A GRADIENT ALGORITHM FOR PHASE-ONLY ADAPTIVE FIR FILTERING

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ABSTRACT

In this paper, we describe a simple gradient algorithm for adapting the phase response of an FIR filter whose magnitude response is already specified. The algorithm is an extension of a previously-developed gradient adaptive algorithm for allpass filtering. The algorithm directly adjusts the impulse response of the FIR filter using input and desired response signals in a simple manner without the need for frequency-domain processing or coefficient monitoring. A stationary point analysis of the algorithm verifies its desirable estimation capabilities. Simulations confirm the capability of the algorithm in a filter modeling task.

1. INTRODUCTION

Since their development over forty years ago, adaptive filters have found wide use in a variety of communications, signal processing, and control applications. The simplest adaptive filters adjust the coefficients of an FIR filter to match its output signal to a desired response signal in a mean-square sense. This adaptation strategy produces a time varying filter whose magnitude and phase characteristics in the frequency domain change according to the statistical relationship between the input and desired response signals.

In some applications, one desires an adaptive filter that estimates the phase relationship between the input and desired response signals. Such a phase-only adaptive algorithm would maintain the magnitude response of the adaptive filter as specified through a target filter. Applications where such a procedure would be useful include (i) phase equalization for spatial audio sound reproduction [1, 2], (ii) plant identification for adaptive control, in which the phase response of the plant model is critical to the success of the control scheme [3], and (iii) null steering for adaptive beamforming [4, 5]. To our knowledge, no procedure for adapting the phase response of an FIR filter with an arbitrary magnitude response has been described in the literature.

In this paper, we present a gradient adaptive algorithm for adjusting the phase response of a frequency-selective FIR filter using input and desired response signals. The algorithm is an extension of a recently-proposed technique for adaptive allpass filtering [6]. The algorithm is simple, requiring about four multiplies per adaptive filter coefficient per iteration to implement. We provide an analysis of the algorithm to show that its only stable stationary point corresponds to the desired frequency-domain magnitude and phase characteristics of the chosen task. Simulations verify that the algorithm performs its designated task.

2. PHASE-ONLY ADAPTATION UNDER AUTOCORRELATION CONSTRAINTS

The adaptive algorithm described in this paper uses extensions and modifications of previously-described gradient techniques for adaptive paraunitary filter banks and allpass filters [6, 7]. The algorithmic basis for the new method is now described. Let w_l denote the possibly-complex-valued impulse response of a doubly-infinite IIR filter, where $-\infty < l < \infty$, and let g_l be any finite-energy sequence. Define the discrete-time convolution of two sequences u_l and v_l as

$$u_l * v_l \quad = \sum_{i=-\infty}^{\infty} u_i v_{l-i}. \tag{1}$$

The algorithm derived in this paper relies on the following continuous-time differential update for the discrete-time impulse response sequence w_l first described in [7]:

$$\frac{dw_l}{dt} \quad = \quad w_l * w_{-l}^{\dagger} * g_l - w_l * g_{-l}^{\dagger} * w_l, \tag{2}$$

where $\cdot^{\dagger}$ denotes complex conjugate. We now prove an important invariance property of the update in (2).

Theorem 1: *The update in (2) satisfies*

$$\frac{d}{dt}\left\{w_l * w_{-l}^{\dagger}\right\} \quad = \quad \frac{d}{dt}\left\{\sum_{i=-\infty}^{\infty} w_i w_{l+i}^{\dagger}\right\} \;=\; 0. \tag{3}$$

Proof: Taking derivatives of the quantity $\{w_l * w_{-l}^{\dagger}\}$,

$$\frac{d}{dt}\left\{w_l * w_{-l}^{\dagger}\right\} \quad = \quad \frac{dw_l}{dt} * w_{-l}^{\dagger} + w_l * \frac{dw_{-l}^{\dagger}}{dt}. \tag{4}$$

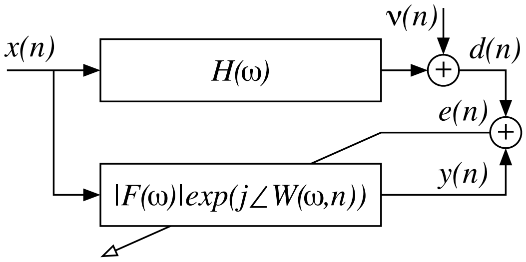

Fig. 1: Phase-only adaptive filtering.

Substituting (2) into (4) and using the distributive and commutative properties of the convolution operation, we obtain

$$\begin{aligned}\frac{d}{dt}\left\{w_l * w_{-l}^{\dagger}\right\} & \\ = \;& \{w_l * w_{-l}^{\dagger} * g_l - w_l * g_{-l}^{\dagger} * w_l\} * w_{-l}^{\dagger} \\ & + w_l * \{w_{-l}^{\dagger} * w_l * g_{-l}^{\dagger} - w_{-l}^{\dagger} * g_l * w_{-l}^{\dagger}\} \quad (5) \\ = \;& w_l * w_{-l}^{\dagger} * w_{-l}^{\dagger} * g_l - w_l * w_l * w_{-l}^{\dagger} * g_{-l}^{\dagger} \\ & + w_l * w_l * w_{-l}^{\dagger} * g_{-l}^{\dagger} - w_l * w_{-l}^{\dagger} * w_{-l}^{\dagger} * g_l \quad (6) \\ = \;& 0. \quad (7)\end{aligned}$$

Discussion: The above theorem can be interpreted as follows. If the impulse response $\{w_l(t)\}$ is adapted over the open interval $t \geq 0$ using (2), its discrete-time autocorrelation function does not change with t; that is,

$$\sum_{l=-\infty}^{\infty} w_l(t) w_{-l}^{\dagger}(t) \;=\; \sum_{l=-\infty}^{\infty} w_l(0) w_{-l}^{\dagger}(0). \qquad (8)$$

Define the time-varying Fourier transform of $\{w_l(t)\}$ as

$$W(\omega, t) \;=\; \sum_{l=-\infty}^{\infty} w_l(t) e^{-j\omega l}. \qquad (9)$$

Then, (8) can be expressed as

$$|W(\omega, t)|^2 \;=\; |W(\omega, 0)|^2. \qquad (10)$$

The frequency response of the filter remains fixed during adaptation; only the phase response of the filter changes.

3. PHASE-ONLY ADAPTIVE FILTERS

We now show how the update in (2) can be used to solve the phase-only adaptive filtering task. Fig. 1 shows the nature of this task, in which a desired response signal $d(n)$ is created from an input signal $x(n)$ as

$$d(n) \;=\; \sum_{i=0}^{\infty} h_i x(n-i) + \nu(n), \qquad (11)$$

where $\{h_i\}$ is the impulse response of an unknown filter with frequency response $H(\omega)$ and $\nu(n)$ is an uncorrelated noise signal with variance σ_ν^2. We desire a phase-only adaptive filter with impulse response $w_l(n)$, $0 \leq l \leq L$ to

$$\begin{aligned} \text{minimize} \quad & E\{|e(n)|^2\} \qquad (12) \\ \text{such that} \quad & |W(\omega, n)|^2 = |F(\omega)|^2 \qquad (13)\end{aligned}$$

where

$$e(n) \;=\; d(n) - y(n) \qquad (14)$$

$$y(n) \;=\; \sum_{l=0}^{L} w_l(n) x(n-l) \qquad (15)$$

$$W(\omega, n) \;=\; \sum_{l=0}^{L} w_l(n) e^{-j\omega l} \qquad (16)$$

$$F(\omega) \;=\; \sum_{m=0}^{M} f_m e^{-j\omega m} \qquad (17)$$

and $\{f_m\}$ is the impulse response of a chosen target filter.

It can be shown under the constraint in (13) that

$$\begin{aligned} E\{|e(n)|^2\} \;=\; & E\{|d(n)|^2\} - 2\Re e E\{d(n) y^{\dagger}(n)\} \\ & + E\{|x_f(n)|^2\} \qquad (18) \end{aligned}$$

$$x_f(n) \;=\; \sum_{m=0}^{M} f_m x(n-m). \qquad (19)$$

Thus, (12)–(13) is equivalent to the following task:

$$\begin{aligned} \text{maximize} \quad & \Re e E\{d(n) y^{\dagger}(n)\} \qquad (20) \\ \text{such that} \quad & |W(\omega, n)|^2 = |F(\omega)|^2. \qquad (21)\end{aligned}$$

The constraint in (21) is identical in form to (10). Moreover, the gradient of the cost function in (20) is

$$g_l \;=\; E\{d(n) x^{\dagger}(n-l)\} \qquad (22)$$

Thus, the differential procedure in (2) is relevant to the phase-only adaptive filtering task.

To develop a practical procedure for the phase-only adaptive filtering task, several issues need to be addressed:

- A numerically-stable discretized version of the differential update in (2) is required.
- The doubly-infinite impulse response $\{w_l\}$ must be truncated to a finite-length.
- The causality of the coefficient updates needs to be maintained.

Similar issues have arisen in adaptive algorithms for spatio-temporal principal and minor subspace analysis [7] single-channel blind deconvolution [8], and adaptive time-delay estimation [9]. We can use similar modifications to develop our desired procedure. In particular,

- We approximate the differential update in (2) with a discrete-time update, in which the finite differences $\{w_l(n+1) - w_l(n)\}/\mu$ replace differentials dw_l/dt, where μ is the step size.
- We truncate the doubly-infinite IIR filter $\{w_l\}$ to finite length, such that $w_l(n)$ is non-zero only for $0 \leq l \leq L$. The output signal $y(n)$ defined in (15) reflects this choice.
- We assume that the adaptation speed of the algorithm is slow enough such that

$$w_l(n) \approx w_l(n-1) \approx \cdots \approx w_l(n-L) \quad (23)$$

within certain filtered gradient update terms.

The complete details of these modifications are omitted for brevity, and only the final algorithm form is given:

$$w_l(n+1) = w_l(n) + \mu[d_f(n-L)x_f^\dagger(n-L-l) - y(n-L)u^\dagger(n-l)] \quad (24)$$

$$d_f(n) = \sum_{m=0}^{M} f_m d(n-m). \quad (25)$$

$$u(n) = \sum_{q=0}^{L} w_{L-q}^\dagger(n) d(n-q). \quad (26)$$

Equations (15), (19), and (24)–(26) describe our phase-only adaptive FIR filtering algorithm. If $M \ll L$, the complexity of this algorithm is approximately four multiply/adds per adaptive filter coefficient at each time instant.

Remark 1: The above algorithm is most-closely related to the modified phase-only differential update given by

$$\frac{dw_l}{dt} = f_l * f_{-l}^\dagger * g_l - w_l * g_{-l}^\dagger * w_l. \quad (27)$$

The only difference between (2) and (27) is the use of the target filter impulse response f_l in place of w_l in the first update term.

Remark 2: Although the target filter impulse response $\{f_m\}$ is described by a finite-impulse response filter, an infinite-impulse response filter can be chosen instead. In such cases, $x_f(n)$ and $d_f(n)$ are generated using an IIR filter defined by the system function $F(z) = B(z)/A(z)$, in which the target magnitude response is $|F(\omega)| = |B(\omega)/A(\omega)|$.

4. ANALYSIS

In this section, we provide a stationary point analysis of the proposed phase-only adaptive FIR filtering algorithm. This analysis uses the ordinary differential equation (ODE) of the algorithm in (27) expressed in the frequency domain, where g_l is as defined in (22). It is straightforward to show for stationary input and desired response signals that

$$g_l = \sum_{i=0}^{\infty} h_i r_{xx}(l-i), \quad (28)$$

where $r_{xx}(k) = E\{x(n)x^\dagger(n-k)\}$. Taking the Fourier transform of both sides of (27), we obtain

$$\frac{dW(\omega)}{dt} = |F(\omega)|^2 G(\omega) - W(\omega)G^\dagger(\omega)W(\omega). \quad (29)$$

Finally, employing the Fourier transform of g_l in (28) and defining $S_{xx}(\omega)$ as the power spectrum of $x(n)$,

$$\frac{dW(\omega)}{dt} = |F(\omega)|^2 H(\omega) S_{xx}(\omega) - W(\omega)S_{xx}(\omega)H^\dagger(\omega)W(\omega). \quad (30)$$

The following two theorems relate to (30).

Theorem 2: If $S_{xx}(\omega) \neq 0$ and $H(\omega) \neq 0$, the stationary points of (30) are

$$W(\omega) = \pm|F(\omega)|e^{j\angle H(\omega)}, \quad (31)$$

where $\angle H(\omega)$ is the phase of $H(\omega)$.

Theorem 3: The only stable stationary point of (30) is

$$W(\omega) = |F(\omega)|e^{j\angle H(\omega)}. \quad (32)$$

Proof of Theorem 2: The stationary points of (30) are defined by $dW(\omega)/dt = 0$. Setting the right-hand-side of (30) to zero yields

$$W(\omega)S_{xx}(\omega)H^\dagger(\omega)W(\omega) = |F(\omega)|^2 H(\omega)S_{xx}(\omega). \quad (33)$$

Assume that $S_{xx}(\omega)$ and $H(\omega)$ are both non-zero. We can simplify (33) to obtain

$$W^2(\omega) = |F(\omega)|^2 e^{j2\angle H(\omega)}. \quad (34)$$

Taking the square root of both sides of (34) produces (31).

Proof of Theorem 3: To determine the stability of (30) at the stationary points in (31), we must evaluate the Hessian of the coefficient update equation. Since the quadratic term within the update in (30) is identical to that of Eqn. (15) of [6], we can use the second-order condition from [6] directly:

$$\Re e\left[W(\omega)|G(\omega)|e^{-j\angle G(\omega)}\right] > 0. \quad (35)$$

Substituting the Fourier transform of g_l and the stationary points in (31) into the left-hand side of (35) and noting that both $S_{xx}(\omega) \neq 0$ and $H(\omega) \neq 0$, we find that only (32) satisfies this condition.

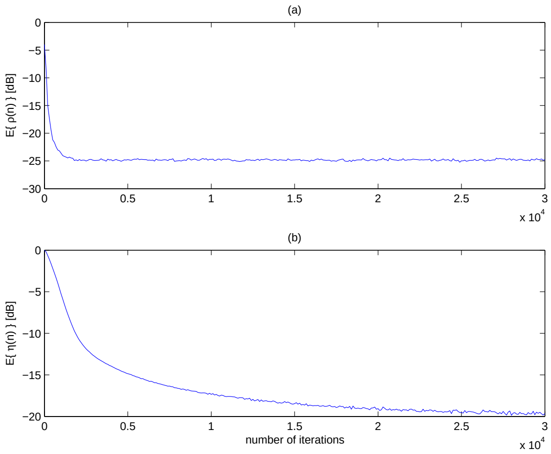

Fig. 2: Convergence of (a) $E\{\rho(n)\}$ and (b) $E\{\eta(n)\}$.

Remark 3: Since $S_{xx}(\omega) \neq 0$ and $H(\omega) \neq 0$, both the input and desired response signals must have non-zero energy at all spectral frequencies for the algorithm to function properly. In situations where either $x(n)$ or $d(n)$ lack signal energy, the proposed algorithm may not accurately identify either $|F(\omega)|$ or $\angle H(\omega)$. Our initial studies of these performance issues indicate that the algorithm accurately estimates $\angle H(\omega)$ in frequency regions where $|F(\omega)H(\omega)|$ is large, a generally desirable characteristic.

5. SIMULATIONS

We now explore the behavior of the proposed phase-only adaptive FIR filter through simulations. In these simulations, the filter $F(\omega)$ is a fourth-order Chebyshev Type 1 lowpass filter with a bandwidth of 0.15π and 0.2dB of passband ripple. We let $x(n)$ and $\nu(n)$ be zero-mean uncorrelated Gaussian signals with $E\{|x(n)|^2\} = 1$ and $E\{|\nu(n)|^2\} = 0.01$, and define $d(n) = x(n-D)+\nu(n)$, where $D = 20$. The goal of the adaptive filter is to approximate the magnitude frequency response of the Chebyshev filter while maintaining linear phase across its passband. The adaptive filter's parameters are $L = 39$, $\mu = 0.001$, and $w_l(0) = 0$. We calculate the performance factors

$$\rho(n) = \frac{1}{P}\left(\sum_{i=0}^{P-1} |\angle W(\omega_i) + [D\omega_i]|^2\right)^{1/2} \quad (36)$$

$$\eta(n) = \left(\sum_{i=0}^{Q-1} [|W(\omega_i)| - |F(\omega_i)|]^2 / \sum_{j=0}^{Q-1} |F(\omega_j)|^2\right)^{1/2} \quad (37)$$

where $P = 20$, $Q = 60$ and $\omega_i = 2\pi i/120$. With these choices, $\rho(n)$ measures the adaptive filter's distance from phase linearity across the filter passband, whereas $\eta(n)$ measures the error in the adaptive filter's magnitude response across the entire frequency band.

Shown in Fig. 2(a) and (b) are the evolutions of $E\{\rho(n)\}$ and $E\{\eta(n)\}$ as estimated from ensemble averages of one hundred simulation runs. The low values of these averaged performance factors near the end of the simulation runs indicates that the proposed adaptive filter does converge to a linear phase filter whose magnitude response accurately matches that of the target filter. Fig. 3(a) and (b) show the magnitude and phase responses of the filter at the end of one of the simulation runs. As can be seen, $\angle W(\omega)$ is linear over the passband, and $|W(\omega)| \approx |F\omega)|$ over the entire frequency range.

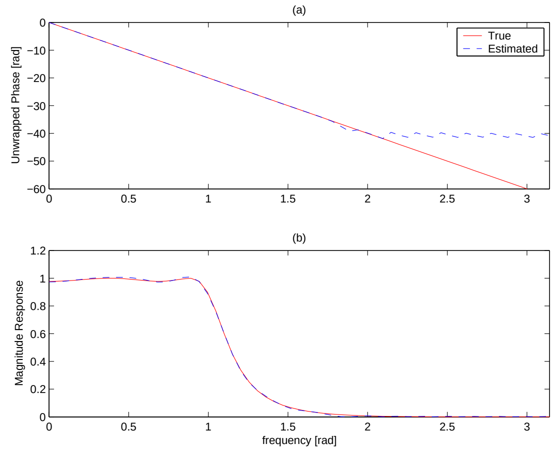

Fig. 3: Comparison of the (a) unwrapped phases and (b) magnitude responses for the true and estimated filters.

6. CONCLUSIONS

This paper presents a novel time-domain algorithm for phase only adaptive filtering, in which the magnitude response of the adaptive filter is specified via a target filter. A stationary point analysis of the algorithm verifies the capabilities of the scheme. Simulations show that the algorithm performs its desired task.

7. REFERENCES

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