RECURSIVE ESTIMATION OF WAVELET BASED TIME VARYING AR MODELS OF SPEECH

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Abstract

Estimation of time varying systems can be done by representing the whole system behaviour in terms of some fixed basis sequences. Available solutions use computationally expensive nonrecursive solutions. In this work we derived an RLS solution for the wavelet based expansion of time varying systems. This method has been applied to speech signals and its performance is studied. A recursive lattice algorithm is derived for the above modeling. Also the performance of the method using both the Haar and Daubechies bases are studied.

1. INTRODUCTION

Time varying system identification has been receiving considerable attention in the recent past in the literature of signal processing. Time Varying systems arise in various applications like speech processing, communication systems,seismic analysis. However conventional methods for system identification make use of stationarity over a short frame of time as assumption. However most of the signals we encountered in practice are nonstationary in nature, which are well described by using a time-varying model.

In the past, different algorithms have been proposed for time varying system identification based upon adaptive algorithm,basis expansion [1]and kalman filter[3]. In this work the nonstationary signal is modeled as a time-varying autoregressive process and the time-varying autoregressive coefficients are modeled as a linear combination of the wavelet bases by parametric expansion. The wavelet bases are obtained from the iterated filter bank using the multiresolution analysis. Then the estimation of the time-varying coefficients becomes the estimation of the unknown coefficients in the expansion. By selecting the model with only a few bases leads to a parsimonious representation of the system.

2. WAVELET BASED AR MODELING

Consider a non-stationary signal $y(n)$ whose AR model of an order p is given by,

$$y(n) = \sum_{k=1}^{p} a_k(n) y(n-k) \tag{1}$$

In the above, let us model each time-varying autoregressive coefficient $a_k(n)$ as a linear combination of some basis sequences $f_l(n)$ for($l = 1 \cdots L$, where L is the total number of basis functions), Then

$$a_k(n) = \sum_{l=1}^{L} \xi_{kl} f_l(n) \tag{2}$$

$$\text{where, } k = 1 \cdots p$$

Now, the identification of $a_k(n)$ reduces to the identification of time-invariant coefficients ξ_{kl}. The wavelet basis functions $f_l\,(n)$ are derived by iterating low pass $H_0\,(z)$ and high pass filters $H_1\,(z)$ using the following relationships.

$$H_0^{j_{max}}(z) = H_0(z) H_0(z^2) \cdots H_0(z^{2^{j_{max}-1}})$$
$$H_1^{j}(z) = H_0(z) H_0(z^2) \cdots H_1(z^{2^{j-1}}), j = 1 \cdots j_{max}$$

The wavelet decompsition of $a_k\,(n)$ using the multiresolution approach is as shown in Fig.1. Now, each time varying coefficient $a_k(n)$ can be expanded as

$$a_k(n) = \sum_{m=1}^{\frac{N}{2^{j_{max}}}} \zeta^k_{j_{max},m} \tilde{h}_0^{j_{max}}(n - 2^{j_{max}} m) + \sum_{j=1}^{j_{max}} \sum_{m=1}^{\frac{N}{2^j}} \xi^k_{j,m} \tilde{h}_1^j (n - 2^j m) \tag{3}$$

In the above equation the first term represents the low resolution decomposition of $a_k\,(n)$ and the second term represents detailed decomposition of $a_k\,(n)$. Arranging (1) and (3) in the matrix form, it can be shown that

$$\underline{y} = H \underline{\xi} + \underline{e} \tag{4}$$

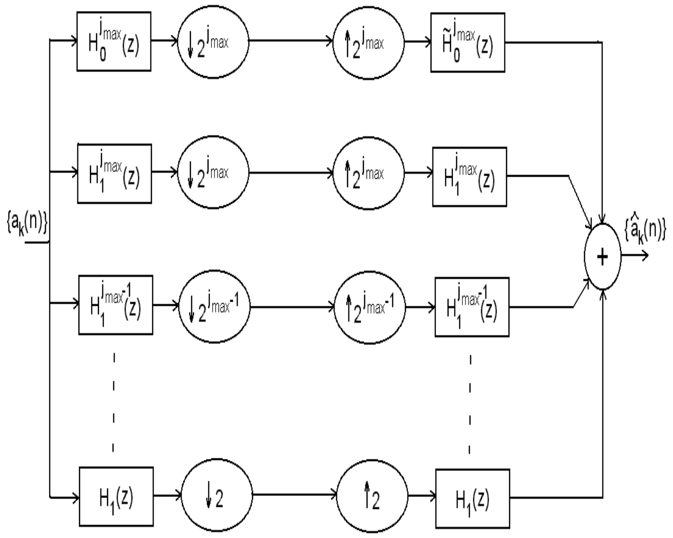

Fig. 1. Wavelet decomposition and reconstruction down to depth j

where

$$\begin{aligned}
\underline{y} &= [y(0)\cdots y(n-1)]^T \\
\underline{e} &= [e(0)\cdots e(N-1)]^T \\
\underline{\xi} &= [\underline{\xi}^1 \cdots \underline{\xi}^p]^T \\
H &= [H_{0c}^{j_{max}} H_{1c}^{j_{max}} \cdots H_{1c}^1] \\
H_{0c}^{j_{max}} &= [\tilde{h}_0^{j_{max}}(n)\tilde{h}_0^{j_{max}}(n-2^{j_{max}})\cdots \\
&\cdots \tilde{h}_0^{j_{max}}(n-2^{j_{max}}\frac{N}{2^{j_{max}}})] \\
H_{1c}^j &= [\tilde{h}_1^j(n)\tilde{h}_1^j(n-2^j)\cdots\tilde{h}_1^j(n-2^j\frac{N}{2^j})]
\end{aligned}$$

Here $\underline{\xi}^k$ is the sequence of all the wavelet coefficients representing the k^{th} parameter.

Equation(4) can be simplified by discarding the lower level coefficients. That is, $\xi_{j,m}^k = 0$ for $0 < j \leq j_{min}$ by removing the corresponding columns from(4). This reduces the size of the parameter set as well as the underdeterminancy provided $j_{min} > \log_2 p$.

Discarding the lower level coefficients yields,

$$\underline{y} = H_\mu \underline{\xi}_\mu + \underline{e} \tag{5}$$

The least square solution of (7) is given by

$$\hat{\underline{\xi}}_\mu = [H_\mu^T H_\mu]^{-1} H_\mu^T \underline{y} \tag{6}$$

However to overcome the computational complexity associated with LS solution we employed a recursive estimation for the estimation of $\hat{\underline{\xi}}_\mu$ in (6) whose proof is given in [4].

3. A WAVELET BASED LATTICE ALGORITHM

Lattice structures are found to be very useful in many signal processing applications. In lattice method the prediction parameters are found out in terms of reflection coefficients by minimizing the forward and the backward prediction error. A time-varying lattice structure is as shown in Fig. 2.

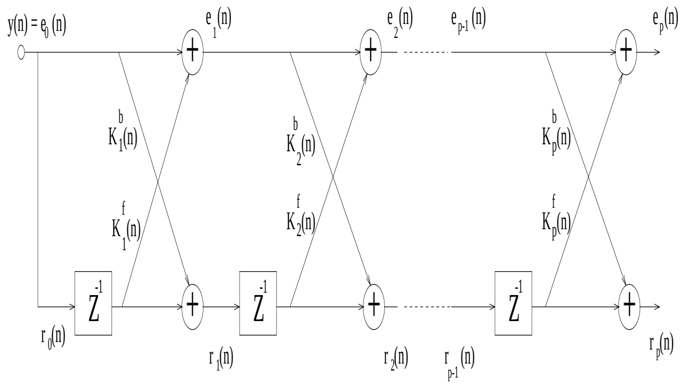

Fig. 2. *Time-varying lattice structure*

The recursions for the forward and backward errors are given by the following relations

$$e_i(n) = e_{i-1}(n) - K_i^f(n)\, r_{i-1}(n-1) \tag{7}$$

$$r_i(n) = r_{i-1}(n-1) - K_i^b(n)\, e_{i-1}(n) \tag{8}$$

In the above recursions $K_i^f(n)$ and $K_i^b(n)$ are the forward and backward reflection coefficients.

3.1. Wavelet Based Modeling

From equation(7) and (8) we have

$$\begin{aligned}
e_i(n) &= e_{i-1}(n) - K_i^f(n)\, r_{i-1}(n-1) \\
r_i(n) &= r_{i-1}(n-1) - K_i^b(n)\, e_{i-1}(n)
\end{aligned}$$

By rearranging the above relations we have

$$e_{i-1}(n) = K_i^f(n)\, r_{i-1}(n-1) + e_i(n) \tag{9}$$

$$r_{i-1}(n-1) = K_i^b(n)\, e_{i-1}(n) + r_i(n) \tag{10}$$

In the process of identification of $K_i^f(n)$ and $K_i^b(n)$,each time-varying reflection coefficient $K_i^f(n)$ and $K_i^b(n)$ are modeled as a linear combination of wavelet basis sequences as we did in section 2.

From an order p lattice structure we have the forward and backward prediction errors $e_i(n)$ and $r_i(n)$ which are related to the time-varying forward and backward autoregressive parameters by the following relationships.

$$e_i\left(n\right) = y\left(n\right) - a_1^i\left(n\right)y\left(n-1\right) - \cdots - a_i^i\left(n\right)y\left(n-i\right) \quad (11)$$

$$r_i\left(n\right) = y\left(n-i\right) - b_1^i\left(n\right)y\left(n\right) - \cdots - b_i^i\left(n\right)y\left(n-\left(i-1\right)\right) \quad (12)$$

Substituting the recursive relationships for $e_i\left(n\right)$ and $r_i\left(n\right)$ given in (7) and (8) it can be shown that,

$$\begin{aligned} & a_1^i\left(n\right)y\left(n-1\right) + \cdots + a_i^i\left(n\right)y\left(n-i\right) = \\ & \quad a_1^{i-1}\left(n\right)y\left(n-1\right) + \cdots + a_{i-1}^{i-1}\left(n\right)y\left(n-\left(i-1\right)\right) + \\ & \quad K_i^f\left(n\right)\left\{y\left(n-i\right) - b_1^{i-1}\left(n-1\right)y\left(n-1\right) - \cdots \right. \\ & \quad \left. - b_{i-1}^{i-1}\left(n-1\right)y\left(n-\left(i-1\right)\right)\right\} \end{aligned} \quad (13)$$

$$\begin{aligned} & b_1^i\left(n\right)y\left(n\right) + \cdots + b_i^i\left(n\right)y\left(n-\left(i-1\right)\right) = \\ & \quad b_1^{i-1}\left(n-1\right)y\left(n-1\right) + \cdots + b_{i-1}^{i-1}\left(n\right)y\left(n-\left(i-1\right)\right) \\ & \quad + K_i^b\left(n\right)\left\{y\left(n\right) - a_1^{i-1}\left(n\right)y\left(n-1\right) - \cdots \right. \\ & \quad \left. - a_{i-1}^{i-1}\left(n\right)y\left(n-\left(i-1\right)\right)\right\} \end{aligned} \quad (14)$$

Comparing the constituents of (13) and (14) on both the sides, the recursions can be given as,

$$a_j^i\left(n\right) = a_j^{i-1}\left(n\right) - K_i^f\left(n\right)b_j^{i-1}\left(n-1\right) \qquad j \in [1, i-1] \quad (15)$$

$$a_i^i\left(n\right) = K_i^f\left(n\right) \quad (16)$$

$$b_1^i\left(n\right) = K_i^b\left(n\right) \quad (17)$$

$$b_j^i\left(n\right) = b_{j-1}^{i-1}\left(n-1\right) - K_i^b\left(n\right)a_{j-1}^{i-1}\left(n\right) \qquad j \in [2, i] \quad (18)$$

4. PERFORMANCE STUDIES

In the initial part of the present work, the above algorithm is tested for synthetic speech, which is generated by driving a time-varying all pole model of the following transfer function with white gaussian noise of variance $\sigma^2 = 1$.

$$H\left(z\right) = \frac{1}{1 - 2r\cos\left(\frac{\pi n}{50}\right)z^{-1} + r^2 z^{-2}} \quad (19)$$

The above system has a pair of complex conjugate poles located at $z = re^{\pm j\left(\frac{\pi n}{50}\right)}$. From the above transfer function given in equation(19), we have two AR coefficients with the first one varying sinusoidally with a magnitude $2r\cos\left(\frac{\pi n}{50}\right)$ and the other is kept constant at $-r^2$.

Figure 3 to 5 shows the performance of this method. These plots consist the original signal along with the predicted one and it's corresponding prediction error.

Figure 6 and 7 shows the true and estimated time-varying autoregressive parameters of the above model. As mentioned earlier the first parameter was varying in a sinusoidal fashion and the next parameter is held at some constant value.

5. CONCLUSIONS

A recursive lattice algorithm is derived based on the wavelet expansion of the time-varying autoregressive parameters. This recursive estimation overcomes the computational complexity of the least squares by the use of matrix inversion lemma which makes it attractive for the on-line implementation.

The simulation results shows the effectivness of the present method in case of estimating the time-varying parameters. However the performance will vary depending upon the type of the wavelet employed. Generally short length wavelets are more suitable than long wavelets for estimating rapidly varying parameters but yield estimates which are very rough due to small vanishing moments (ex: Haar, Daub4). With long length wavelets the results are smooth but they cant handle rapid variations of the time varying parameters (ex: Daub20). If the system parameters changes abruptly then Haar wavelet is the best.

6. REFERENCES

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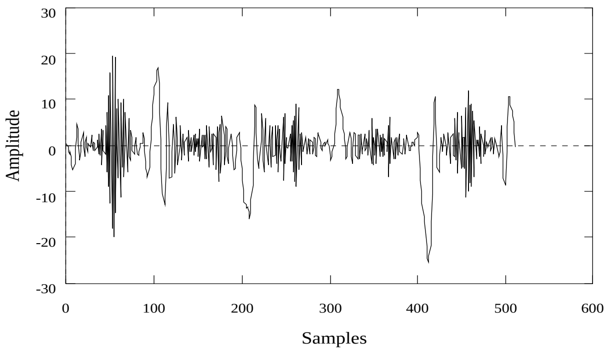

Fig. 3. *The synthetic speech*

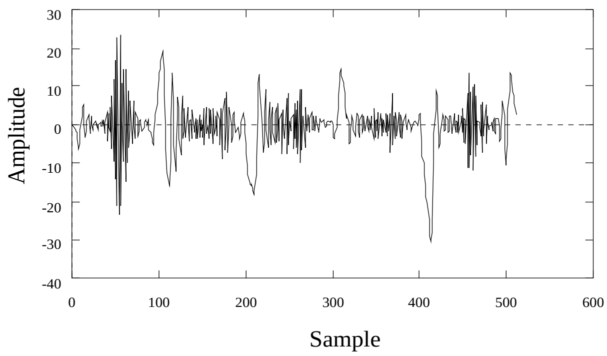

Fig. 4. *The forward predicted speech*

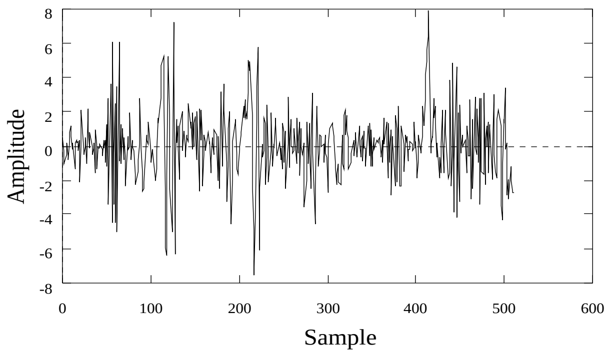

Fig. 5. *The forward prediction error*

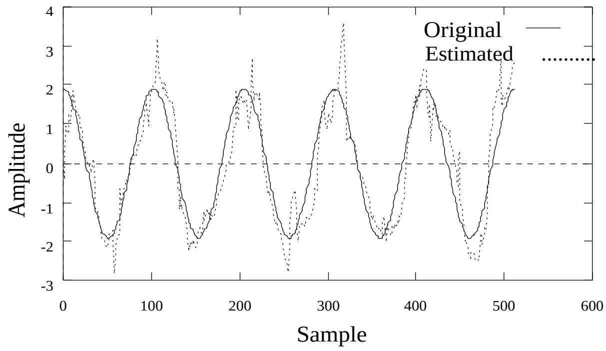

Fig. 6. *True and estimated coefficient* $a_1(n)$ *using Daub4 wavelet*

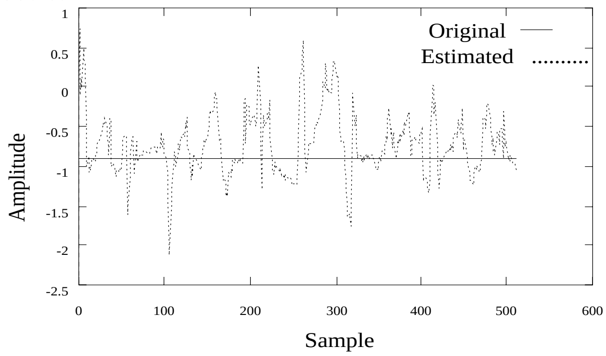

Fig. 7. *True and estimated coefficient* $a_2(n)$ *using Daub4 wavelet*