ON THE DESIGN OF VARIABLE FRACTIONAL DELAY FILTERS WITH LAGUERRE AND KAUTZ FILTERS

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ABSTRACT

In this paper, we generalize the well-known variable fractional delay filter structure of Farrow to include Laguerre and Kautz filters. We also show how one can incorporate amplitude response shaping into the optimization design formulation, and extend the range of variable delay to beyond the usual one sampling period. Some salient features of the new structures are demonstrated through three design examples based on the simple least squares criterion.

1. INTRODUCTION

Fractional delay filters have found applications in many fields of signal processing such as digital modems, microphone and sonar array processing, and speech and music signal processing [1]. These filters allow the user to delay sampled signals by amounts that are not integer multiples of the sampling period. An excellent discussion on the various design methods for fractionally delay filters can be found in the tutorial article [1].

One particularly important class of fractional delay filters is the *variable fractional delay filters*. Apart from the usual signal input, these filters have one additional input through which the user can adjust the amount of delay synthesized by the filters. The best known structure for implementing variable fractional delay filters is the one proposed by Farrow in [2] and shown in Fig. 1. It consists of M parallel FIR filters, whose coefficients are fixed, and a multiplier chain through which the user adjusts the filter delay. The FIR filters have K taps each. Methods to design the fixed filters have been summarized in [1], and some more recent work are reported in [3]-[6].

Here, we note that, in the design methods described thus far in the literature, (i) they all aim to design all-pass filters; and (ii) the range of delays synthesized by the filter is always taken to be one sampling period. In some applications, e.g. beam steering of the microphone array of a teleconferencing system [7], it may be desirable to incorporate some form of frequency response shaping, e.g. a bandpass characteristic from 300 Hz to 3000 Hz, into the beam steering fractional delay filters. Also, to track moving speech sources, it may be desirable if the range of delays that is synthesized by the filters can be easily adjusted over more than one sampling period. For speech, the sampling rate is typically 8000 Hz and in [7], it is seen that to cover the range of delays that can arise in a microphone array, it may be necessary to switch unit delay elements in and out of the Farrow filters. This can be awkward in practice.

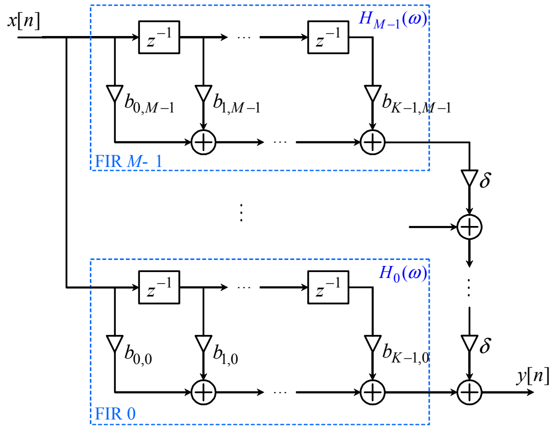

Fig. 1. The Farrow structure

In this paper, we show how the optimum design formulations of variable fractional delay filters can be reformulated trivially to include frequency response shaping and an extended range of variable delays. We also show how the conventional Farrow structure can be generalized to incorporate Laguerre and Kautz filters [8]. Laguerre and Kautz filters are higher order forms of the unit delay elements of an FIR filter. By replacing the unit delays with Laguerre and Kautz filter sections, an extra one (Laguerre) or two (Kautz) degrees of freedom are introduced into the design. It will be shown that this extra freedom can yield a better design.

2. THE LAGUERRE-FARROW AND KAUTZ-FARROW STRUCTURES

It can be easily shown that the transfer function of the

Farrow filter structure of Fig. 1 is given by [1],[4]

$$H(\omega,\delta) = \sum_{k=0}^{K-1}\left[\sum_{m=0}^{M-1} b_{k,m}\delta^m\right]e^{-jk\omega}. \qquad (1)$$

Eq. (1) shows that $H(\omega,\delta)$ can be thought of as being just a K-tap FIR filter except that each of its tap coefficients is an $(M-1)$th order polynomial of δ. In other words, $H(\omega,\delta)$ approximates the dependence of each of its coefficients on δ by an $(M-1)$th order polynomial.

We next express (1) as follows

$$H(\omega,\delta) = \sum_{k=0}^{K-1}\left[\sum_{m=0}^{M-1} b_{k,m}\delta^m\right]\phi_k(\omega) \qquad (2)$$

where $\{\phi_k(\omega),\, k=0,\ldots,(K-1)\}$ is a set of K ortho-normal basis functions in ω. Other orthonormal basis functions are the Laguerre and Kautz functions

Laguerre

$$\phi_k(\omega) = \sqrt{1-\alpha^2}\cdot\frac{(z^{-1}-\alpha)^k}{(1-\alpha z^{-1})^{k+1}} \qquad (3)$$

Kautz

$$\phi_{2k}(\omega) = c_0\cdot\frac{(1-z^{-1})\left[(z^{-1}-\beta)(z^{-1}-\beta^*)\right]^k}{\left[(1-\beta z^{-1})(1-\beta^* z^{-1})\right]^{k+1}} \qquad (4)$$

$$\phi_{2k+1}(\omega) = c_1\cdot\frac{(1+z^{-1})\left[(z^{-1}-\beta)(z^{-1}-\beta^*)\right]^k}{\left[(1-\beta z^{-1})(1-\beta^* z^{-1})\right]^{k+1}} \qquad (5)$$

where $z=e^{j\omega}$, $-1<\alpha<1$, $|\beta|<1$,

$$c_0 = \sqrt{\frac{1}{2}(1+\beta)(1+\beta^*)(1-\beta\beta^*)}, \qquad (6)$$

and

$$c_1 = \sqrt{\frac{1}{2}(1-\beta)(1-\beta^*)(1-\beta\beta^*)}. \qquad (7)$$

The Laguerre-Farrow and Kautz-Farrow structures are shown, respectively, in Figs. 2 and 3 where

$$L_0(z) = \sqrt{1-\alpha^2}\cdot\frac{1}{1-\alpha z^{-1}}, \qquad (8)$$

$$L(z) = \frac{z^{-1}-\alpha}{1-\alpha z^{-1}}, \qquad (9)$$

$$K_0(z) = \frac{c_0(1-z^{-1})}{(1-\beta z^{-1})(1-\beta^* z^{-1})}, \qquad (10)$$

$$K_1(z) = \frac{c_1(1+z^{-1})}{(1-\beta z^{-1})(1-\beta^* z^{-1})}, \qquad (11)$$

and

$$K(z) = \frac{(z^{-1}-\beta)(z^{-1}-\beta^*)}{(1-\beta z^{-1})(1-\beta^* z^{-1})}. \qquad (12)$$

Note that a Laguerre filter reduces to an FIR filter when $\alpha=0$, while a Kautz filter reduces to a Laguerre filter when β is real and to an FIR filter when $\beta=0$.

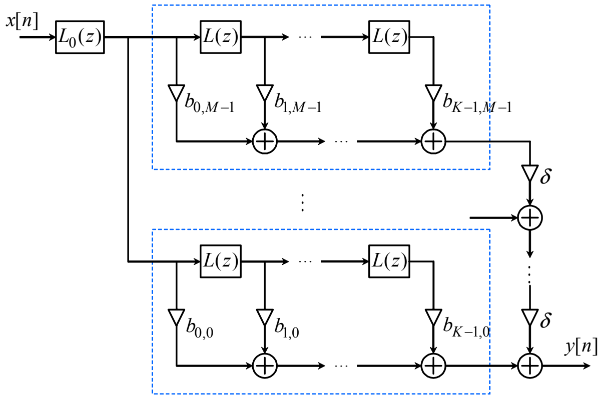

Fig. 2. The Laguerre-Farrow structure

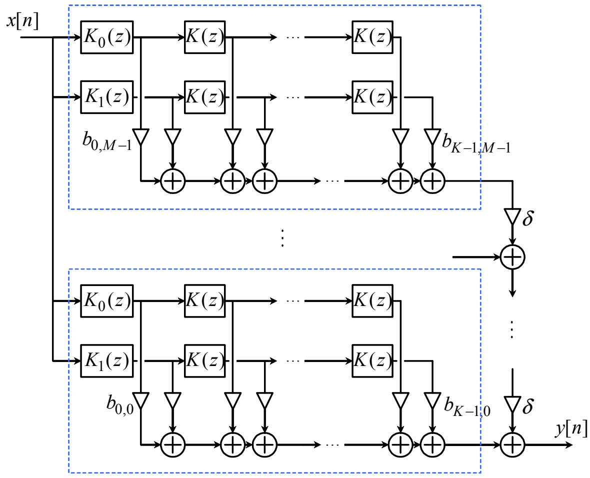

Fig. 3. The Kautz-Farrow structure

3. OPTIMUM DESIGN FORMULATIONS

Suppose the desired transfer function is given by

$$H_d(\omega,\bar{D},\delta) = A_d(\omega)e^{-j\omega(\bar{D}+\delta)} \qquad (13)$$

where $-\pi\le\omega\le\pi$, $\bar{D}$ is the nominal or mean delay to be synthesized by the variable delay filter, $A_d(\omega)$ is the desired amplitude response, $-\delta_m\le\delta\le\delta_m$, and δ_m is the maximum delay variation. For the Farrow structure, design methods reported thus far all assumed $A_d(\omega)=1$ and $\delta_m=0.5$ [1].

Clearly, many optimum design problems can be formulated. For example, the minimax design for the Laguerre-Farrow structure is given by

$$\min_{\substack{\mathbf{h}\in\mathbb{R}^{MK}\\ -1<\alpha<1\\ \delta_m\le\bar{D}\le(K-1-\delta_m)}}\ \max_{\substack{-\pi\le\omega\le\pi\\ -\delta_m\le\delta\le\delta_m}} \left|E(\omega,\bar{D},\delta)\right| \qquad (14)$$

where

$$E(\omega,\bar{D},\delta) = H(\omega,\delta) - H_d(\omega,\bar{D},\delta), \qquad (15)$$

while the least squares design is given by

$$\min_{\substack{-1<\alpha<1 \\ \delta_m \le \bar{D} \le (K-1-\delta_m)}} \left\{ \min_{\mathbf{h}\in\mathbb{R}^{MK}} \int_{-\delta_m}^{\delta_m} \int_{-\pi}^{\pi} \left|E(\omega,\bar{D},\delta)\right|^2 d\omega\, d\delta \right\}. \quad (16)$$

A similar set of design formulations can be written for the Kautz-Farrow structure if we replace the constraint $-1<\alpha<1$ in (14) and (15) with $|\beta|<1$.

Other design formulations can also be posed, for example, those involving constraints in the amplitude and/or phase responses [9]. In the sequel we will consider only the least squares formulation (16).

4. LEAST SQUARES DESIGN

Define firstly the vectors

$$\mathbf{h} = \left[b_{0,0} \ \cdots \ b_{K-1,0} \quad \cdots \quad b_{0,M-1} \ \cdots \ b_{K-1,M-1} \right]^T, \quad (17)$$

$$\boldsymbol{\delta} = \left[\delta^0 \ \ \delta^1 \ \cdots \ \delta^{M-1} \right]^T, \quad (18)$$

$$\boldsymbol{\varphi}(\omega) = \left[\phi_0(\omega) \ \ \phi_1(\omega) \ \cdots \ \phi_{K-1}(\omega) \right]^T, \quad (19)$$

and

$$\boldsymbol{\varphi}_\delta(\omega) = \boldsymbol{\delta} \otimes \boldsymbol{\varphi}(\omega). \quad (20)$$

where $\otimes$ denotes Kronecker product. Using the results of [6], it can be shown that the optimum solution to the inner minimization of (16) is given by

$$\mathbf{h}(\bar{D},\alpha) = \mathbf{R}^{-1}\mathbf{p}(\bar{D},\alpha) \quad (21)$$

where

$$\mathbf{R} = 2\pi \cdot \mathbf{R}_{\boldsymbol{\delta}} \otimes \mathbf{I}_{K\times K}, \quad (22)$$

$$\left[\mathbf{R}_{\boldsymbol{\delta}}\right]_{pq \atop p,q=1,\ldots,M} = \begin{cases} \dfrac{2\delta_m^{p+q-1}}{p+q-1}, & (p+q) \text{ even} \\ 0, & (p+q) \text{ odd} \end{cases}, \quad (23)$$

and

$$\mathbf{p}(\bar{D},\alpha) = \int_{-\delta_m}^{\delta_m} \int_{-\pi}^{\pi} \mathrm{Re}\left[H_d^*(\omega,\bar{D},\delta)\cdot \boldsymbol{\varphi}_\delta(\omega) \right] d\omega\, d\delta \quad (24)$$

The optimum solution to (16) is then found by searching for the best $\bar{D}$ and α.

Note that, with an appropriate re-definition of $\phi_k(\omega)$, (21) applies also to the Farrow and Kautz-Farrow filters.

4. DESIGN EXAMPLES

In this section, we present and compare three band pass variable fractional delay filter designs based on the Farrow, Laguerre-Farrow, and Kautz-Farrow structures. All three filters have the same desired amplitude response (stopband $[0, 0.25\pi] \cup [0.75\pi, \pi]$, passband $[0.4\pi, 0.6\pi]$, and linear in the transition bands), the same filter length $K=20$, and the same delay variation of 2 sampling periods, i.e. $\delta_m = 1$. Concerning M, it was found experimentally that for $\delta_m = 1$, increasing M beyond 5 will give only a very small decrease in optimum cost. We thus set $M=5$. It was also found experimentally that M depends only very slightly on K, which is expected since M gives the degree of the polynomial approximation of the filter coefficients on δ as δ varies from $-\delta_m$ to δ_m.

The results of the designs are summarized in Table 1. As expected, in terms of the achieved optimum cost, the Kautz-Farrow filter outperforms the Laguerre-Farrow filter which, in turn, outperforms the Farrow filter. Fig. 4 shows the magnitude responses while Fig. 5 shows the group delay responses for $\omega \in [0.4\pi, 0.6\pi]$. It is intuitively satisfying to see the pole of the Kautz-Farrow filter is located at almost 90° which corresponds to the centre frequency of the passband.

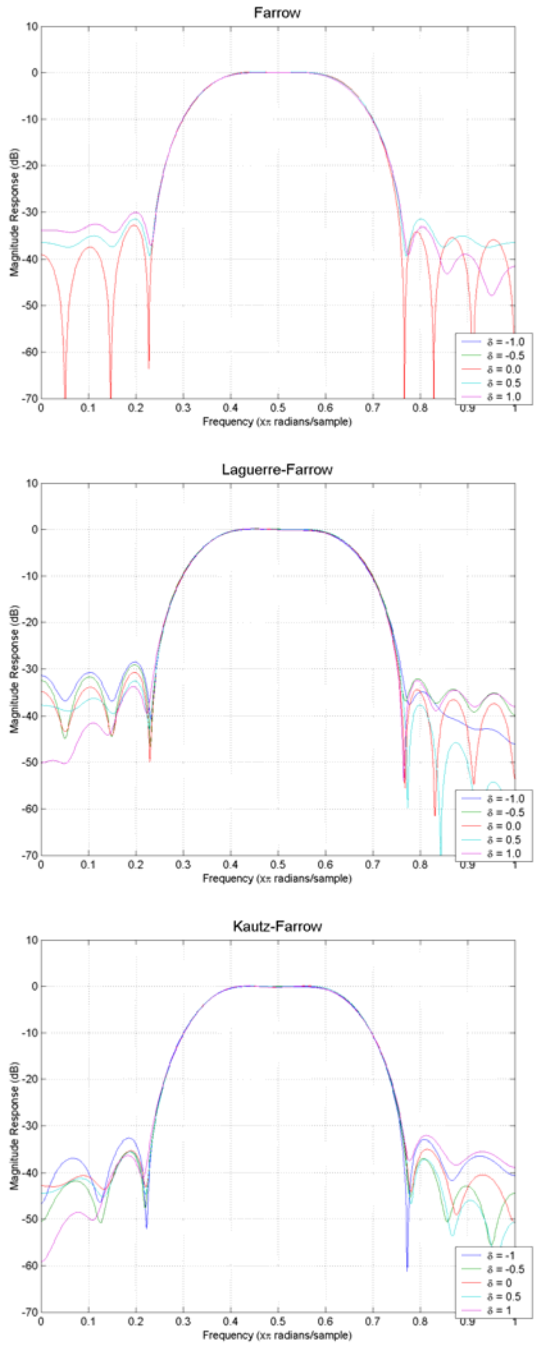

Fig. 4. Magnitude responses

	$\bar{D}$	Pole Posn.	Opt. Cost
Farrow	9.5	0	0.00476220
Laguerre	9.50225	-0.0306339	0.00428066
Kautz	10.2480	0.381983∠90.5393°	0.00349160

Table 1. Summary of filter design results

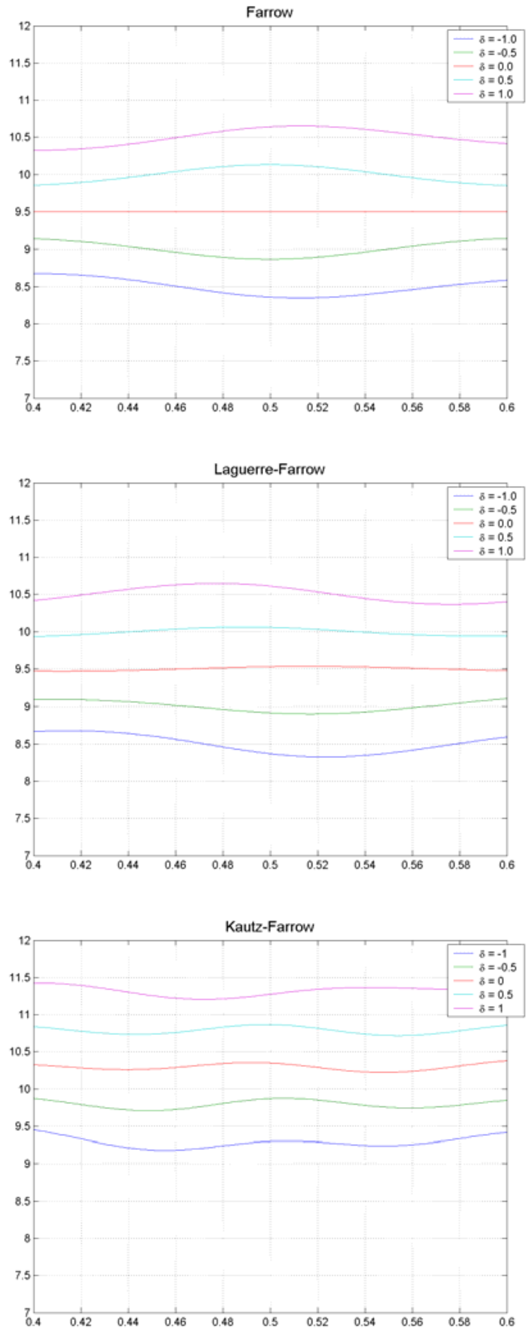

Fig. 5. Group delay responses

Comparing the magnitude responses shown in Fig. 4, we see that there appears to be little difference between the three filters in the passband from 0.4π to 0.6π. However, in the stopband, the Kautz-Farrow filter gives better suppression. As for the group delay responses shown in Fig. 5, we see that the Laguerre-Farrow filter gives better group delay linearity than the Farrow filter, while the Kautz-Farrow filter is better still.

5. CONCLUSIONS

In this paper, we have generalized the Farrow variable fractional delay structure to include Laguerre and Kautz filters, and using a simple least squares design formulation, we have shown that the Laguerre-Farrow and Kautz-Farrow structures can offer performance gains. It is expected that by a more appropriate formulation, one can extract better performance gains from the higher order variable fractional delay filters. We also showed how one can include amplitude response shaping into the design formulation and extend the range of delays that the filters can synthesize.

6. REFERENCES

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