BLIND ESTIMATION OF BAND LIMITED CHANNELS: A LOW COMPLEXITY APPROACH

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ABSTRACT

We consider the problem of blind estimation of a band-limited channel excited by a cyclostationary input. We propose a channel estimation method that, like several existing ones, exploits the relationship between the channel frequency response and the cyclic spectrum. However, the novelty here is an approximation made for the discretized phase of the cyclic spectrum, which, under certain conditions, results in a significant simplification of the aforementioned relationship. The result is a channel estimation method with complexity equal to that of an IDFT. The proposed approach is applied to simulated data, and real recordings obtained at our wireless communications testbed, and is compared to existing methods.

1. INTRODUCTION

Blind channel estimation and equalization plays a very important role in high-speed communications, where the use of a training sequence is considered to be a waste of bandwidth. In general, estimation of an LTI system excited by an unknown stationary non-Gaussian input, based on the system output, is possible using higher-order statistics (HOS) of the output. However, when the input is cyclostationary, which is the case of communications systems, system estimation can be carried out using second-order cyclic statistics of the output [1, 3]. Pioneering work done in [3, 6] followed by [7] and [5] has led to several approaches that exploit second-order statistics for channel estimation. Basically these fall into two categories: time domain and frequency domain, with reference to the transform domain in which the estimate is obtained.

In this paper we consider estimation of the channel in frequency domain, with a prior knowledge that the channel is bandlimited. The proposed method will always have an error, however it will be negligible if the channel has negligible energy within the stopband.

2. PROBLEM FORMULATION

Let us consider the baseband representation of a digital data communication system. Assuming that the channel is LTI and BIBO stable, the received signal is given by:

$$x(t) = \sum_{m=-\infty}^{\infty} s_m h(t-mT) + w(t) \quad (1)$$

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where s_m is the transmitted signal, which is assumed to be zero-mean i.i.d. with unit variance; $h(t)$ is the combined response of the shaping filter, channel and receiver filter; T is the symbol interval, and $w(t)$ is stationary, zero-mean, white noise with variance N_o, independent of s_m. The signal $x(t)$ is cyclostationary with cyclic period T [1]. Sampling $x(t)$ with sampling interval $\Delta = \frac{T}{p},\ p \in \mathbb{Z}$, yields:

$$x(n) = \sum_{m=-\infty}^{\infty} s_m h(n-mp) + w(n), n \in \mathbb{Z} \quad (2)$$

where $h(n) = h(n\Delta),\ w(n) = w(n\Delta)$. The discrete time signal $x(n)$ is a cyclostationary process with cyclic period p. Cyclostationarity implies that the autocorrelation function is periodic with period p. The Fourier series coefficients of the autocorrelation are referred to as cyclic autocorrelation, $R_x^{k\alpha}(m)$, and the Fourier Transform of those as the cyclic spectrum, $S_x^{k\alpha}(e^{j\omega})$ [1], i.e.,

$$R_x^{k\alpha}(m) = \sum_{n=0}^{p-1} R_x(n+m,n) e^{-j2\pi k\alpha n} \quad (3)$$

$$S_x^{k\alpha}(e^{j\omega}) = \sum_{m=-\infty}^{\infty} R_x^{k\alpha}(m) e^{-jm\omega} \quad (4)$$

where $\alpha = \frac{1}{p},\ n,m,k \in \mathbb{Z}$ and $R_x(m,n)$ denotes autocorrelation of $x(n)$. It holds that:

$$S_x^{k\alpha}(j\omega) = H(e^{j\omega}) H^*(e^{j(\omega - 2\pi k\alpha)}) + pN_o\delta(k), k \in \mathbb{Z} \quad (5)$$

This is a key relationship that can lead to the estimation of the channel frequency response $H(e^{j\omega})$. The magnitude response of the channel can be easily recovered by evaluating (5) for $k = 0$. Evaluating (5) for $k = 1$ and considering phases, we get:

$$\psi(\omega) = \phi(\omega + \pi\alpha) - \phi(\omega - \pi\alpha) \quad (6)$$

where $\psi(\omega),\ \phi(\omega)$ are the phases of $S_x^{k\alpha}(\omega)$ and $H(e^{j(\omega)})$, respectively. Several methods have been proposed to employ (6) in obtaining $\phi(\omega)$. In [2], it was shown that (6) cannot recover the phase response of an arbitrary channel, since when evaluated for different ω's it always leads to an underdetermined system of equations. According to [6], discrete channels with zeros uniformly $\frac{2\pi}{T}$ spaced on a circle are not identifiable. Parametric approaches for obtaining $\phi(\omega)$ from (6) have been proposed in [4] as well as approaches for identifiable channels, that exploit the periodicity of $\phi(\omega)$. In [9], it was shown that an appropriately discretized version of (6), i.e., $\omega = \frac{2\pi}{N}k,\ k = 0,\ldots,N-1$, with N and p co-prime numbers, can lead to the phase within a scalar ambiguity.

3. PROPOSED METHOD

For bandlimited channels, the estimation of phase for the full frequency range is unnecessary because within the stopband the amplitude is very small. An estimate of phase of such a channel for the whole frequency interval $[0, 2\pi)$ would be highly error prone.

Let us consider a channel that has no zeros uniformly spaced on a circle at angle $2\pi/p$ where p is the over sampling ratio, and which is bandlimited with cutoff frequency ω_c.

Let us discretize $\psi(\omega)$, i.e., $\omega = (2\pi/N)k,\ k = 0, ..., N-1$, and select N and p so that whenever $\phi(\omega)$ falls in the passband, $\phi(\omega + \pi\alpha)$ falls in the stopband. This can be achieved as long as,

$$L_c < m < N/2 - L_c \tag{7}$$

where L_c and m are the closest integers to $\frac{2\pi}{\omega_c}N$ and $\frac{N}{2p}$ respectively. Closer inspection reveals that in order to find a valid value for p, we need $N > 4L_c$, i.e. the channel cutoff should be less than $\pi/2$ rad/cycle.

For the above values of N and p, let us define:

$$\widehat{\phi(k)} = \begin{cases} -\psi\widehat{(k+m)}; 0 \leq k \leq L_c - 1,\ \ N - L_c \leq k \leq N-1 \\ 0;\ \ L_c \leq k \leq N - L_c - 1 \end{cases} \tag{8}$$

It can be shown (see Appendix A) that the channel impulse response constructed based on $\hat{\phi}(k)$ and the correct frequency response magnitude is related to the true channel impulse response as follows:

$$\begin{aligned} &\widehat{h(n)} \\ &\stackrel{\triangle}{=}\ IDFT\{|H(k)|e^{j\widehat{\phi(k)}}\} \\ &=\ ah(n+b) \otimes \{-2\frac{N}{\Delta}\frac{e^{j\frac{2\pi}{N}k_o n}}{(1-e^{j\frac{2\pi}{N}n})}\sum_{i=0}^{\Delta}\delta(n-(2i+1)\frac{N}{2\Delta})\} \\ &\quad + E(n) \end{aligned} \tag{9}$$

where

$$a = e^{-jN_z(N/2-m)},\ \ b = \frac{N_z}{2},\ \ k_o = 2m - L_c \tag{10}$$
$$\Delta = \lfloor \frac{N-2L_c}{N_z - 1} \rfloor,\ \ E(n) \leq \frac{2}{N}\sum_{k=L_c}^{N-L_c}|H(k)|$$

where $\otimes$ denotes N point circular convolution. Eq (9) suggests that $\widehat{h(n)}$ will contain delayed copies of the original channel separated be $\frac{N}{\Delta}$. If $N_z > L\frac{N-2L_c}{N}$, where L is the channel length, these copies will be separated well enough for us to extract the true channel. Note also that the error E will be small if the energy within the stopband is negligible.

4. SIMULATION RESULTS

We consider the channel

$$(c(t, 0.11) + 0.8c(t-1, 0.11) - 0.4c(t-3, 0.11))W_{6T}(t) \tag{11}$$

where $c(t-\tau, \beta)$ gives a raised cosine with delay τ and rolloff β, $W_{6T}(t)$ is a rectangular window of width 6 symbol intervals T. The channel was exited by a 16 QAM signal. The cyclic spectrum was estimated as a DFT of cyclic correlation windowed by a rectangular window. The DFT length for estimation was 128 and the oversampling ratio was 4. We varied the assumed channel length, which entered the estimation as the size of the cyclic correlation window. The NMSE for various combinations of SNR and data length is given in Fig. 4. We also did a comparison with the high complexity subspace based method [7] as seen in Figs. 5 and 6.

Although the subspace approach gives lower NMSE when the channel length is exactly known, when the length is unknown (which is the practical case) it gives an NMSE of the same order or higher than the proposed approximate approach, while the computational complexity of the proposed approach is significantly lower.

5. PERFORMANCE WITH EXPERIMENTAL DATA

We used real data obtained from our wireless communications testbed to verify the performance of the algorithm. The setup included an Agilent ESG 4431B Vector Signal Generator (operating range 250 kHz to 6.0 GHz), Agilent VSA 89640 Vector Signal Analyzer (operating range D.C. to 2.7 GHz), VSA 89640 Analyzer software on a laptop and two omni-directional antennae.

A Fujitsu MB86060 chip internal to the Vector Signal Generator was used for pulse shaping (square root raised cosine pulse with rolloff 0.08). We transmitted a known 4 QAM data sequence of 2500 symbols length at a carrier frequency of 2.4 GHz and at a data rate of 12 Msps.

We obtained the channel using our algorithm and for comparison, we correlated the received signal with the known input sequence to obtain another estimate. In each estimate, the taps from 16 to 32 were extracted as the channel. The DFT length used was 175 and the oversampling ratio was 4. We have given the results in Fig. 7.

6. CONCLUSIONS

We presented a method for blindly identifying bandlimited pulse shaping systems, within a scalar ambiguity and an unknown delay. We have seen that the subspace approach gives lowest NMSE when the channel length is exactly known. However, when this is unknown, the proposed method yields an NMSE of the same order, or smaller, at a much lower computational complexity, i.e., the complexity of an inverse DFT.

APPENDIX A

Let us consider an ideal bandlimited channel with zero rolloff and cutoff frequency ω_c. The zero magnitude within the stopband is achieved by designing the shaping function to have a set of uniformly spaced zeros. We assume that the number of these zeros, N_z is large compared to the remaining zeros of the channel.

The channel Frequency Responce can then be given as:

$$H(\omega) = \prod_{i=0}^{N_{total}}(1 - z_i z^{-1})|_{z=e^{j\omega}} \tag{12}$$

Here the channel has N_{total} zeros, out of which N_z lie on the unit circle.

Let us refer to Fig. 1 to calculate the phase of the terms of (12) that contains the zeros on the unit circle. As ω increases, the set of zeros rotate anticlockwise. Let ϕ_i be the phase of $(1 - z_i e^{-j\omega})$ where $|z_i| = 1$. It holds:

$$\phi_i(\omega) = \frac{\pi}{2} + \frac{\omega_c - \omega}{2} + i\frac{\pi - \omega_c}{N_z - 1}, 0 \le i < N_z \quad (13)$$

The sum of the phases of all terms $(1 - z_i e^{-j\omega})$ for which $|z_i| = 1$, i.e. $\phi_z(\omega)$ will then be equal to:

$$\phi_z(\omega) = N_z(\pi - \frac{\omega}{2}) \quad (14)$$

When $\omega \approx \theta_i$, we can show that

$$\frac{d\phi_i(\omega)}{d\omega} = \frac{r}{(1-r)^3}((1-r)^2 - \frac{1}{2}(1+r)(\theta_i - \omega)^2), \quad \omega \approx \theta_i \quad (15)$$

We see that as $r \to 1$, $\frac{d\phi_i(\omega)}{d\omega}$ becomes infinite indicating discontinuity of ϕ_i at $\omega = \theta_i$.

We see from Fig. 1 that at this discontinuity, the overall change in $\phi_i(\omega)$ is π. Hence, the overall phase response is linear with jumps of π at frequencies corresponding to each zero on the unit circle. Note that these jumps will not occur within the passband of the channel. Hence, within the stopband, if N_z is large enough, $\phi_z(\omega)$ will dominate the overall phase response because all its derivatives become large whenever we have $\omega \approx \theta_i$ for any $i \in [0, N_z - 1]$.

If the phase due to zeros away from the unit circle is $\phi_t(\omega)$, the phase of the channel can be given as

$$\phi(\omega) = N_z(\pi - \frac{\omega}{2}) + \phi_t(\omega) + \pi \sum_{i=0}^{N_z-1} u(\omega - \omega_c - i\, 2\frac{\pi - \omega_c}{N_z - 1}) \quad (16)$$

where $u(\omega)$ is the step function. Then in discrete form (8) can be re-written using (16) as:

$$\widehat{\phi(k)} = \begin{cases} \phi(k) - \frac{N_z \pi}{N} k - N_z(N/2 - m) \\ +\pi \sum_{i=0}^{N_z-1} u(k + 2m - L_c - i\Delta); \\ 0 \le k \le L_c - 1, \;\; N - L_c \le k \le N - 1 \\ 0; \;\; L_c \le k \le N - L_c \end{cases} \quad (17)$$

Then $\widehat{h(n)} = IDFT\{|H(k)|e^{j\widehat{\phi(k)}}\}$ yields:

$$\begin{aligned} \widehat{h(n)} = & \quad (18) \\ & \frac{a}{N} \sum_{k=0}^{L_c-1} H(k) e^{j\{\frac{2\pi}{N}(b+n)k + \pi \sum_{i=0}^{N_z-1} u(k+k_o-i\Delta)\}} \\ & + \frac{a}{N} \sum_{k=L_c}^{N-L_c-1} H(k) e^{j\{\frac{2\pi}{N}(b+n)k + \pi \sum_{i=N_z}^{(N-1)/\Delta-1} u(k+k_o-i\Delta)\}} \\ & + \frac{a}{N} \sum_{k=N-L_c}^{N-1} H(k) e^{j\{\frac{2\pi}{N}(b+n)k + \pi \sum_{i=0}^{N_z-1} u(k-N+1+k_o-i\Delta)\}} \\ & + E \end{aligned}$$

$$\widehat{h(n)} = \frac{a}{N} \sum_{k=0}^{N-1} H(k) e^{j\{\frac{2\pi}{N}(b+n)k\}} G(k) + E \quad (19)$$

where

$$G(k) = 2\sum_{i=0}^{M} rect(\frac{k + k_o - i2\Delta}{\Delta}) - 1 \;\; M \in \mathbb{Z} \quad (20)$$

$$(21)$$

and $rect(\frac{\tau}{\Delta})$ is a rectangular pulse of width Δ.

We see that the IDFT of $G(k)$ is

$$g(n) = \frac{e^{j\frac{2\pi}{N}k_o n}(1 - e^{-j\frac{2\pi}{N}\Delta n})}{(1 + e^{-j\frac{2\pi}{N}\Delta n})} \frac{(1 - e^{j2\pi n})}{(1 - e^{j\frac{2\pi}{N}n})} \quad (22)$$

where $g(n) \neq 0$ for $n = \frac{N}{2\Delta}, 3\frac{N}{2\Delta}, \ldots$. By simplification, (9) follows.

7. REFERENCES

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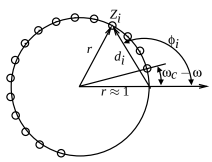


Fig. 1. Calculation of Phase Due to Channel Zeros Corresponding to the Stopband

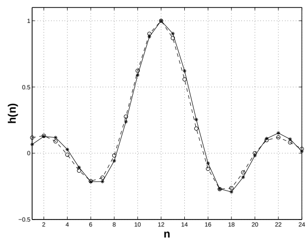
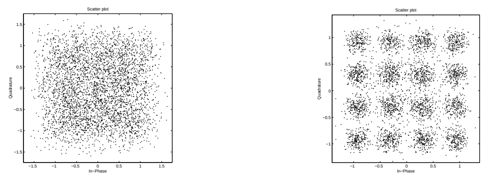


Fig. 2. Channel estimates for 50 runs, true channel is given in stars/solid line. The mean of estimates is given in circles/broken line. The SNR was 30 dB and 400 symbols were used in each estimation. N was kept at 128



(a) (b)

Fig. 3. Output of the channel (a) Before and (b) After equalization for one run with an SNR of 30 dB for a 16 QAM signal. 400 symbols were used for channel estimation and 4000 symbols were used in equalization. A zero forcing equalizer was used.

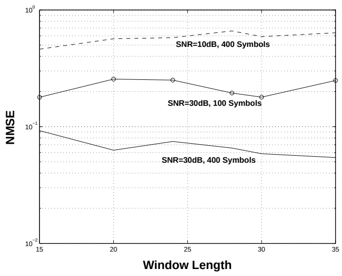


Fig. 4. Variation of NMSE with assumed channel length used in windowing for the channel given in (11). For each point, 50 Monte-Carlo runs were used to obtain the NMSE.

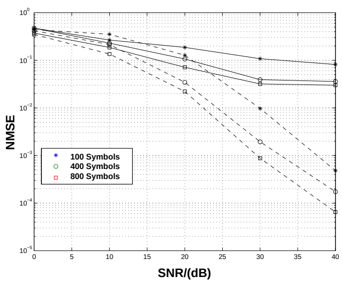


Fig. 5. Variation of NMSE with SNR for the channel given in (11). Broken Lines are for subspace based method [7]. Solid lines are for proposed method. For each point, 25 Monte-Carlo runs were used to obtain the NMSE.

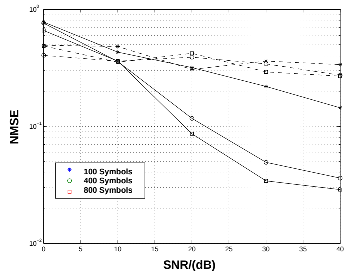


Fig. 6. Variation of NMSE with SNR for the channel given in (11) when the channel length was overestimated by two symbol intervals. Broken Lines are for subspace based method [7]. Solid lines are for proposed method. For each point, 25 Monte-Carlo runs were used to obtain the NMSE.

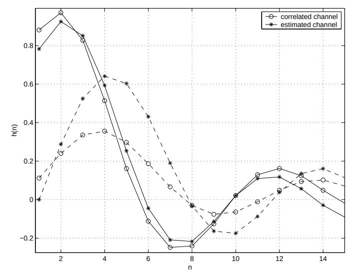
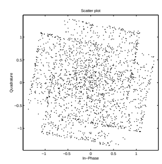
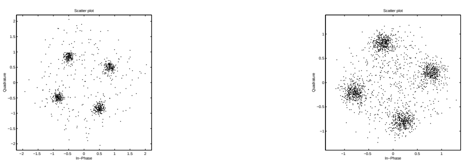


Fig. 7. Channel obtained using experimental data. Lines with circles gives the channel obtained by input-output correlation. Lines with stars gives the mean of the estimated channel using 400 symbols. Solid lines give the real part while broken lines give the imaginary part.



(a)



(a) (b)

Fig. 8. Equalized experimental data (a) using (b) channel obtained using correlation and (c) channel obtained using proposed method 2000 symbols were used in equalization. 400 symbols were used in estimation.