The lattice structures in [4, 5] for such a system can be summarized as

$$\mathbf{E}(z) = \mathbf{G}_{K-1}(z) \mathbf{G}_{K-2}(z) \cdots \mathbf{G}_1(z) \mathbf{E}_0(z), \quad (1)$$

where the size of $\mathbf{G}_i(z)$ ($i = 1, \dots, K-1$) and $\mathbf{E}_0(z)$ are $P \times P$ and $P \times M$, respectively.

For simplicity purpose, we focus on situations with $P = 2p$, $M = 2m$, and $N_s = N_a = p$, where p and m are integers. In this case, the following choices of $\mathbf{E}_0(z)$ and $\mathbf{G}_i(z)$ can cover a large family of OSFB:

$$\begin{aligned} \mathbf{E}_0(z) &= \mathbf{E}_0 = \text{diag}(\mathbf{U}_0, \mathbf{V}_0) \hat{\mathbf{W}}_M, \\ \mathbf{G}_i(z) &= \text{diag}(\mathbf{I}_p, \mathbf{T}_i) \mathbf{W}_P \mathbf{\Lambda}_P(z) \mathbf{W}_P, \end{aligned} \quad (2)$$

where $\mathbf{\Lambda}_P(z) = \text{diag}(\mathbf{I}_p, z^{-1} \mathbf{I}_p)$ and

$$\hat{\mathbf{W}}_M = \frac{\sqrt{2}}{2} \begin{bmatrix} \mathbf{I}_m & \mathbf{J}_m \\ \mathbf{I}_m & -\mathbf{J}_m \end{bmatrix}, \quad \mathbf{W}_P = \frac{\sqrt{2}}{2} \begin{bmatrix} \mathbf{I}_p & \mathbf{I}_p \\ \mathbf{I}_p & -\mathbf{I}_p \end{bmatrix}.$$

In (2), \mathbf{U}_0 and \mathbf{V}_0 are two $p \times m$ tall matrices that are left-invertible, and \mathbf{T}_i in each building stage $\mathbf{G}_i(z)$ is a $p \times p$ free invertible matrix. The implementation is depicted in Fig. 1 (a).

It was observed in [5] that the above structure is not minimal for some OSFB. In this paper, we will show when this situation occurs and more importantly, the minimal lattice structure for such kinds of OSFB.

This research has been supported by NSF under CAREER Grant CCR-0093262.

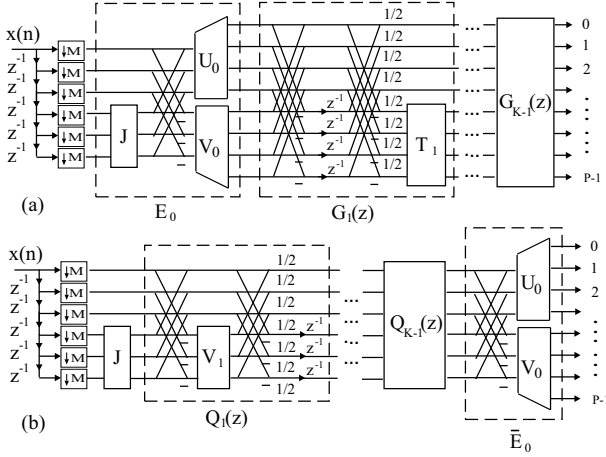


Fig. 1. (a) The existing structure of oversampled filter banks; (b) Minimal factorization of a subset of (a).

3. AN ALTERNATIVE STRUCTURE

Recall that any linear combination of a group of symmetric (anti-symmetric) filters is still symmetric (anti-symmetric) [2]. Suppose $\mathbf{A}_M(z)$ is the polyphase matrix of an M -channel, KM -tap critically sampled LPPRFB. Define

$$M_s = \lceil M/2 \rceil, \quad M_a = \lfloor M/2 \rfloor. \quad (3)$$

It is shown in [2] that $\mathbf{A}_M(z)$ has M_s symmetric filters and M_a anti-symmetric filters. Without loss of generality, we assume that the first M_s rows of $\mathbf{A}_M(z)$ correspond to symmetric filters. We can thus obtain an oversampled FB by applying a postprocessing to $\mathbf{A}_M(z)$ as follows:

$$\mathbf{E}(z) = \text{diag}(\mathbf{U}_0, \mathbf{V}_0) \mathbf{A}_M(z), \quad (4)$$

where \mathbf{U}_0 is an $N_s \times M_s$ left invertible matrix. It is applied to the symmetric filters of $\mathbf{A}_M(z)$ and N_s symmetric filters can be generated. Similarly, N_a anti-symmetric filters can be obtained after applying the $N_a \times M_a$ left invertible matrix \mathbf{V}_0 to the anti-symmetric filters of $\mathbf{A}_M(z)$.

By applying the lattice structure of critically sampled LPPRFB in [2, 8, 9, 7], Eq. (4) leads to another implementation of OSFB. In particular, when $M = 2m$, we can write

$$\mathbf{E}(z) = \mathbf{E}_0(z) \mathbf{Q}_{K-1}(z) \mathbf{Q}_{K-2}(z) \cdots \mathbf{Q}_1(z) \mathbf{Q}_0, \quad (5)$$

where $\mathbf{Q}_0 = \text{diag}(\mathbf{I}_m, \mathbf{J}_m)$ and

$$\begin{aligned} \mathbf{E}_0(z) &= \bar{\mathbf{E}}_0 = \text{diag}(\mathbf{U}_0, \mathbf{V}_0) \mathbf{W}_M, \\ \mathbf{Q}_i(z) &= \mathbf{\Lambda}_M(z) \mathbf{W}_M \text{diag}(\mathbf{I}_m, \mathbf{V}_i) \mathbf{W}_M. \end{aligned} \quad (6)$$

Here $\mathbf{\Lambda}_M(z) = \text{diag}(\mathbf{I}_m, z^{-1} \mathbf{I}_m)$. The term \mathbf{V}_i is an $m \times m$ invertible matrix. The corresponding lattice structure is given in Fig. 1 (b).

Both (1) and (5) can generate P -channel, KM -tap OSFB, but their approaches are different. In (1), an $M \times 1$ data block is first expanded into a $P \times 1$ block. Various preprocessing stages are then applied to the augmented block. In (5), an M -channel critically sampled FB is first generated before being expanded into a P -channel oversampled system. The operations in the M -sample block side can be viewed as preprocessing to the expander $\bar{\mathbf{E}}_0$.

Clearly, the complexity of (5) is lower than that of (1). Hence, some immediate questions are: what is the relationship between the two structures? Which gives better performance? How much is the difference between them? These questions will be addressed in this paper.

4. RELATIONSHIP OF THE TWO STRUCTURES

The following theorem shows that the OSFBs represented by (5) belong to a subset of (1).

Theorem 1 *The OSLPPRFB given by (5) and (6) with $P = 2p$, $M = 2m$, and $N_s = N_a = p$ can always be factorized in terms of (1) and (2).*

Proof: First, notice that any $p \times m$ matrix \mathbf{U}_0 has the following properties:

$$\begin{aligned} \text{diag}(\mathbf{U}_0, \mathbf{U}_0) \mathbf{W}_M &= \mathbf{W}_P \text{diag}(\mathbf{U}_0, \mathbf{U}_0), \\ \text{diag}(\mathbf{U}_0, \mathbf{U}_0) \mathbf{\Lambda}_M(z) &= \mathbf{\Lambda}_P(z) \text{diag}(\mathbf{U}_0, \mathbf{U}_0), \end{aligned} \quad (7)$$

i.e., the matrix $\text{diag}(\mathbf{U}_0, \mathbf{U}_0)$ commutes with the butterfly and the delay chain, with appropriate adjustment of their sizes. To convert (5) into (1), we need to find a $p \times p$ matrix \mathbf{T}_1 such that

$$\text{diag}(\mathbf{U}_0, \mathbf{V}_0) = \text{diag}(\mathbf{I}_p, \mathbf{T}_1) \text{diag}(\mathbf{U}_0, \mathbf{U}_0), \quad (8)$$

i.e.,

$$\mathbf{T}_1 \mathbf{U}_0 = \mathbf{V}_0. \quad (9)$$

Notice that for left-invertible \mathbf{U}_0 and \mathbf{V}_0 , we can always find two $p \times (p-m)$ matrices $\bar{\mathbf{U}}_0$ and $\bar{\mathbf{V}}_0$ such that $\mathbf{U} \triangleq [\mathbf{U}_0, \bar{\mathbf{U}}_0]$ and $\mathbf{V} \triangleq [\mathbf{V}_0, \bar{\mathbf{V}}_0]$ are invertible. We can thus choose $\mathbf{T}_1 = \mathbf{V} \mathbf{U}^{-1}$, which satisfies $\mathbf{T}_1 \mathbf{U} = \mathbf{V}$ and therefore $\mathbf{T}_1 \mathbf{U}_0 = \mathbf{V}_0$. This allows us to write

$$\begin{aligned} \bar{\mathbf{E}}_0 &= \text{diag}(\mathbf{U}_0, \mathbf{V}_0) \mathbf{W}_M \\ &= \text{diag}(\mathbf{I}_p, \mathbf{T}_1) \text{diag}(\mathbf{U}_0, \mathbf{U}_0) \mathbf{W}_M \\ &= \text{diag}(\mathbf{I}_p, \mathbf{T}_1) \mathbf{W}_P \text{diag}(\mathbf{U}_0, \mathbf{U}_0). \end{aligned} \quad (10)$$

By (7), the term $\text{diag}(\mathbf{U}_0, \mathbf{U}_0)$ in (10) can be further moved across $\mathbf{\Lambda}_M(z)$ and $\mathbf{W}_M(z)$ in $\mathbf{Q}_{K-1}(z)$. Therefore

$$\bar{\mathbf{E}}_0 \mathbf{Q}_{K-1}(z) = \mathbf{G}_1(z) \text{diag}(\mathbf{U}_0, \mathbf{U}_0 \mathbf{V}_{K-1}) \mathbf{W}_M, \quad (11)$$

where

$$\mathbf{G}_1(z) = \text{diag}(\mathbf{I}_p, \mathbf{T}_1) \mathbf{W}_P \mathbf{\Lambda}_P(z) \mathbf{W}_P. \quad (12)$$

This shows that by moving the tall matrices into the lattice structure, we can turn the $M \times M$ preprocessing stage $\mathbf{Q}_{K-1}(z)$ into a $P \times P$ preprocessing stage $\mathbf{G}_1(z)$.

Successively applying the procedure in (8) to (11), all $M \times M$ stages $\mathbf{Q}_i(z)$ can be converted into $P \times P$ stages $\mathbf{G}_i(z)$. Finally we can move $\text{diag}(\mathbf{U}_0, \mathbf{U}_0)$ into $\mathbf{Q}_1(z)$. Let

$$\mathbf{E}_0 \triangleq \text{diag}(\mathbf{U}_0, \mathbf{U}_0 \mathbf{V}_1) \mathbf{W}_M \mathbf{Q}_0, \quad (13)$$

a structure in the form of (1) can be obtained. ■

Theorem 1 states that (5) covers a subset of (1). Clearly, for OSFB in this subset, the implementation in (5) is preferred, since it has lower complexity than (1) has. In fact, the structure in (5) is the minimal factorization, since it uses the same number of delay units as an M -channel, KM -tap critically sampled FB, and the minimality of the latter has been well established [2].

A counter example was given in [5] to illustrate that (1) is not minimal for some cases. However, no minimal solution was found in [5]. It is easy to verify that our proposed structure in (5) provides the minimal implementation of the example in [5].

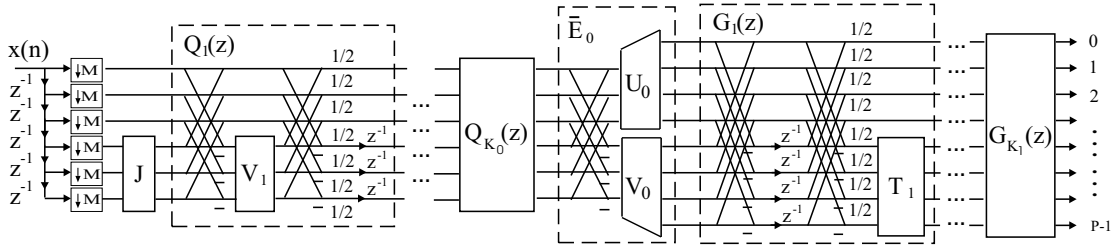


Fig. 2. Efficient implementation of the class of OSLPPRFB in (1).

5. EFFICIENT IMPLEMENTATION FOR GENERAL CASE

Although an $M \times M$ preprocessing stage can always be converted into a $P \times P$ postprocessing stage, the converse is true only in some special cases. Consider the lattice given in (1) and (2). In order to move $\text{diag}(\mathbf{U}_0, \mathbf{U}_0)$ into $\mathbf{G}_1(z)$, we have to find an $m \times m$ matrix \mathbf{V}_1 such that

$$\text{diag}(\mathbf{U}_0, \mathbf{V}_0) = \text{diag}(\mathbf{U}_0, \mathbf{U}_0) \text{diag}(\mathbf{I}_m, \mathbf{V}_1), \quad (14)$$

i.e.,

$$\mathbf{U}_0 \mathbf{V}_1 = \mathbf{V}_0. \quad (15)$$

Such a \mathbf{V}_1 exists if and only if \mathbf{U}_0 and \mathbf{V}_0 span the same column space. In this case, the structure can be simplified into

$$\mathbf{G}_1(z) \mathbf{E}_0 = \text{diag}(\mathbf{U}_0, \mathbf{T}_1 \mathbf{U}_0) \mathbf{W}_M \mathbf{Q}_1(z) \mathbf{Q}_0, \quad (16)$$

where

$$\mathbf{Q}_1(z) = \mathbf{\Lambda}_M(z) \mathbf{W}_M \text{diag}(\mathbf{I}_m, \mathbf{V}_1) \mathbf{W}_M. \quad (17)$$

If \mathbf{U}_0 and $\mathbf{T}_1 \mathbf{U}_0$ also span the same column space, we can extract the term $\text{diag}(\mathbf{U}_0, \mathbf{U}_0)$ in (16) and further simplify the lattice. This procedure can be repeated until \mathbf{U}_0 and $\mathbf{T}_i \mathbf{U}_0$ do not have the same column space for an $i \in [1, \dots, K-1]$.

This suggests the following efficient implementation of an OSFB given by (1):

Theorem 2 When $P = 2p$, $M = 2m$, and $N_s = N_a = p$, an OSLPPRFB given by (1) can be implemented as

$$\mathbf{E}(z) = \prod_{i=K_1}^1 \mathbf{G}_i(z) \bar{\mathbf{E}}_0 \prod_{j=K_0}^1 \mathbf{Q}_j(z) \mathbf{Q}_0, \quad (18)$$

where $K_0 + K_1 = K - 1$, and $\mathbf{G}_i(z)$ and $\mathbf{Q}_j(z)$ are given in (2) and (6), respectively.

The structure is illustrated in Fig. 2. Theorem 2 reduces the complexity by maximizing the number of preprocessing stages. As a result, the number of delay units is reduced from $(K-1)p$ in (1) to $K_0 m + K_1 p$. However, whether this structure is minimal or not is still an open problem.

The results in Theorem 1 and Theorem 2 reveal the fundamental difference between oversampled FB and critically sampled FB. For critically sampled system, it is shown in [7] that an FB of given length can be implemented via both pre- and postprocessing of a core transform. Moreover, the pre- and postprocessing stages can be freely converted to each other without changing the complexity of the implementation. Whereas in oversampled system, although preprocessing stages can be converted into postprocessing stages, the converse is not true in general. Moreover, a conversion between pre- and postprocessing affects the complexity of the structure.

6. DESIGN EXAMPLES

By Theorem 2, the OSFBs covered by (1) can be classified into some subsets, according to the number of possible preprocessing stages in their implementations. In this section, we compare the performance of OSFBs in different subsets. The design criterion is to minimize the stopband energy, which is given by

$$\epsilon = \sum_{i=0}^{P-1} \int_{\omega \in \Omega_i} \left(|H_i(e^{j\omega})|^2 + |F_i(e^{j\omega})|^2 \right) d\omega, \quad (19)$$

where $H_i(e^{j\omega})$ and $F_i(e^{j\omega})$ are the frequency responses of the i -th analysis filter and synthesis filter, respectively, and Ω_i denotes the stopband of the i -th subband. The choice of stopbands is similar to that in [8].

Fig. 3 shows two 16-tap OSFBs with $P = 8$, $M = 4$, $K = 4$, and $N_s = N_a = 4$. All free matrices are chosen to be orthogonal. An $n \times n$ square orthogonal matrix is modeled by $\binom{n}{2}$ rotation angles, whereas an $n \times m$ ($n > m$) tall orthogonal matrix is modeled by $\binom{n}{2} - \binom{n-m}{2}$ rotation angles. Fig. 3 (a) gives the optimized result when there are three stages of 8×8 postprocessing. Fig. 3 (b) corresponds to a structure with two stages of 4×4 preprocessing and one stage of 8×8 postprocessing. The two structures involve 28 and 18 rotations, respectively. In addition, the second structure also saves 8 butterflies by using preprocessing. Their stopband energy as defined in (19) are 0.84 and 1.14, respectively. Although the first result has less stopband energy, Fig. 3 shows that their worst stopband attenuations are quite close. Hence the structure with preprocessing provides a fast approximation of the optimal solution with satisfactory performance.

On the other hand, when there is only one postprocessing stage and no preprocessing stage, an 8-tap OSFB is generated. It has 16 rotation angles, but the stopband energy increases to 2.31. This also suggests that the structure with preprocessing is an economical way of boosting the system performance.

7. APPLICATION IN ERROR-RESILIENT DATA TRANSMISSION OVER OFDM SYSTEM

Apart from the stopband energy, some other design criteria for OSFB have been proposed. For example, to improve the robustness of the system against data loss in transmission, the combination of coding gain and MSE of linear prediction was adopted in [1], where the linear prediction was used to estimate the lost data from the available subband coefficients.

Since the tall matrices \mathbf{U}_0 and \mathbf{V}_0 in (5) are at the end of the FB signal flow, the idea in the linearly precoded OFDM (LP-OFDM) in [10] can be applied to take full advantage of the redundancy such that small amount of data loss can be perfectly re-

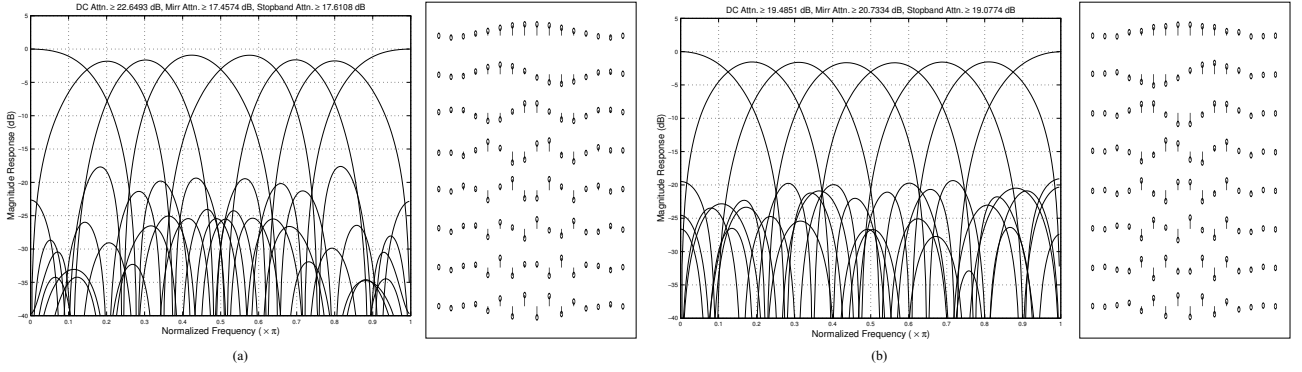


Fig. 3. Design examples of paraunitary OSFB with $P = 8$, $M = 4$, $K = 4$, and $N_s = N_a = 4$: (a) Optimized result with three stages of postprocessing; (b) Optimized result with two stages of preprocessing and one stage of postprocessing.

covered. This method can be useful for transmission over OFDM-based wireless communication systems, such as the wireless LAN.

The idea in LP-OFDM is to choose \mathbf{U}_0 and \mathbf{V}_0 as special Vandermonde matrices:

$$\mathbf{U}_0(i, j) = \alpha_i^j, \quad \mathbf{V}_0(i, j) = \beta_i^j, \quad (20)$$

where the two sets $\{\alpha_i\}_{i=1}^{N_s}$ and $\{\beta_i\}_{i=1}^{N_a}$ contain distinct elements, respectively. This guarantees that any M_s rows of \mathbf{U}_0 and any M_a rows of \mathbf{V}_0 are nonsingular.

Suppose an $M_s \times 1$ vector \mathbf{x} is obtained before \mathbf{U}_0 , then after the insertion of redundancy and transmission over the OFDM system, the received signal can be written as

$$\mathbf{y} = \mathbf{D} \mathbf{U}_0 \mathbf{x} + \mathbf{e}, \quad (21)$$

where \mathbf{e} is the additive channel noise, and \mathbf{D} is a $N_s \times N_s$ diagonal matrix containing the attenuation of the involved OFDM subchannels.

If \mathbf{D} is invertible, we can find the estimate $\hat{\mathbf{x}}$ of \mathbf{x} by

$$\hat{\mathbf{x}} = (\mathbf{U}_0^T \mathbf{U}_0)^{-1} \mathbf{U}_0^T \mathbf{D}^{-1} \mathbf{y}. \quad (22)$$

This is the best linear unbiased estimate (BLUE) of \mathbf{x} when the noise is white.

When there are deep fading subchannels in the OFDM system, some diagonal entries of \mathbf{D} could be very small, generating large diagonal entries in \mathbf{D}^{-1} . This can seriously amplify the noise and cause decoding errors. However, since \mathbf{U}_0 introduces redundancy in the transmitted data, we can afford to discard the corresponding elements in \mathbf{y} when deep fading subchannels are detected in \mathbf{D} . The original signal can still be recovered as long as there are M_s reliable subchannels in \mathbf{D} . A similar approach can be applied to the anti-symmetric filter coefficients as well.

This method can tolerate up to $N_s - M_s$ and $N_a - M_a$ lost data in each block of symmetric coefficients and anti-symmetric coefficients, respectively. Interleaving technique can be applied to further improve the error-correction capability.

To ensure satisfactory performance for both source coding and channel coding, we can optimize the coding gain of the overall OSFB under the Vandermonde constraint of \mathbf{U}_0 and \mathbf{V}_0 . The design and application of this method is under our investigation.

8. CONCLUSION

We present an alternative method of generating oversampled LP-PRFB. Its relationship to existing OSFB is established. The result

leads to efficient implementations of a large class of OSFB. Some design examples are demonstrated, and the potential application of the new scheme in error-resilient data transmission over OFDM is discussed. Although this paper focuses on OSFB with even channel, even decimation factor and same number of symmetric and anti-symmetric filters, the result can be generalized to other cases with appropriate modifications.

9. REFERENCES

- [1] F. Labeau and L. Vandendorpe, "Structure, factorizations, and design criteria for oversampled paraunitary filterbanks yield linear phase filters," *IEEE Trans Signal Processing*, vol. 48, pp. 3062–3071, Nov. 2000.
- [2] A. K. Soman, P. P. Vaidyanathan, and T. Q. Nguyen, "Linear phase paraunitary filter banks: theory, factorizations and designs," *IEEE Trans. Sig. Proc.*, vol. 41, pp. 3480–3496, Dec. 1993.
- [3] T. Tanaka and Y. Yamashita, "The generalized lapped pseudo-biorthogonal transform," *Proc. ICASSP'02*, pp. 1273–1276, May 2002.
- [4] L. Gan and K.-K. Ma, "Theory and lattice factorization of oversampled linear-phase perfect reconstruction filter banks," *Proc. ICASSP'02*, pp. 1161–1164, May 2002.
- [5] L. Gan and K.-K. Ma, "Oversampled linear-phase perfect reconstruction filter banks: Theory, lattice structure and parameterization," *to appear on IEEE Trans. Sig. Process.*
- [6] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Prentice Hall, 1993.
- [7] J. Liang, T. D. Tran, and R. L. de Queiroz, "DCT-based general structure for linear phase paraunitary filter banks," *to appear on IEEE Trans Signal Processing*.
- [8] X. Gao, T. Q. Nguyen, and G. Strang, "On factorization of M-channel paraunitary filterbanks," *IEEE Trans. Signal Processing*, vol. 49, pp. 1433–1446, July 2001.
- [9] L. Gan and K.-K. Ma, "A simplified lattice factorization for linear-phase perfect reconstruction filter bank," *IEEE Signal Processing Letters*, vol. 8, pp. 207–209, July 2001.
- [10] Z. Wang and G. B. Giannakis, "Linearly precoded or coded OFDM against wireless channel fades?," *Third IEEE Sig. Proc. Workshop on Sig. Proc. Advances in Wireless Comm.*, pp. 267–270, Taiwan, March 2001.