EFFICIENT SHARPENING OF CIC DECIMATION FILTER

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ABSTRACT

We propose an efficient sharpening of a CIC decimation filter for an even decimation factor. The proposed structure consists of two main sections: a section composed of a cascade of the first-order moving average filters, and a sharpening filter section. The proposed decomposition scheme allows a sharpening section to operate at half of the input rate. In addition, the sharpened CIC filter is of length that is half of that of the original CIC filter. With the aid of the polyphase decomposition, the polyphase subfilters of the first section can also be operated at the half of the input rate.

1. INTRODUCTION

A commonly used decimation filter is the cascaded integrator comb (CIC) filter, consisting of an integrator section and a comb section [1], [2]. The former is composed of a cascade of *K* integrators while the latter is a cascade of *K* comb filters. The transfer function of the resulting decimation filter is given by

$$H(z) = \left[\frac{1}{M} \frac{1 - z^{-M}}{1 - z^{-1}} \right]^K , \qquad (1)$$

where *M* is the decimation ratio, and *K* is the number of stages. The scale factor 1/*M* in Eq. (1) ensures a dc gain of 0 dB. The CIC filter is followed by a down-sampler with a down-sampling factor *M*. The above decimation filter is attractive in many applications because of its very low complexity. The frequency response of the CIC filter is

$$H(e^{j\omega}) = \left(\frac{\sin(\omega M/2)}{M \sin(\omega/2)} e^{-j\omega[(M-1)/2]} \right)^K , \qquad (2)$$

which exhibits a linear-phase, lowpass *sinMx/sinx* characteristics with a droop in the desired frequency range in the passband that is dependent upon the decimation factor *M*. In addition, there are nulls at normalized angular frequencies $\omega = \ell / M$, $\ell = 1, 2, \ldots$ As a result, the signal components aliased into the baseband, which is caused by the down-sampling, appear on both sides of the nulls. The worst-case aliasing occurs in the passband near the first null at 1/*M*.

Several schemes have been proposed to design computationally efficient decimation filter with improved frequency response [3]-[5]. One of them made use of the filter sharpening approach [5]. This technique sharpens the frequency response of a linear-phase finite impulse response (FIR) filter using multiple copies of the same filter [6]. In the simplest case the transfer function of the sharpened filter is given by

$$H_{sh}(z) = 3H^2(z) - 2H^3(z) , \qquad (3)$$

where $H(z)$ denotes the transfer function of the original linear-phase FIR filter. It has been shown that when the original filter has the form of a CIC filter given by Eq. (1), the resulting sharpened filter has a significantly improved frequency response, i.e., the reduced passband droop and improved alias rejection. The main drawback of this structure is that the filtering is performed at the high input rate. In addition, for a given decimation factor *M*, the complexity increases with the increase of the number of stages *K*. This paper presents one modification of the sharpened CIC structure, which overcomes these two problems. In the proposed scheme the filter sharpening is implemented in two stages allowing filtering at a rate lower than the high input rate while reducing the length of the sharpened CIC filter to one-half of that of the original CIC filter. In Section 2 we first examine the property of a sharpened CIC filter and then in Section 3 we describe how this property can be exploited for efficient implementation of a sharpened CIC decimation filter.

2. SHARPENED CIC FILTER

We consider the case where *M*=2*N*. In this case we can rewrite Eq. (1) as

$$H(z) = \left[H_1(z^2) H_2(z) \right]^K , \qquad (4)$$

where

$$H_1(z^2) = \frac{1}{N}\frac{1-z^{-2N}}{1-z^{-2}}, \tag{5}$$

$$H_2(z) = \frac{1}{2}(1+z^{-1}). \tag{6}$$

Figure 1 shows the corresponding z-plane pole-zero plots for $K = 1$ and $N = 8$. As it can be seen from these plots, $H_1(z^2)$ has zeros at the same locations as $H(z)$ except $z = -1$, which is provided by $H_2(z)$. Therefore we can apply the sharpening to $H_1(z^2)$ instead of $H(z)$ and use $H_2(z)$ to achieve the necessary attenuation of the last side lobe. This suggests that the decimation filter can be constructed using different number of stages for two sections as indicated below:

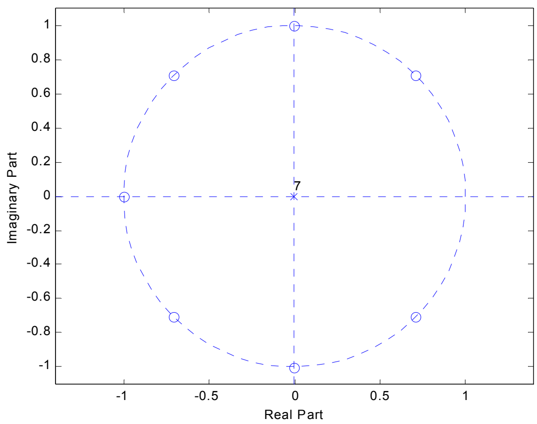
$$H_n(z) = \left[H_1(z^2)\right]^K \left[H_2(z)\right]^L, \tag{7}$$

where $H_1(z^2)$ and $H_2(z)$ are given by Eqs. (5) and (6), respectively.

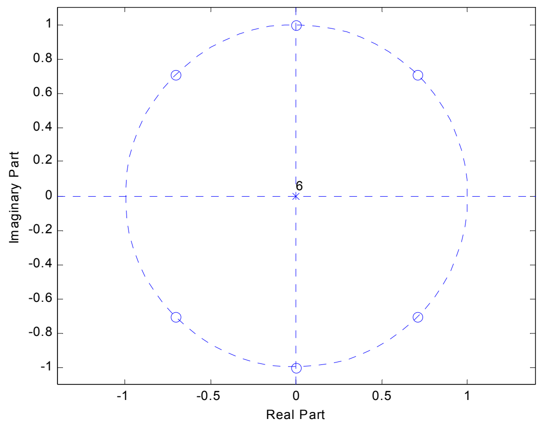
Applying the sharpening to $[H_1(z^2)]^K$ we arrive at the transfer function of the proposed decimation filter as

$$H_{sh,n}(z) = \{[3H_1(z^2)]^{2K} - [2H_1(z^2)]^{3K}\}[H_2(z)]^L. \tag{8}$$

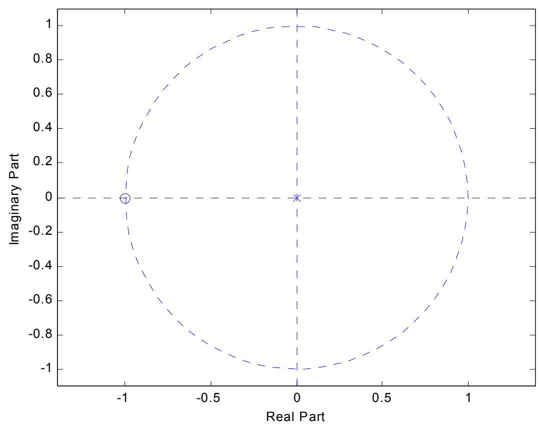
Figure 2 shows the gain response plots of the original sharpened CIC filter obtained using Eq. (3) and of the filter designed using the proposed sharpening scheme using Eq. (8) for several values of the parameters K and L.



(a) $H(z)$.

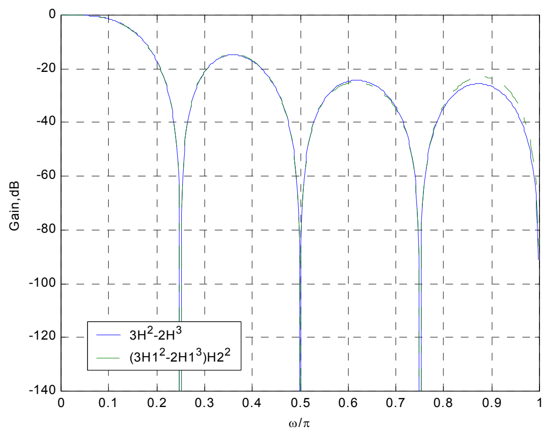


(b) $H_1(z^2)$.

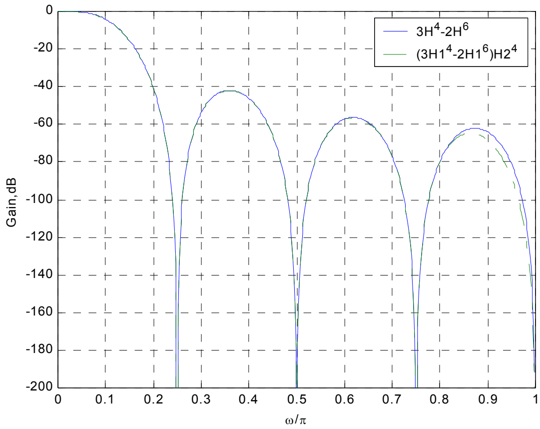


(c) $H_2(z)$.

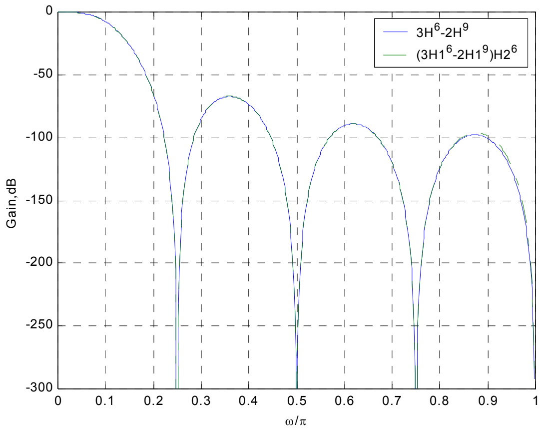
Figure 1: Pole-zero plots.



(a) K=1; L=2.



(b) K=2; L=4.

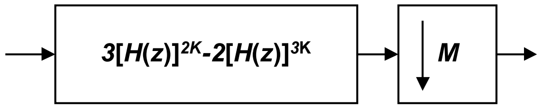


(c) K=3; L=6.

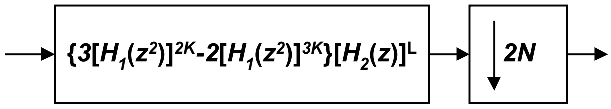
Figure 2: Gain responses of sharpened CIC filters.

3. PROPOSED STRUCTURE

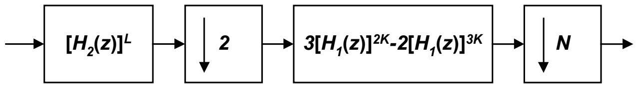
Figure 3(a) shows the sharpened CIC filter structure of Kwentus et. al. [5]. The structure proposed in this paper is shown in Figure 3 (b). A computationally efficient realization of the proposed structure obtained using the cascade equivalence [2] is shown in Figure 3 (c). Note that the sharpening in the new structure operates at half of the input rate and that the sharpened filter $H_1(z)$ has half the number of the coefficients of the original CIC filter $H(z)$.



(a)



(b)



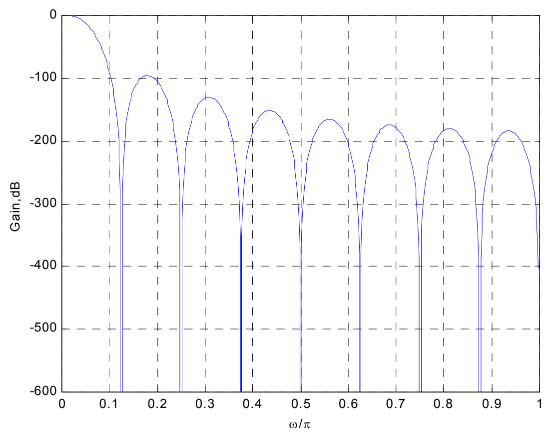
(c)

Figure 3: CIC structures.

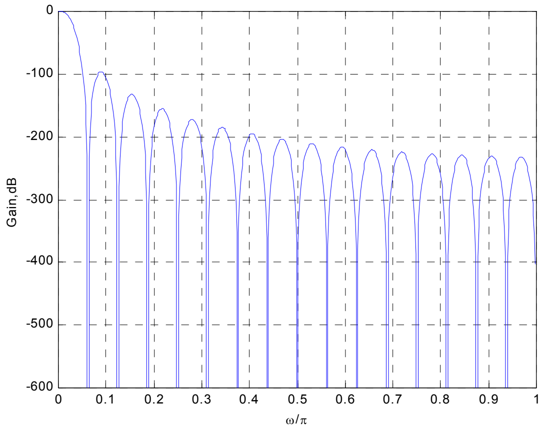
The first section consists of the cascade of L first order moving average (MA) filters. Using the polyphase decomposition the first section can also be moved to a lower rate and implemented efficiently by using dedicated shift-and-add multipliers and eliminating redundant operations using the data broadcast structure [7]- [9].

Example 1

The gain responses of the proposed CIC filter for M=16, K=4 and L=8 and for M=32, K=4 and L=8 are shown in Figures 4 (a) and 4 (b). Figures 5 (a) and 5 (b) show the same plots in the frequency range (0, 0.3) and dB scale (-200,0).

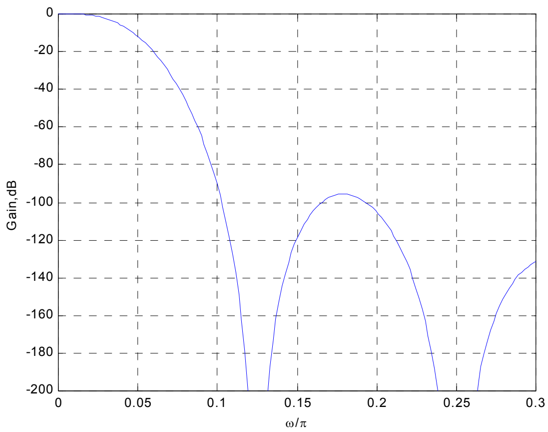


(a) M=16, K=4, L=8.

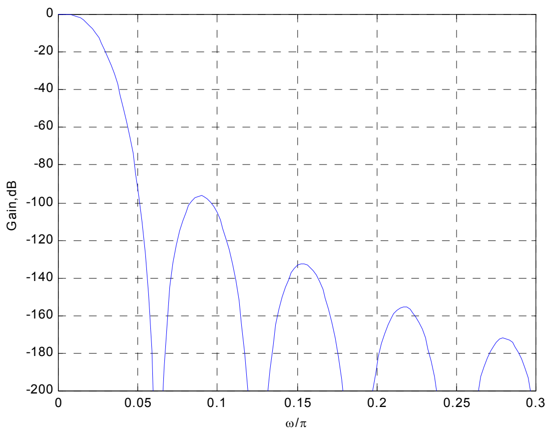


(b) M=32, K=4, L=8.

Figure 4: Gain responses of the proposed filter.



(a) M=16, K=4, L=8.



(b) M=32, K=4, L=8.

Figure 5: Gain responses of the proposed filter in the range (0, 0.3).

CONCLUDING REMARKS

A new computationally efficient structure for a sharpened CIC decimation filter is proposed for an even decimation factor. The structure consists of two main sections: a cascade of the first-order moving average filters, and a sharpening filter. The sharpening is moved to lower section, which operates at the half of the high input rate and has only one half of the original sharpened CIC filter length.

With the aid of the polyphase decomposition, the polyphase subfilters of the first section can also be operated at the half of the high input rate.

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