# INTERFERENCE SUPPRESSION WITH MINIMAL SIGNAL DISTORTION

*A. A. (Louis) Beex & James R. Zeidler*

DSPRL – ECE 0111
Virginia Tech
Blacksburg, VA 24061-0111, USA

Communications & Information Systems, 28505
SPAWAR Systems Center
San Diego, CA 92152, USA

## ABSTRACT

We propose a two-stage adaptive filtering process for canceling narrowband interference from a wideband signal process, based on a reference signal consisting of a Doppler shifted version of the interference. The first stage uses the time-varying behavior of NLMS with large step-size to produce an estimate of the interference signal, in which the Doppler shift is mitigated. The second stage uses NLMS with small step-size, as an approximation to the Wiener filter, in order to modify amplitude and phase of the Doppler-mitigated reference signal so that an estimate of the interference signal is produced suitable for interference cancellation. Simulations with a wideband QPSK signal corrupted by strong sinusoidal interference, for which a Doppler shifted reference is available, show the performance of the proposed approach in terms of signal-to-interference ratio and bit-error rate improvement.

## 1. INTRODUCTION

One of the important and interesting applications of adaptive filtering is in adaptive noise, or interference, canceling (ANC). Fig. 1 shows the classical setup [1]. With sinusoidal interference, and a reference that can be derived from that sinusoidal interference by linear filtering, the adaptive filter weights tend to the Wiener filter weights as the step-size approaches zero. The overall filter acts like a notch filter centered at the interference frequency, with a notch bandwidth “proportional” to the adaptation step-size [2]. In order to benefit from adaptation to changes in the environment, the step-size has to be large enough to preserve a significant portion of the instantaneous error signal in the filter update. Consequently, the notch width entails a distortion of the wideband signal component. When the frequency of the reference and interference are not equal, as when the reference is a Doppler-shifted version of the interference, the Wiener filter weights are zero, and no cancellation of the interference takes place. The time-varying behavior of the normalized least-mean-square (NLMS) weights at large step-size [3] allows us to better estimate the desired signal in the ANC scenario. However, the desired signal for the ANC scenario, wideband signal plus strong interference, is not actually the signal-of-interest (SOI) to us. When the desired signal consists of a wideband SOI and an additive interference, it is difficult to remove the interference without signal distortion. In this paper a two stage filtering process is used to first improve the estimate of the interference frequency using the Doppler shifted reference signal, followed by using the improved estimate in the second stage to remove the interference with minimal signal distortion.

The paper is organized as follows. Section II reviews the NLMS algorithm and some of its properties and relevant interpretations. Section III treats sinusoidal ANC and the inherent time-varying behavior of NLMS with large step-size in that scenario. Section IV defines the proposed two-stage ANC-NLMS approach, and simulation results illustrating the efficacy of the proposed approach are presented in Section V. A conclusion is provided in Section V.

## 2. NLMS & PROPERTIES

The NLMS adaptation algorithm is as follows:

$$\begin{aligned} \hat{d}_n &= \mathbf{w}_n^H \mathbf{u}_n \\ e_n &= d_n - \hat{d}_n \\ \mathbf{w}_{n+1} &= \mathbf{w}_n + \bar{\mu} \frac{e_n^*}{\mathbf{u}_n^H \mathbf{u}_n} \mathbf{u}_n \end{aligned} \qquad (1)$$

When the desired signal $d_n$ has the same structure as that used in the modeling process, i.e.

$$d_n = \mathbf{w}_o^H \mathbf{u}_n \qquad (2)$$

for some fixed weight vector $\mathbf{w}_o$, then the NLMS adaptation converges to that weight vector (it produces *a posteriori* errors of zero and weight vector increment norms of zero). When there is noise added in (2), the weight vector converges to a neighborhood of $\mathbf{w}_o$, with the size of that neighborhood proportional to the stepsize used. The latter means that for small stepsizes the NLMS weight vector will be close to the best possible constant weight vector, i.e. the Wiener solution.

Prominent time-varying NLMS weight behavior has been observed in situations where a large stepsize is chosen, for example $\bar{\mu} = 1$, so that adaptation is the fastest. For the latter, it can be shown that the *a posteriori* error equals zero [4], i.e.

$$\begin{aligned} \varepsilon_n &= d_n - \mathbf{w}_{n+1}^H \mathbf{u}_n \\ &= 0 \end{aligned} \qquad (3)$$

Another interpretation of the NLMS algorithm is that the *a posteriori* weight vector minimizes the norm of the weight vector increment, with (3) serving as a constraint [4]. Note from (1) that the change from $\mathbf{w}_n$ to $\mathbf{w}_{n+1}$ is always in the direction of $\mathbf{u}_n$.

## 3. SINUSOIDAL ANC

The classical ANC scenario is indicated in Fig. 1. The desired signal $d_n$ consists of a wideband signal-of-interest $q_n$ that is corrupted by strong sinusoidal interference $i_n$. The reference signal $r_n$ is the input to the adaptive filter. Consequently, the NLMS input vector $\mathbf{u}_n$ has as its elements $r_n$, $r_{n-1}$, …, $r_{n-M+1}$ for an *M*-tap adaptive filter.

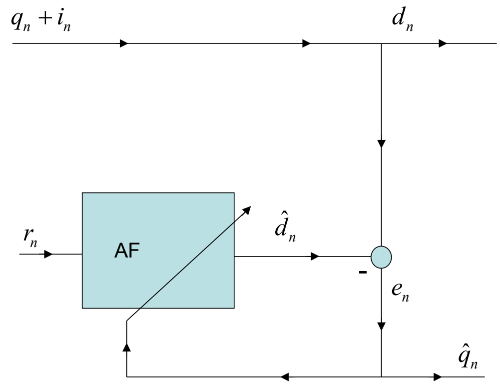


Fig. 1: Classical ANC Configuration.

For sinusoidal interference and reference signals, we have

$$\begin{aligned} i_n &= A_i e^{j\phi_i} e^{j\omega_i n} \\ r_n &= A_r e^{j\phi_r} e^{j\omega_r n} \end{aligned} \tag{4}$$

Note that we can rewrite the interference in terms of its past.

$$\begin{aligned} i_n &= A_i e^{j\phi_i} e^{j\omega_i (n-1)} e^{j\omega_i} \\ &= e^{j\omega_i} i_{n-1} \end{aligned} \tag{5}$$

Let's assume a hypothetical NLMS input vector $\tilde{\mathbf{u}}_n$, consisting of the immediate past of the interference signal and a single sample of the reference. The output of this filter is now:

$$\begin{aligned} \hat{d}_n &= \tilde{\mathbf{w}}^H \tilde{\mathbf{u}}_n \\ &= \tilde{\mathbf{w}}^H \begin{bmatrix} i_{n-1} \\ r_n \end{bmatrix} \end{aligned} \tag{6}$$

Using (5) we can produce an NLMS output that cancels the interference, as follows.

$$\begin{aligned} \hat{d}_n &= \tilde{\mathbf{w}}^H \begin{bmatrix} e^{-j\omega_i} i_n \\ r_n \end{bmatrix} \\ &= \begin{bmatrix} e^{j\omega_i} & 0 \end{bmatrix} \begin{bmatrix} e^{-j\omega_i} i_n \\ r_n \end{bmatrix} \\ &= i_n \end{aligned} \tag{7}$$

Next we use the concept of the linking sequence [3], to express the relationship between the immediate past of the interference and the reference signal.

$$\begin{aligned} \rho_n^{(-1)} &= \frac{i_{n-1}}{r_n} \\ &= \frac{A_i e^{j\phi_i} e^{j\omega_i (n-1)}}{A_r e^{j\phi_r} e^{j\omega_r n}} \\ &= \frac{A_i}{A_r} e^{j(\phi_i - \phi_r)} e^{j(\omega_i - \omega_r) n} e^{-j\omega_i} \end{aligned} \tag{8}$$

Using (8) to substitute for $i_{n-1}$ in (6) and (7) yields

$$\begin{aligned} \hat{d}_n &= \begin{bmatrix} e^{j\omega_i} & 0 \end{bmatrix} \begin{bmatrix} \rho_n^{(-1)} r_n \\ r_n \end{bmatrix} \\ &= e^{j\omega_i} \rho_n^{(-1)} r_n \\ &= \hat{\mathbf{w}}_{\mathrm{TV}}^H (n) \mathbf{r}(n) \end{aligned} \tag{9}$$

Together with (7), the latter shows that a time-varying weight exists, which – operating on the reference – produces the interference signal. Using (8), that time-varying weight (vector) is as follows.

$$\begin{aligned} \mathbf{w}_{\mathrm{TV},\,n} &= \left[ e^{j\omega_i} \rho_n^{(-1)} \right]^H \\ &= \frac{A_i}{A_r} e^{-j(\phi_i - \phi_r)} e^{-j(\omega_i - \omega_r) n} \end{aligned} \tag{10}$$

From the adaptive filtering point of view, the desired signal can therefore be written as

$$\begin{aligned} d_n &= w_{\mathrm{TV},n}^H r_n + q_n \\ &= \frac{A_i}{A_r} e^{j(\phi_i - \phi_r)} e^{j(\omega_i - \omega_r) n} r_n + q_n \end{aligned} \tag{11}$$

For maximum NLMS stepsize, i.e. $\bar{\mu} = 1$, the *a posteriori* weight vector forces the *a posteriori* error to zero. This implies

$$\begin{aligned} \hat{d}_n &= w_{\mathrm{AF},n+1}^H r_n \\ &= d_n \end{aligned} \tag{12}$$

Comparing (11) and (12), we note that when the signal-to-interference ratio (SIR) is low, the *a posteriori* adaptive filter weight tracks the hypothetical – and optimal – time-varying weight in (11).

$$w_{\mathrm{AF},n} = \frac{A_i}{A_r} e^{-j(\phi_i - \phi_r)} e^{-j(\omega_i - \omega_r)(n-1)} \tag{13}$$

With the latter time-varying weight, the adaptive filter operation amounts to a modulation of the reference [5], together with a correction in its amplitude and phase, in an attempt to cancel the interference signal.

Under the above circumstances, the error incurred by the adaptive filtering operation results from the fact that the tracking is very good *a posteriori*, i.e. mostly lag error is incurred. An expression for the steady-state *a priori* error can be found [3], which includes the dependence on stepsize $\bar{\mu}$.

$$
\begin{aligned}
e_n &= i_n - w_{\mathrm{AF},n}^H r_n \\
&= A_i e^{j\phi_i} e^{j\omega_i n} \frac{1 - e^{-j(\omega_i - \omega_r)}}{1 - (1 - \bar{\mu}) e^{-j(\omega_i - \omega_r)}} \qquad (14) \\
&= i_n \frac{1 - e^{-j(\omega_i - \omega_r)}}{1 - (1 - \bar{\mu}) e^{-j(\omega_i - \omega_r)}}
\end{aligned}
$$

Two important cases can be distinguished here, depending on whether the interference and reference frequencies are the same or different. When the interference and reference frequencies are the same, i.e. no Doppler-shift is present, the NLMS weight loses its time dependence. Consequently, NLMS with small stepsize can converge to the optimal solution given in (10) and the corresponding error in (14) goes to zero. For large stepsize, NLMS is subject to misadjustment, caused by the presence of the wideband signal-of-interest $q_n$ acting as noise, and the interference cancellation is less effective.

When the interference and reference frequencies are different, as when Doppler-shift is present, and the stepsize is small, the *a priori* error in (14) approaches the interference itself, while the adaptive filter output approaches the Wiener solution to the problem, i.e. weights equal to zero. Consequently, the signal-of-interest can not be recovered from the adaptive filter error. For a stepsize equal to one, the adaptive filter output lags one step behind in producing an estimate of the interference. As a result, the interference is not canceled effectively and – being large relative to the SOI – the latter still cannot be recovered.

## 4. TWO-STAGE ANC-NLMS

Under the Doppler-shifted condition, we saw that the NLMS adaptive filter with $\bar{\mu} = 1$ produced an estimate of the interference signal, albeit at a lag of one sample. While this estimate cannot be used to cancel the interference, it can be used as the reference signal for an ANC in the classical mode, as shown in Fig. 1. The latter realization leads to the two-stage ANC-NLMS approach proposed here: first, get a good estimate of the interference in spectral terms, i.e. one that is frequency-locked to the interference; second, use the classical ANC setup to produce an adaptive filter output that is also amplitude and phase locked to the interference. The latter then provides for interference cancellation. Fig. 2 depicts the proposed approach.

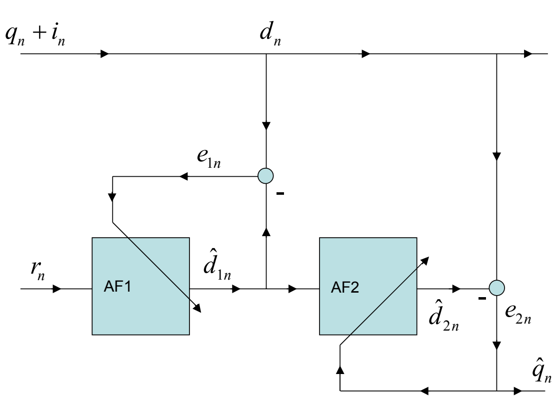


Fig. 2: Two-Stage ANC-NLMS.

AF1, the first adaptive filter, produces an estimate which is a modulated version of the Doppler-shifted reference signal. As seen above, AF1 is used with stepsize $\bar{\mu} = 1$ and therefore rapidly adapting to any changes in the interference signal. In fact, under the given circumstances, AF1 adapts in a single step [3]. AF2 is used in the conventional ANC mode, with small stepsize.

## 5. SIMULATION RESULTS

In order to show the efficacy of the proposed approach, let the wideband signal-of-interest, $q_n$, be a QPSK signal. The interference signal is a complex exponential with frequency $f_i = 0.1\overline{66}$, while the Doppler-shifted reference signal has frequency $f_r = 0.17\overline{6}$. The AF1 stepsize, $\bar{\mu}_1$, equals one, while the AF2 stepsize, $\bar{\mu}_2$, is set to 0.01 for this simulation. Both AF1 and AF2 are 10-tap filters. A simple detector is used; one that assigns as detected (estimated) symbol the one from the signal constellation that is closest to it.

For (signal-to-interference ratio) SIR = –40 dB, the behavior of the AF2 error minus the corresponding QPSK estimate is shown in Fig. 3, indicating - in the top figure - that it takes about 500 symbols to convergence, i.e. to reach steady-state. We find a 0.0722 bit-error-rate (BER) based on symbols 1 through 5,000.

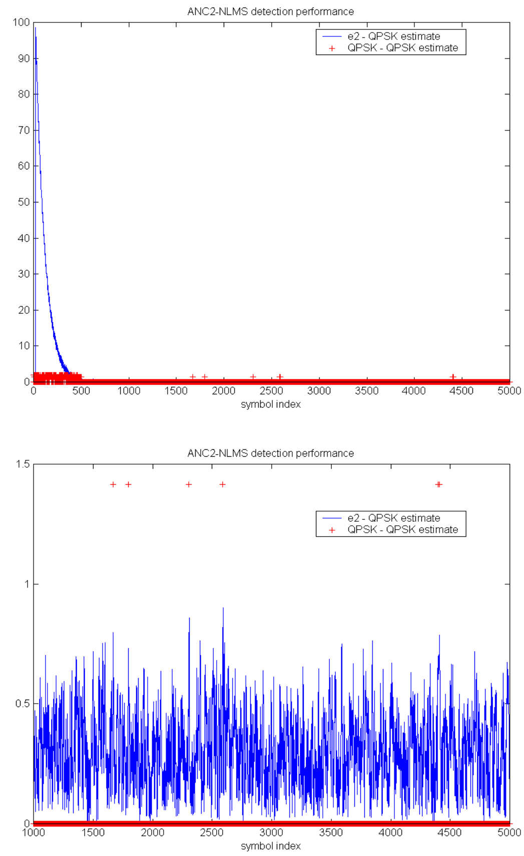


Fig. 3: Two-Stage ANC-NLMS Performance, SINR = –40 dB.

In order to readily observe the occurrence of bit errors, we also graph QPSK–QPSK estimate. In the bottom graph of Fig. 3 we note that only a few bit errors occur (QPSK–QPSK estimate ≠ 0) during steady-state; in fact, steady-state BER = 0.0025.

Analogously, for SINR = –20 dB, the results in Fig. 4 were obtained. In the top figure we note that steady-state behavior is reached sooner, in approximately 300 symbols. A BER of 0.0376 is found over symbols 1 through 5,000. Over the steady-state interval with symbols 1,000 through 5,000 BER is $9.9975_{10}$-4.

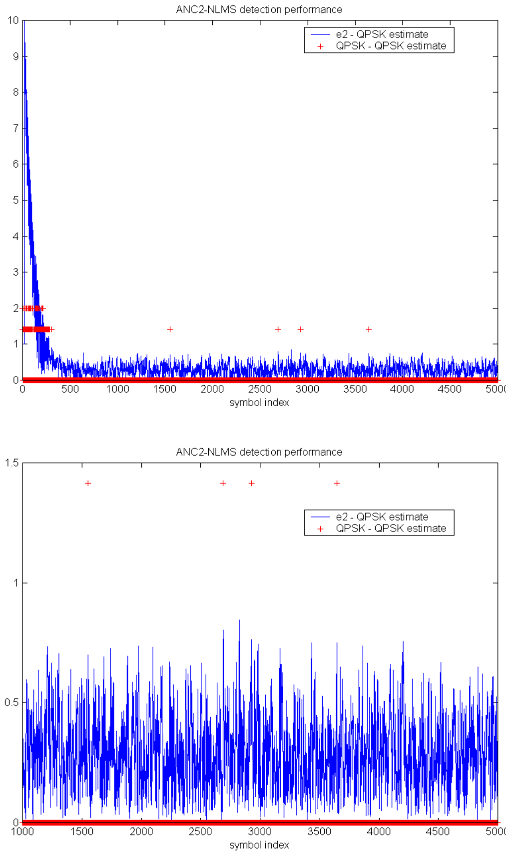


Fig. 4: Two-Stage ANC-NLMS Performance, SINR = –20 dB.

The results for SIR = 0 dB are shown in Fig. 5. Steady-state is reached in approximately 150 symbols. The overall BER was 0.0076, while over the steady-state interval of symbols 1,000 through 5,000 we found a BER of $4.9988_{10}$-4.

Based on these limited experiments, we saw the convergence rate cut by a factor of 2 for every 20 dB increase in SIR. We observed also that the steady-state BER is cut by a factor of two for every 20 dB increase in SIR.

Note that the above BER are associated with individual bits, i.e. the symbols represent individual bits. Consequently, if the wideband signal-of-interest (SOI) is a CDMA type signal, we can expect the symbol-error-rate to be much less than the BER.

To illustrate the dependence of the two-stage approach on the AF2 stepsize, we repeat the SIR = –40 dB experiment with $\bar{\mu}_2$ changed to 0.1. While steady-state behavior is now reached very quickly, in fewer than 100 symbols, the overall and steady-state BER are now 0.0536 and 0.0437 respectively. The latter may be a result of the wider notch bandwidth [5] producing more severe distortion of the SOI component.

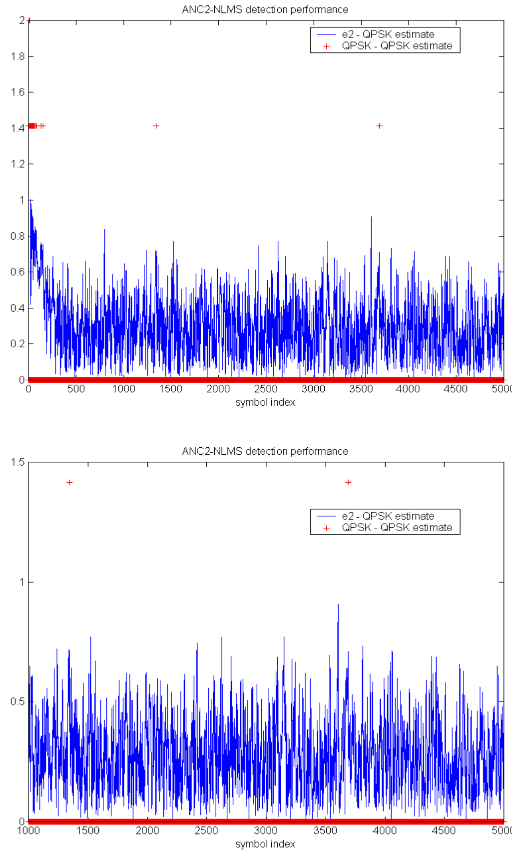


Fig. 5: Two-Stage ANC-NLMS Performance, SINR = 0 dB.

## 6. CONCLUSION

We proposed to use the time-varying weight behavior of NLMS, when its stepsize is maximum, to produce a reference signal that is frequency-locked to an interference impinging on a signal-of-interest, for the case where the available reference is a Doppler-shifted version of the interference signal. Simulations show the efficacy of the proposed two-stage ANC-NLMS approach for QPSK symbols subject to strong sinusoidal interference.

## 7. REFERENCES

[1] J. R. Glover, "Adaptive noise canceling applied to sinusoidal interference," IEEE Trans. Acoust., Speech, Signal Processing, ASSP-25, pp. 484-491, Dec. 1977.

[2] B. Widrow, J. Glover, J. McCool, J. Kaunitz, C. Williams, R. Hearn, J. Zeidler, E. Dong, Jr., and R. Goodin, "Adaptive noise canceling: principles and applications", Proc. IEEE, 63, pp. 1692-1716, December 1975.

[3] A. A. (Louis) Beex and James R. Zeidler, "Steady-State Dynamic Weight Behavior in (N)LMS Adaptive Filters," in Advances in LMS Adaptive Filters, eds. S. Haykin and B. Widrow, John Wiley & Sons, to appear 2002/3.

[4] G. C. Goodwin and K. S. Sin, Adaptive Filtering, Prediction, and Control, Prentice-Hall, 1984.

[5] M. J. Shensa, "Non-Wiener solutions of the adaptive noise canceler with a noisy reference," IEEE Trans. Acoust., Speech, Signal Processing, ASSP-28, pp. 468-473, August 1980.