



# IMPROVING FREQUENCY RESOLUTION FOR CORRELATION-BASED SPECTRAL ESTIMATION METHODS USING SUBBAND DECOMPOSITION

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## ABSTRACT

Subband decomposition has already been shown to increase the performance of spectral estimators but induced frequency overlapping may be troublesome, bringing edge effects at subband borders. A recent paper proposed a method (SDFW) allowing to perform subband decomposition without aliasing. In the present paper, this subband decomposition is modified in order to improve frequency resolution for any correlation-based spectral estimator when applied to the subband outputs. Three main improvements are proposed: subband decomposition is based on comb filters, the warping operation suggested in the SDFW method is performed using a complex frequency modulation and the autocorrelation is estimated using all sub-series from each subband. Simulation results demonstrate the anticipated performance of the proposed method.

## 1. INTRODUCTION

Some authors recently showed that subband decomposition can be a powerful tool for spectral estimation [1], [2]. Actually, performances of traditional spectral estimation methods can increase when applied to signals filtered by an appropriate filterbank rather than applied to the corresponding fullband signal. In case of sinusoids embedded in noise, this has been theoretically explained by the following properties:

- ( $P_1$ ) local SNR increases by the decimation ratio (due to filtering operation) [2].
- ( $P_2$ ) frequency spacing increases by the decimation ratio (due to decimation) [2].
- ( $P_3$ ) the condition number of autocorrelation matrices decreases due to local order reduction [3].

In practical applications, properties  $P_1$  and  $P_2$  have to take into account that decimation also brings a decrease

of the sample number. Moreover, these properties have been established in the case of a bank of ideal infinitely sharp bandpass filters. However, experimental results show that they are still valid when using non-ideal filterbanks such as modified Quadrature-Mirror Filters (QMF's) [1] or cosine modulated filterbank [2]. Property  $P_3$  is one of the most noticeable contributions of subband decomposition and can be explained by the eigenvalues interleaving theorem (see [3]). The problem of *spectral overlapping* has already been addressed in a recent paper [4] where a (non real-time) procedure was proposed to perform spectral estimation from subbands without aliasing even at subband's borders. The main contribution of the present paper is to propose a procedure based on [4] able to improve the performances of any correlation-based spectral estimation method. Compared to [4], three main improvements are proposed in order to simplify the decomposition method and to increase the frequency resolution. A comparison of the proposed method performances and of the performances of a "classical" fullband spectral estimation is given, in terms of frequency estimation variance and compared to the Cramer-Rao lower bound. The paper is organized as follows. Section 2 presents problem formulation and section 3 is devoted to the proposed method. Simulation results are presented in section 4 and conclusions are reported in section 5.

## 2. PROBLEM FORMULATION

The observed signal is the sum of  $K$  sinusoids corrupted by additive white Gaussian noise:

$$u_n = \sum_{l=1}^K A_l e^{i2\pi f_l n + \phi_l} + b_n, \quad (1)$$

where  $n = 1, \dots, N$ . The problem of estimating the frequencies  $f_l$  from the observed samples  $u_n, n = 1, \dots, N$

has received considerable attention in the signal processing literature (see for instance [5] and references therein). As a consequence, many algorithms have been studied to solve this spectral estimation problem. As a matter of example, some algorithms are based on linear least squares (LS), High-Order Yule-Walker, Pisarenko and MUSIC methods [5]. Algorithms based on the Singular Value Decomposition (SVD) of the autocorrelation matrix have become very popular because of their high resolution properties and their insensitivity to model order overestimation. In this paper, without any loss of generality, we choose to use one of the correlation-based frequency estimation approach: the high-order Yule-Walker (HOYW) frequency estimation method ([5], p. 151). This method is summarized below:

- estimate the  $N \times N$  autocorrelation matrix of  $u_n$  denoted  $\hat{R}_u$ ,
- compute the SVD of  $\hat{R}_u$ ,
- solve the rank-truncated HOYW system of equations in the LS sense, which yields the estimated AR parameter vector denoted  $\hat{a}$ ,
- determine the peaks of the pseudospectrum,

$$S(e^{i2\pi f}) = \frac{1}{|A(e^{i2\pi f})|^2},$$

where  $A(z)$  is the estimated AR polynomial.

This method is chosen for its good frequency resolution properties.

### 3. IMPROVING SPECTRAL ESTIMATION USING SUBBAND DECOMPOSITION

Let us divide the frequency interval  $[0, 1]$  into  $2M$  equal subbands and denote  $\nu_j = (j - 1)\frac{0.5}{M} + \frac{0.25}{M}$ ,  $j = 1, \dots, 2M$  the middle of the  $j^{th}$  subband. Aliasing occurs due to decimation of the signals by the filterbank. Considering the  $j^{th}$  branch of the bank represented in figure 1, the Fourier Transform of the decimated signal versus the input of the filter bank  $u_\Delta(n)$  can be written as (do not pay attention to the modulation operation of this figure, it will be explained later):

$$X_j(e^{i2\pi Mf}) = \frac{1}{M} \sum_{l=0}^{M-1} H_j(e^{i2\pi(f - \frac{l}{M})}) U_\Delta(e^{i2\pi(f - \frac{l}{M})}). \quad (2)$$

This expression highlights spectral overlapping given by the terms for  $l = 1, \dots, M - 1$ . Obviously, ideal infinitely sharp bandpass filters bring no aliasing. In order to cancel aliasing in practical conditions, we propose to use a bank of comb filters. The  $j^{th}$  comb filter

(whose order is taken equal to the decimation ratio  $M$ ) has a frequency response equal to 1 in the middle of the  $j^{th}$  subband and 0 in the middle of others. This allows aliasing cancellation not in the whole subband but only in its middle  $f = \nu_j$  so that:

$$X_j(e^{i2\pi M\nu_j}) = \frac{1}{M} U_\Delta(e^{i2\pi\nu_j}). \quad (3)$$

Comb filters are usual in the field of subband decomposition (see for example [6], p.227). However, the proposed way of using them in this paper is somehow original. As aliasing is cancelled on only one frequency bin, a frequency modulation device is set prior to the filterbank (see figure 1). Hence, any frequency under interest  $f$  is shifted in the middle of its corresponding subband  $\nu_j$  such that:

$$U(e^{i2\pi f}) = U_\Delta(e^{i2\pi\nu_j}) \text{ with } \Delta_f = \nu_j - f. \quad (4)$$

Thus, the use of comb filters and of appropriate modulation operation leads to get rid of the main drawback of subband decompostion, i.e. frequency aliasing. Therefore, any spectral estimation method can be applied on the outputs of each subband. In order to improve frequency resolution, we propose to use a correlation-based method (HOYW for example) and to pay a particular attention on the correlation estimation. The whole proposed method is summarized as follows:

for  $f = 0$  to 1

1 - subband selection: find subband number  $j$  knowing  $f$ .

2 - complex modulation: shift the input signal's spectrum by the quantity  $\Delta_f = \nu_j - f$  using:

$$u_\Delta(n) = e^{i2\pi\Delta_f n} u(n). \quad (5)$$

3 - Subband decomposition: use the  $j^{th}$  filter of the comb filterbank on the modulated signal  $u_\Delta$  to get  $M$  filtered and decimated sub-series  $x_{j_m}$ .

4 - Estimate the autocorrelation sequence  $\hat{R}_{x_{j_m}}(k)$  on each sub-series and the autocorrelation of the output of the  $j^{th}$  branch as:

$$\hat{R}_{x_j}(k) = \frac{1}{M} \sum_{m=0}^{M-1} \hat{R}_{x_{j_m}}(k), \quad (6)$$

5 - Spectral estimation : apply HOYW method using  $\hat{R}_{x_j}(k)$  to estimate the amplitude of the spectrum at only one frequency  $\nu_j$ .  $U(e^{i2\pi f})$  is derived using (3) and (4).

end for.

Obviously, this algorithm can be written in a parallel

way, allowing the spectrum estimation on  $M$  simultaneous frequencies, using a spectral estimation procedure on each branch of the filterbank. Moreover, it is important to note that the use of the different sub-series of the decimated signal in each branch is the key point in order to improve results when using subbands rather than a classical fullband spectral estimation. Indeed, as mentioned in the introduction, property  $P_1$  ensures that the filtering operation will increase  $SNR$  but the use of  $M$  times less samples for autocorrelation estimation would cancel this gain. Property  $P_2$  shows that the frequency spacing will increase by the decimation ratio but at the same time, the number of samples decreases by the same factor. Thus, from a Fourier resolution point of view, there would be no improvement.

#### 4. SIMULATION RESULTS AND PERFORMANCES OF THE METHOD

Simulations are done in the case of a sum of 2 complex sinusoids:

$$u_n = A_1 e^{i2\pi f_1 n + \phi_1} + A_2 e^{i2\pi f_2 n + \phi_2} + b_n, n = [0, \dots, N-1] \quad (7)$$

$b_n$  is a Gaussian zero-mean white noise whose power is  $\sigma_b^2$ . The signal to noise ratio is  $SNR = \frac{A_1^2 + A_2^2}{\sigma_b^2}$ . An uniform modulated filterbank is used, in which all filters are derived from only one comb filter of order  $M$ :

$$h_0(n) = 1, \forall n = 0, \dots, M-1 \text{ and } 0 \text{ elsewhere,} \quad (8)$$

$$\Leftrightarrow H_0(e^{i2\pi f}) = \frac{1}{M} e^{-i\pi f(M-1)} \frac{\sin \pi f M}{\sin \pi f}, \quad (9)$$

The filterbank frequency response is plotted on fig. 2. First, performances of the proposed method are compared to a fullband classical spectral estimation, in terms of frequency estimation variance. The Cramer-Rao lower bound is also plotted (derived in [7], p.414 for a sum of  $p$  sinusoids). Figure 3 presents these results for both frequencies  $f_1$  and  $f_2$ . These simulations have been run for  $A_1 = A_2 = 1$ ,  $f_1 = 0.15$ ,  $f_2 = 0.35$  and  $N = 1000$ . The autocorrelation estimation method was chosen unbiased. The variance of the estimator of parameters  $f_1$  and  $f_2$  is estimated using 50 Monte-Carlo runs for each SNR. Moreover, it is important to note that for fair comparison, the ratio "model order" / "number of samples" is kept constant. As a consequence, the model order is  $p = 750$  for fullband simulations and  $p_{sub} = 24$  for subband modelling with  $M = 32$  subbands so that  $p \simeq M p_{sub}$ . Figure 3 highlights the improvement brought by the HOYW method applied to subband signal (following the proposed procedure) rather than to the fullband signal. In order to clearly show this improvement, fig. 4 presents

50 realizations of the model spectrum around frequencies  $f_1$  and  $f_2$  using subband decomposition or not at  $SNR = -17$  dB (subband spectrum has been moved forward for better readability). Spurious peaks appearing on fullband spectral estimation disappeared using subband decomposition.

Dealing with numerical accuracy (property  $P_3$ ), simulations were run at  $SNR = 0dB$  on 50 realizations using the above parameters in order to compare condition number of autocorrelation matrices. For fullband spectral estimation, there is only one autocorrelation matrix  $\forall f \in [0, 1]$ . Using the subband method, to each frequency  $f$  corresponds a different subband signal, so that condition number is varying versus the frequency. In fig. 5, condition numbers are plotted in logarithmic scale. This figure shows that property  $P_3$  is verified experimentally. A gain varying from 30 to 70 dB is observed, depending on the frequency  $f$ .

#### 5. CONCLUSION

The aim of this paper was to propose a spectral estimation method based on subband decomposition and leading to better performances than spectral estimation applied on the fullband signal. Subband decomposition induces frequency aliasing (using non-ideal filters) which is a main drawback for spectral estimation. A previous paper [4] has already presented a subband decomposition method allowing to cancel aliasing. In this paper, the previously proposed method is simplified by the mutual use of comb filters and frequency modulation. Moreover, when using correlation-based spectral estimation methods, it is pointed out that the autocorrelation estimation in each subband has to be performed using all information, in order to reach better performances than fullband frequency estimation. Simulations have been given, showing the improvement brought by subband spectral estimation over the classical fullband one.

#### 6. REFERENCES

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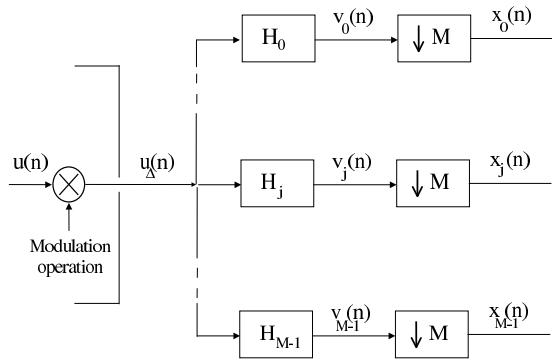


Figure 1: Filterbank and modulation operation.

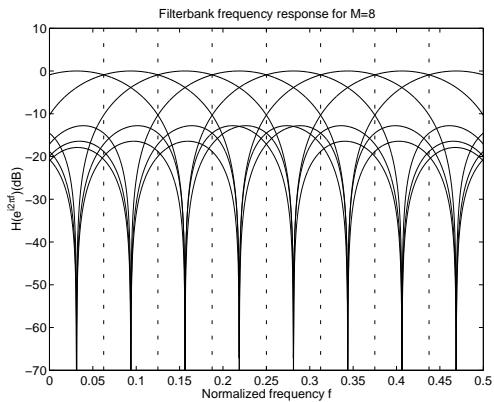
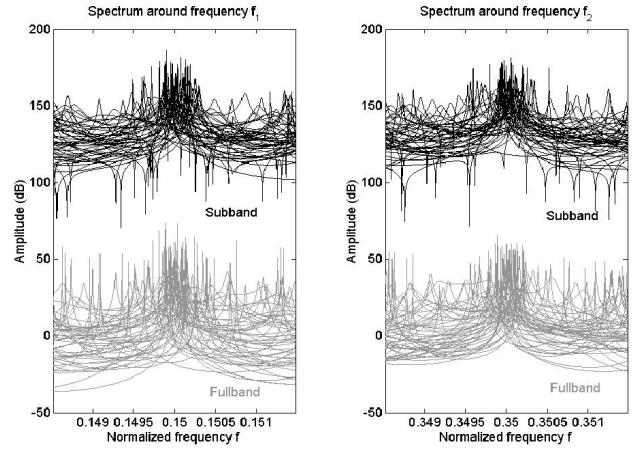


Figure 2: Frequency response of comb filters for  $M = 8$ .

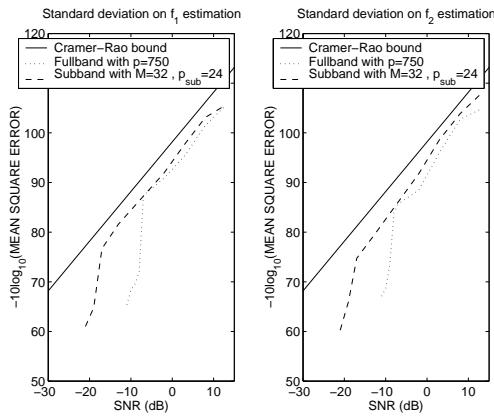


Figure 3: Comparison of subband versus fullband spectral estimation.

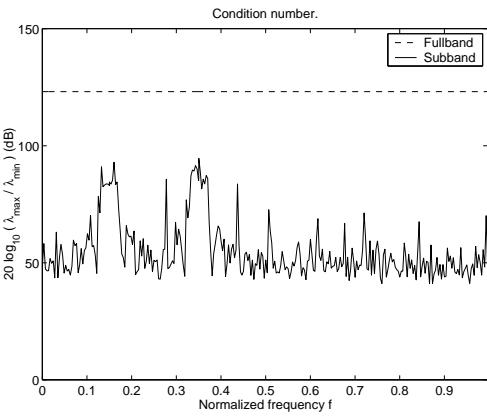


Figure 5: Subband and fullband condition numbers.