KERNEL WITH BLOCK STRACTURE FOR SAMPLING RATE CONVERTER

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ABSTRACT

Sampling rate conversion using the filter banks is proposed in the field of digital signal processing. However, in sampling rate conversion with a rational factor, the computational complexity may become very large. Moreover whenever the sampling rate is changed, it is necessary to design the filter again. In this paper, hence, we present a kernel with block structure for sampling rate conversion. As the filter proposed has the impulse response approximated by polynomials, it is unnecessary redesign whenever the sampling rate is changed. The kernel proposed has the block structure that the impulse response of the sampling section since the third is represented by the polynomial used for the second sampling section. Therefore, the filter has less memory. Moreover, the filter has the advantage that it is possible to correspond to arbitrary fractional sampling rate conversion.

1. INTRODUCTION

Sampling rate conversion widely used in subband coding [6], A/D and D/A transitions [4] etc. is an important techniques. The technique using filter banks has been popularly used for digital sampling rate conversion for a fixed ratio M/N, where M and N are positive integers [8]. In the application of sampling rate conversion from compact disc (CD) to digital audio tape (DAT) [1]-[3], because M and N are 160 and 147 respectively, the computational complexity of this method become very large. Moreover this method is necessary to redesign the filter, whenever the sampling rate is changed.

Then, we proposed an interpolation kernel (filter) approximated by using any quadratic functions for piecewise the sinc function [5]. This method is unnecessary to redesign the filter, whenever the sampling rate is changed, and the kernel obtained is easy to implement. However, to obtain a large attenuation in the stopband, a lot of quadratic functions are needed. Therefore the computational complexity of this method increases.

In this paper, hence, we present kernel with block structure for sampling rate conversion. The kernel proposed has the block structure that the impulse response of the sampling section since the third is represented by the polynomial used for the second sampling section. Therefore, the filter has less memory. As the filter proposed has the impulse response approximated by polynomials, it is unnecessary to redesign the filter whenever the sampling rate is changed. Moreover, the filter has the advantage that it is possible to correspond to arbitrary fractional sampling rate conversion.

2. KERNEL FOR INTERPOLATION

In this section, the structure and the design method of the filter proposed are described.

2.1 Kernel Structure

Reconstruction of a piecewise continuous function from discrete data is taken to be a linear combination of input data and a reconstruction kernel. For unit spaced samples, this is

$$f(x) = \sum_{i=-\infty}^{\infty} f_i \, y(x-i) \tag{1}$$

where f_i are the sample values and y(x) is the reconstruction kernel. In [5], we proposed a kernel approximated to each sampling section piece by any quadratic functions as follows.

$$y(x)=\begin{cases} a_{1,1}x^2+b_{1,1}x+c_{1,1} & \left(0\le|x|\le\frac{1}{N}\right)\\ \vdots & \\ a_{1,n}x^2+b_{1,n}x+c_{1,n} & \left(\frac{n-1}{N}\le|x|\le 1\right)\\ \vdots & \\ a_{s,n}x^2+b_{s,n}x+c_{s,n} & \left(s-1+\frac{n-1}{N}\le|x|\le s-1+\frac{n}{N}\right)\\ \vdots & \\ a_{S,N}x^2+b_{S,N}x+c_{S,N} & \left(S-1+\frac{N-1}{N}\le|x|\le S\right) \end{cases} \tag{2}$$

Because the coefficient of piecewise local quadratic functions is different, an amount of memory to compose the kernel increase when the sampling section and number of quadratic functions increase. Therefore, to obtain a large attenuation in the stopband, the computational complexity of this method increases.

Hence, we present a filter with block structure. This filter uses the coefficient of the same quadratic function as the second sampling section from the third sampling section. Figure 1 shows the proposed structure of filtre with 5 sampling sections and figure 2 shows the outline of the proposed filter and. Moreover, the proposed kernel of the general form is

$$
g(x)=\begin{cases}
a_{1,1}x^2+b_{1,1}x+c_{1,1} & (0\le|x|\le\frac{1}{N})\\
\vdots & \\
a_{1,n}x^2+b_{1,n}x+c_{1,n} & (\frac{n-1}{N}\le|x|\le 1)\\
e_1(a_{2,1}x^2+b_{2,1}x+c_{2,1}) & (1\le|x|\le 1+\frac{1}{N})\\
\vdots & \\
e_1(a_{2,N}x^2+b_{2,N}x+c_{2,N}) & (1+\frac{N-1}{N}\le|x|\le 2)\\
\vdots & \\
e_{s-1}(a_{2,n}x^2+b_{2,n}x+c_{2,n}) & (s-1+\frac{n-1}{N}\le|x|\le s-1+\frac{n}{N})\\
\vdots & \\
e_{S-1}(a_{2,N}x^2+b_{2,N}x+c_{2,N}) & (S-1+\frac{N-1}{N}\le|x|\le S)
\end{cases} \tag{3}
$$

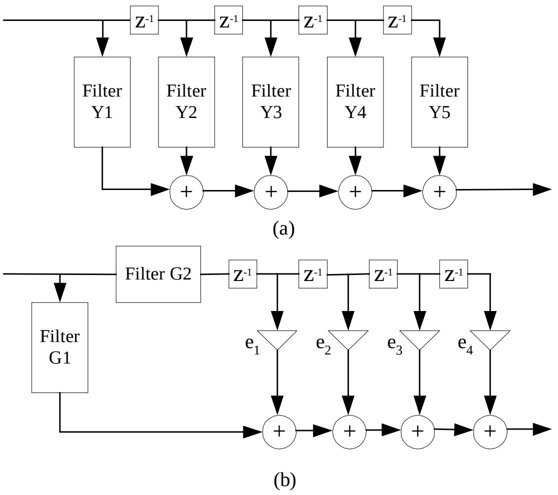

Fig. 1 The structure (a) no block structure (b) block structure

where N and S are the number of polynomials for one sampling section and the number of sampling sections. e_i is arbitrary constant of proportionality.

2.2 Kernel design

To produce a useful filter from the prposed general form, we need to apply restrictions to eq. (3)

1) $g(x)=g_{1,0}$ for $x=0$
2) $g(x)=g_{2,n}$ for $x=\frac{n}{N}$
3) $g(x)=0$ for $x=s$
4) C0-Continuous

We notice from conditions 1) and 3) that the kernel is zero

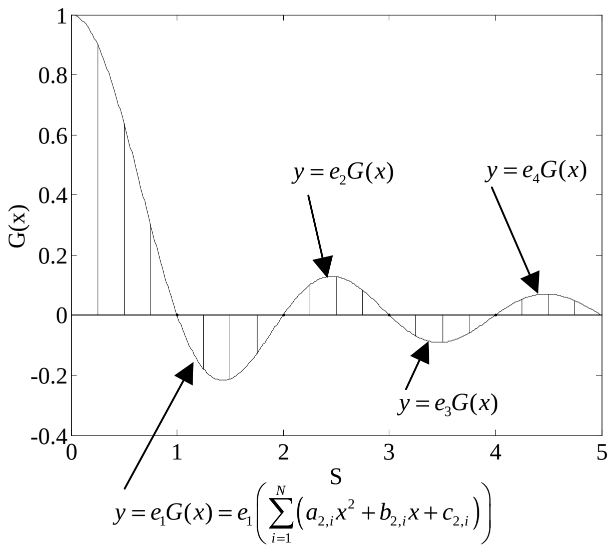

Fig. 2 The outline of the proposed filter

$$
g(x)=\begin{cases}
a_{1,1}\left(x^2-\frac{x}{N}\right)+g_{1,1}Nx-g_{1,0}(Nx-1) & (0\le x\le\frac{1}{N})\\
\vdots & \\
a_{1,n}\left[x^2-N\left\{(\frac{n}{N})^2-(\frac{n-1}{N})^2\right\}(x-\frac{n}{N})-(\frac{n}{N})^2\right]+g_{1,n}(Nx+1-n)-g_{1,n-1}(Nx-n) & (\frac{n-1}{N}\le x\le\frac{n}{N})\\
e_1\left[a_{2,1}\left[x^2-N\left\{(1+\frac{1}{N})^2-1\right\}\left\{x-(1+\frac{1}{N})\right\}-(1+\frac{1}{N})^2\right]+g_{2,1}\left\{Nx+1-N(1+\frac{1}{N})\right\}-g_{2,0}\left\{Nx-N(1+\frac{1}{N})\right\}\right] & (1\le x\le 1+\frac{1}{N})\\
\vdots & \\
e_1\left[a_{2,N}\left[x^2-N\left\{4-(1+\frac{N-1}{N})^2\right\}\{x-2\}-4\right]+g_{2,N}\{Nx+1-2N\}-g_{2,N-1}N\{x-2\}\right] & (1+\frac{N-1}{N}\le x\le 2)\\
\vdots & \\
e_{s-1}\left[a_{2,n}\left[x^2-N\left\{(1+\frac{n}{N})^2-(1+\frac{n-1}{N})^2\right\}\left\{x-(1+\frac{n}{N})\right\}-(1+\frac{n}{N})^2\right]+g_{2,n}\left\{Nx+1-N(1+\frac{n}{N})\right\}-g_{2,n-1}\left\{Nx-N(1+\frac{n}{N})\right\}\right] & (s-1+\frac{n-1}{N}\le x\le s-1+\frac{n}{N})\\
\vdots & \\
e_{S-1}\left[a_{2,N}\left[x^2-N\left\{4-(1+\frac{N-1}{N})^2\right\}\{x-2\}-4^2\right]+g_{2,N}\{Nx+1-2N\}-g_{2,N-1}N(x-2)\right] & (S-1+\frac{N-1}{N}\le x\le S)
\end{cases} \tag{4}
$$

intersymbol interference. Moreover, we notice from conditions 2) that each quadratic function starts and ends at same points. Substituting the above four restrictions in eq. (2), we obtain eq. (4).

Now, to enable the achievement of a large attenuation in the stopband, it approximates in the frequency domain. The frequency characteristic of eq. (3) is

$$G(\omega)=\sum_{j=0}^{S-1} e_j \sum_{i=jL}^{(j+1)L} g(iT)\cos(i\omega T) \tag{5}$$

where T=1/L and L is a number of evaluation points in one sampling section. e_i is coefficient multiplied by the amplitude in the second sampling section to obtain the amplitude for each sampling section since the third and $e_0 = 1$. Moreover, $g(jL)$ is all equal excluding $j \neq 0$.

Now, an ideal frequency characteristic is obtained by

$$D(\omega)\begin{cases} 1 & Passband \\ 0 & Stopband \end{cases} \tag{6}$$

Moreover, let $W(\omega)$ and δ be the weighting function and the maximum allowable approximation error, respectively. Then, the design problem of the filter proposed is to find coefficients e_j, $a_{i,j}$ and $g_{i,j}$ satisfying

$$-\delta \le W(\omega)[D(\omega) - G(\omega)] \le \delta \tag{7}$$

on the evaluation frequency points $0 \le \omega \le \pi$. By the way, Because e_j and the quadratic coefficients, $a_{i,j}$ and $g_{i,j}$, can not be optimized at the same time, eq. (7) can not be solved by using a standard linear programming. Therefore, in the proposed method, e_j is fixed by arbitrary value and the quadratic coefficients, $a_{i,j}$ and $g_{i,j}$, are optimized by using a standard linear programming as follows:

Minimize δ^1

Subject to

$$-W(\omega)\sum_{j=0}^{S-1} E_j \sum_{i=jL}^{(j+1)L} g(iT)\cos(i\omega T) - \delta^1 \le -W(\omega)D(\omega) \tag{8}$$

$$W(\omega)\sum_{j=0}^{S-1} E_j \sum_{i=jL}^{(j+1)L} g(iT)\cos(i\omega T) - \delta^1 \le W(\omega)D(\omega)$$

Next the obtained $a_{i,j}$ and $g_{i,j}$ are fixed and e_j is optimized by using a standard linear programming as follows:

Minimize δ^2

Subject to

$$-W(\omega)\sum_{j=0}^{S-1} e_j \sum_{i=jL}^{(j+1)L} G(iT)\cos(i\omega T) - \delta^2 \le -W(\omega)D(\omega) \tag{9}$$

$$W(\omega)\sum_{j=0}^{S-1} e_j \sum_{i=jL}^{(j+1)L} G(iT)\cos(i\omega T) - \delta^2 \le W(\omega)D(\omega)$$

The above operation continues until the difference δ^i and δ^{i+1} is less than a value of 10^{-5}. Then, with coefficients, e_j, $a_{i,j}$, and $g_{i,j}$, the filter proposed can be formed as FIR filter. In this method, we are confirming that the same filter can be designed though a variety initial value of e_j is given.

3. EXAMPLE

In this section, to show the proposed filter effectiveness, we consider about the following designs.

3.1 Example 1

We think about the filter design of the following specification.

[Specifications]

N : 3, S : 5, L : 9, roll-off rate : 0.25

Passband edge: 0.75, Stopband edge : 1.25,

Weight : 1 (All the frequencies)

The time response and its amplitude characteristics of the filter obtained are shown in figs. 3 and 4, respectively.

It's clear from fig. 3 that the time response of the proposed filter is the zero intersymbol interference because $g(x)$ exactly crosses zero except at the point $x = 0$. In fig. 4, solid line and dash line indicate the amplitude response of the proposed filter and the filter in the same specification without the block processing, that is, filter using ref [5], respectively. The number of the polynomial coefficients of the former filter is 15 and one of latter filter is 26. Consequently, the filter using the proposed block processing method can achieve an equal characteristic by less memory.

3.2 Example 2

This example shows that the amplitude response of the proposed filter dose not change if the sampling rate is changed into the rational number.

[Specifications]

N : 3, S : 5, roll-off rate : 0.25

Passband edge: 0.75, Stopband edge : 1.25,

Weight : 1 (All the frequencies)

In fig. 5, the amplitude response of the obtained filter to be upsampling 9 to 12.5 is shown by solid line and one of the obtained filter to be downsampling 15 to 12.5 is shown by dash line. It is clear from fig. 5 that the amplitude response obtained even if the sampling rate changes very looks like. That is, even if the sampling rate changes, the proposed filter need not be designed. Moreover, the filter proposed has the advantage that it is possible to correspond to arbitrary fractional sampling rate conversion.

4. SUMMARY

In this paper, we present a kernel with block structure for sampling rate conversion. The kernel proposed has the block structure that the impulse response of the sampling section since the third is represented by the polynomial used for the second sampling section. Therefore, the filter has less memory. As the filter proposed has the impulse response approximated by polynomials, it is unnecessary to redesign whenever the sampling rate is changed. Moreover, the filter has the advantage that it is possible to correspond to arbitrary fractional sampling rate conversion.

5. REFERENCES

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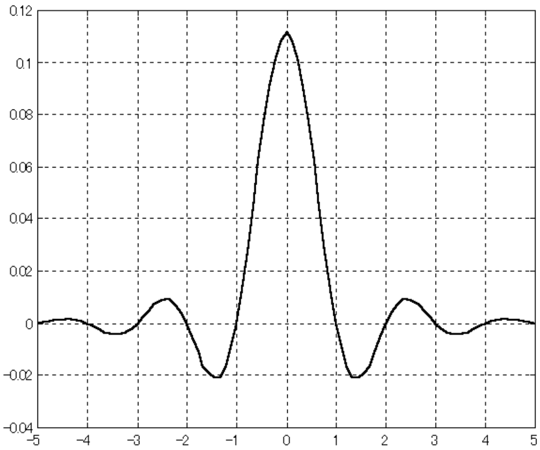

Fig. 3 The time response

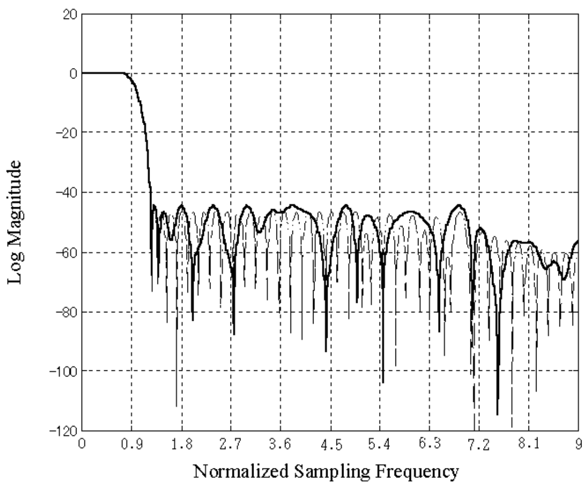

Fig. 4 Amplitude response of the proposed filter

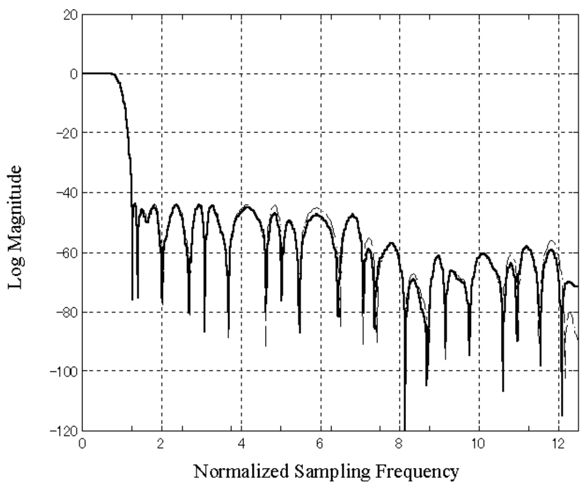

Fig. 5 The comparison of the amplitude responses of the different sampling rate