ADAPTIVE ACTIVE NOISE CONTROL WITHOUT SECONDARY PATH IDENTIFICATION

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ABSTRACT

Fully adaptive feedforward control algorithm is proposed for general active noise control (ANC) when all the path dynamics are uncertain. To reduce the actual canceling error, two virtual errors are introduced and are forced into zero by adjusting three adaptive FIR filters in an on-line manner, which can result in the canceling at the objective point. Unlike other conventional approaches, the algorithm does not need exact identification of the secondary paths, and requires neither any dither signal nor the PE property of the source noise. The effectiveness of the proposed algorithm is validated in experiments of ANC using an air duct.

1. INTRODUCTION

Active noise control (ANC) is efficiently used to suppress unwanted low frequency noises generated by primary sources by emitting artificial secondary sounds to objective points [1][2]. Adaptive feedforward control schemes using primary noises information measured by reference microphones are effective to ANC, since the noise path dynamics cannot be precisely modeled and may be uncertainly change. A variety of filtered-x algorithm have been adopted to attain the feedforward adaptation [2]-[6]. On the assumption that the secondary path dynamics are known a priori, the stability and convergence of the adaptive algorithm has also been investigated in [4][6].

To deal with a general case when the secondary path dynamics are also unknown, almost previous works are based on indirect adaptive approaches which employ on-line identification of the secondary path dynamics. By utilizing the identified path models, the filtered-x algorithms are updated [3]-[8], or the feedforward controller can also be redesigned in a real-time manner [6]. However, the former schemes are not stability-guaranteed and the latter suffers from computational burden. Thus the indirect adaptive approaches need the exact identification of the secondary paths, and then additional dither sounds are needed to secure the persistently exciting (PE) condition for the identification.

The aim of this paper is to propose a fully direct adaptive control approach which does not need explicit identification of the secondary path dynamics. To reduce the actual canceling error, two virtual errors are introduced and are forced into zero by adjusting three adaptive FIR filters in an on-line manner, which enables the noise cancellation at the objective point. Unlike the previous methods, neither dither signals nor the PE property of the source noise are required.

2. STRUCTURE OF NEW ADAPTIVE ANC

2.1. Feedforward active noise control

Fig.1 illustrates a new active noise control system which includes the proposed adaptive algorithm. The primary source noise $s(k)$ is detected by the reference microphone as $r(k)$, which is used as the input to the adaptive feedforward controller $\hat{C}(z,k)$. The controller calculates the artificial control sound $u(k)$ to be emitted from the the secondary loudspeaker to cancel the source noise at the position of the error microphone. The canceling error at the point is denoted by $e_c(k)$. All of the signals are characterized by four path dynamics, in which $G_1(z)$ and $G_2(z)$ represent the primary path dynamics, and $G_3(z)$ and $G_4(z)$ the secondary path dynamics, respectively. Since all of the path dynamics may contain model uncertainty and changeability, the adaptive approaches are essentially needed in such uncertain situations. It follows from Fig.1 that

$$e_c(k) = G_1(z)s(k) - G_4(z)u(k) \quad (1a)$$
$$u(k) = C(z)r(k) \quad (1b)$$
$$r(k) = G_2(z)s(k) + G_3(k)u(k) \quad (1c)$$

where $C(z)$ be an FIR type of feedforward controller. Let $\hat{C}(z.k)$ be an adaptive version of $C(z)$, then the secondary control sound is generated by

$$u(k) = \hat{C}(z,k)r(k) \quad (2)$$

The canceling error $e_c(k)$ can be expressed from (1) as

$$e_c(k) = \bar{G}_1(z)r(k) - \bar{G}_4(z)u(k) \quad (3)$$

where $\bar{G}_1(z) \equiv G_1(z)/G_2(z)$ and $\bar{G}_4(z) \equiv G_4(z) + G_1(z) \cdot G_3(z)\,/G_2(z)$.

2.2. Key idea for new adaptation algorithms

We give a new direct adaptive algorithm which does not need explicit identification of the secondary path dynamics, unlike the ordinary filtered-x algorithms using the identified model of $\bar{G}_4(z)$. The basic structure of the proposed adaptive feedforward control algorithm is illustrated in Fig.1. From the figure, $e_c(k)$, $e_1(k)$, $e_2(k)$ can be expressed as:

$$e_c(k) = \overline{G}_1(z)r(k) - \overline{G}_4(z)u(k) \quad (4a)$$
$$e_1(k) = e_c(k) + \hat{K}(z,k)u(k) - \hat{D}(z,k)r(k) \quad (4b)$$
$$e_2(k) = \hat{D}(z,k)r(k) - \hat{C}(z,k)x(k) \quad (4c)$$

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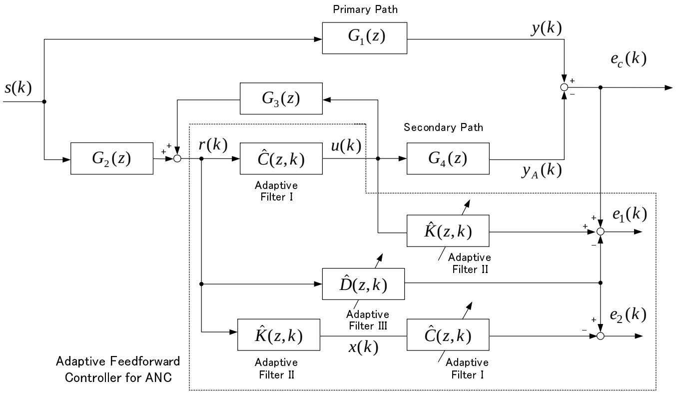


Fig. 1. Direct fully adaptive algorithm for ANC

where $\overline{G}_1(z)$ and $\overline{G}_4(z)$ are defined in the previous section, and the control input $u(k)$ and the auxiliary signal $x(k)$ are also defined as

$$u(k) = \hat{C}(z,k)r(k) \tag{5}$$

$$x(k) = \hat{K}(z,k)r(k) \tag{6}$$

It follows from Fig.1 and the above definitions that

$$\begin{aligned} e_1 + e_2(k) &= [e_c(k) + \hat{K}(z,k)u(k) - \hat{D}(z,k)r(k)] \\ &\quad + [\hat{D}(z,k)r(k) - \hat{C}(z,k)x(k)] \\ &= e_c(k) + [\hat{K}(z,k)\hat{C}(z,k) - \hat{C}(z,k)\hat{K}(z,k)]r(k) \end{aligned} \tag{7}$$

Thus if $e_1(k)$ and $e_2(k) \to 0$ for $k \to \infty$ is satisfied and the FIR parameters of $\hat{C}(z,k)$ and $\hat{K}(z,k)$ converge to any constants, then the second and third terms in the right hand side of (7) can be cancelled, then by the relation $e_1(k) + e_2(k) = e_c(k)$, thus it can also be attained that $e_c(k) \to 0$.

It seems that $\hat{D}(z,k)$ and $\hat{K}(z,k)$ are the identified models for $\overline{G}_1(z)$ and $\overline{G}_4(z)$ respectively and the adaptive controller $\hat{C}(z,k)$ is adjusted according to the identified models of the secondary path dynamics. However, even when the source noise does not satisfy the PE property, the proposed algorithm does not require the convergence of the adjusted parameters to their true values, but only the convergence of their parameters to any constants such that the errors $e_1(k)$ and $e_2(k)$ can converge to zero. Therefore, any probing signal is not needed unlike the indirect adaptive algorithm. The degradation and complexity caused by the dither signals can also be overcome by the proposed direct adaptive algorithm.

3. NEW DIRECT ADAPTIVE ALGORITHMS

3.1. Expression of error system

We here give the details of the direct adaptive algorithm to update the parameters of three FIR adaptive filters:

$$\hat{C}(z,k) = \hat{c}_1(k)z^{-1} + \cdots + \hat{c}_{L_C}(k)z^{-L_C} \tag{8a}$$

$$\hat{K}(z,k) = \hat{k}_1(k)z^{-1} + \cdots + \hat{k}_{L_K}(k)z^{-L_K} \tag{8b}$$

$$\hat{D}(z,k) = \hat{d}_1(k)z^{-1} + \cdots + \hat{d}_{L_D}(k)z^{-L_D} \tag{8c}$$

where the parameter vectors and regressor vectors are defined by $\hat{\boldsymbol{\theta}}_C(k) = [\hat{c}_1(k), \ldots, \hat{c}_{L_C}(k)]^T$, $\hat{\boldsymbol{\theta}}_K(k) = [\hat{k}_1(k), \ldots, \hat{k}_{L_K}(k)]^T$, $\hat{\boldsymbol{\theta}}_D(k) = [\hat{d}_1(k), \ldots, \hat{d}_{L_D}(k)]^T$, $\boldsymbol{\varphi}(k) = [x(k-1), \ldots, x(k-L_C)]^T$, $\boldsymbol{\zeta}(k) = [u(k-1), \ldots, u(k-L_K)]^T$ and $\boldsymbol{\xi}(k) = [r(k-1), \ldots, r(k-L_D)]^T$.

From the notations, $e_1(k)$ and $e_1(k)$ are expressed as

$$\begin{aligned} e_1(k) &= e_c(k) + \hat{K}(z,k)u(k) - \hat{D}(z,k)r(k) \\ &= e_c(k) - \left[\begin{array}{cc} \boldsymbol{\xi}^T(k) & -\boldsymbol{\zeta}^T(k) \end{array}\right] \left[\begin{array}{c} \hat{\boldsymbol{\theta}}_D(k) \\ \hat{\boldsymbol{\theta}}_K(k) \end{array}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} e_2(k) &= \hat{D}(z,k)r(k) - \hat{C}(z,k)x(k) \\ &= -\left[\begin{array}{cc} -\boldsymbol{\xi}^T(k) & \boldsymbol{\varphi}^T(k) \end{array}\right] \left[\begin{array}{c} \hat{\boldsymbol{\theta}}_D(k) \\ \hat{\boldsymbol{\theta}}_C(k) \end{array}\right] \end{aligned} \tag{10}$$

The control input $u(k)$ and the auxiliary signal $x(k)$ are given by the filters with the delayed updated parameters $\hat{\boldsymbol{\theta}}_C(k-l_c)$ and $\hat{\boldsymbol{\theta}}_K(k-l_k)$ to avoid the correlation of the current updated parameters, as

$$u(k) = \hat{\boldsymbol{\theta}}_C^T(k-l_c)\boldsymbol{\xi}(k), \quad x(k) = \hat{\boldsymbol{\theta}}_K^T(k-l_k)\boldsymbol{\xi}(k)$$

Rewriting (9) and (10) into a compact form, the error system can be described as

$$\boldsymbol{e}(k) = \mathbf{e}_0(k) - \boldsymbol{\Phi}^T(k)\hat{\boldsymbol{\theta}}(k) + \boldsymbol{v}(k) \tag{11}$$

where

$$\boldsymbol{e}(k) = \left[\begin{array}{c} e_1(k) \\ e_2(k) \end{array}\right] \quad \mathbf{e}_0(k) = \left[\begin{array}{c} e_c(k) \\ 0 \end{array}\right],$$

$$\boldsymbol{\Phi}^T(k) = \left[\begin{array}{ccc} \boldsymbol{\xi}^T(k) & -\boldsymbol{\zeta}^T(k) & 0 \\ -\boldsymbol{\xi}^T(k) & 0 & \boldsymbol{\varphi}^T(k) \end{array}\right],$$

$$\hat{\boldsymbol{\theta}}(k) = \left[\begin{array}{c} \hat{\boldsymbol{\theta}}_D(k) \\ \hat{\boldsymbol{\theta}}_K(k) \\ \hat{\boldsymbol{\theta}}_C(k) \end{array}\right] \quad \boldsymbol{v}(k) = \left[\begin{array}{c} v_1(k) \\ v_2(k) \end{array}\right]$$

where $\boldsymbol{v}(k)$ is an uncertain term including the truncation error in the FIR models, which is assumed to be bounded for stability assurance, as

$$|v_1(k)| \le \beta_1, \quad |v_2(k)| \le \beta_2 \tag{12}$$

3.2. New robust adaptive algorithm

The following adaptive algorithm is derived from the error system (11):

$$\hat{\boldsymbol{\theta}}(k+1) = \hat{\boldsymbol{\theta}}(k) + \gamma(k)\boldsymbol{\Phi}(k)\bar{\boldsymbol{e}}(k) \tag{13a}$$

$$\gamma(k) = \frac{2\alpha\|\bar{\boldsymbol{e}}(k)\|^2}{d + \|\boldsymbol{\Phi}(k)\bar{\boldsymbol{e}}(k)\|^2} \tag{13b}$$

$$\bar{\boldsymbol{e}} = [\bar{e}_1(k) \;\; \bar{e}_2(k)]^T$$

$$\bar{e}_i(k) = \begin{cases} e_i(k) - \beta_i & : \; e_i(k) > \beta_i \\ 0 & : \; |e_i(k)| \le \beta_i \\ e_i(k) + \beta_i & : \; e_i(k) < -\beta_i \end{cases} \tag{13c}$$

where $i = 1$ and 2, $0 < \alpha < 1$ and $d > 0$.

Thus, the coefficient parameters of the three adaptive filters in Fig.1 are updated in the above algorithm. The feedforward controller $\hat{C}(z, k)$ generating the artificial sound $u(k)$ can be updated by the delayed parameter estimate or the parameter estimate averaged during a time interval. The auxiliary filter $\hat{K}(z, k)$ is also updated by the delayed or averaged parameter vector of $\hat{\boldsymbol{\theta}}_K(k)$. The proposed algorithm satisfies the local convergence and boundedness of the canceling error on the assumption of slow adaptation.

Property: The above robust adaptive algorithm has the convergence property:

$$\text{(a)} \quad \lim_{k\to\infty} \left\| \hat{\boldsymbol{\theta}}(k+1) - \hat{\boldsymbol{\theta}}(k) \right\| = 0 \qquad (14)$$

$$\text{(b)} \quad \lim_{k\to\infty} \sup |e_i(k)| = \beta_i, \quad i = 1,\ 2 \qquad (15)$$

The prrof is omitted here, but from the above property it follows that the boundedness of the actual canceling error $e_c(k)$ is also be assured.

3.3. Simplified adaptive algorithms for implementation

In order to reduce the computational load of the above algorithm, we will give a simplified algorithm such that the norm of the artificial error vector $\|\boldsymbol{e}(k)\|^2$ is minimized. By using the gradient vector $\partial\|\boldsymbol{e}(k)\|^2/\partial\boldsymbol{\theta}$, the normalized LMS type of algorithm can be expressed as follows:

$$\hat{\boldsymbol{\theta}}_D(k+1) = \hat{\boldsymbol{\theta}}_D(k) + \frac{2\alpha_D}{\beta_D + \|\boldsymbol{\xi}(k)\|^2}\boldsymbol{\xi}(k)[e_1(k) - e_2(k)] \quad (16a)$$

$$\hat{\boldsymbol{\theta}}_K(k+1) = \hat{\boldsymbol{\theta}}_K(k) + \frac{2\alpha_K}{\beta_K + \|\boldsymbol{\zeta}(k)\|^2}\boldsymbol{\zeta}(k)e_1(k) \quad (16b)$$

$$\hat{\boldsymbol{\theta}}_C(k+1) = \hat{\boldsymbol{\theta}}_C(k) + \frac{2\alpha_K}{\beta_K + \|\boldsymbol{\varphi}(k)\|^2}\boldsymbol{\varphi}(k)e_2(k) \quad (16c)$$

where $0 < \alpha_D, \alpha_K, \alpha_C < 1$, and $\beta_D, \beta_K, \beta_C > 0$. It can be interpreted that the parameter vectors $\hat{\boldsymbol{\theta}}_K(k)$ and $\hat{\boldsymbol{\theta}}_C(k)$ are updated to reduce the artificial errors $e_1(k)$ and $e_2(k)$ respectively, and the parameter vector $\hat{\boldsymbol{\theta}}_D(k)$ is updated so that the error $e_1(k)$ is equal to $e_2(k)$. Then, if $e_1(k)$ and $e_2(k)$ can converge to zero, the canceling error $e_c(k)$ can also converge to zero as stated before. The computational quantity can be reduced compared to (13a) and (13b).

An alternative algorithm can also be given as follows:

$$\hat{\boldsymbol{\theta}}_D(k+1) = \hat{\boldsymbol{\theta}}_D(k) + \gamma_D\boldsymbol{\xi}(k)\varepsilon_1(k) \quad (17a)$$

$$\hat{\boldsymbol{\theta}}_K(k+1) = \hat{\boldsymbol{\theta}}_K(k) - \gamma_K\boldsymbol{\zeta}(k)\varepsilon_1(k) \quad (17b)$$

$$\hat{\boldsymbol{\theta}}_C(k+1) = \hat{\boldsymbol{\theta}}_C(k) + \gamma_C\boldsymbol{\varphi}(k)\varepsilon_2(k) \quad (17c)$$

$$\varepsilon_1(k) = \frac{e_1(k)}{1 + \gamma_D\|\boldsymbol{\xi}(k)\|^2 + \gamma_K\|\boldsymbol{\zeta}(k)\|^2}$$

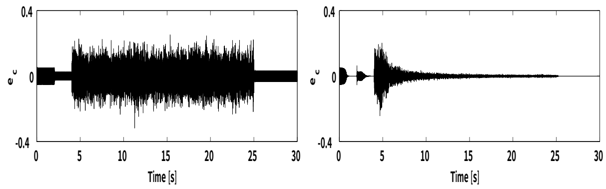
$$\varepsilon_2(k) = \frac{e_2(k)}{1 + \gamma_C\|\boldsymbol{\varphi}(k)\|^2}$$

The algorithm was derived so that $\hat{\boldsymbol{\theta}}_K(k)$ and $\hat{\boldsymbol{\theta}}_D(k)$ are updated to minimize $e_1^2(k)$ and $\hat{\boldsymbol{\theta}}_C(k)$ is updated to minimize $e_2^2(k)$ respectively, which is reduced to a simplified algorithm which is robust for initial conditions.

4. SIMULATION AND EXPERIMENTAL RESULTS

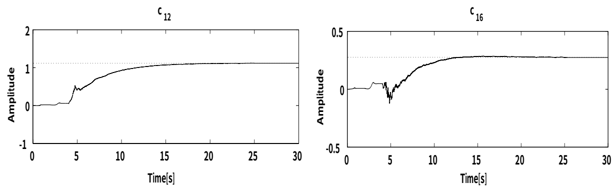
4.1. Advanteges of new algorithm

One of the advantages of the proposed algorithm is that the direct identification of the secondary paths is not needed unlike other approaches, so dither signals are not also needed for assurance of the PE property. The proposed method is applied to simulations in which the models of $G_1(z) \sim G_4(z)$ obtained from the experiment using an air duct are used. Fig.2 shows the ANC results when the source noise has line spectrum in the intervals (0s, 4s) and (25s, 30s) and has low-pass continuous spectrum in (4s, 25s). Even in the absence of the PE property of the source noise and any dither signals, the proposed scheme can attain the canceling satisfactorily, as shown in Fig.2. Fig.3 shows the behavior of two controller FIR parameters of $\hat{C}(z, k)$ which do not take true values (given by the dotted line) in the first interval (0s, 4s) with insufficient PE property of $s(k)$, but converge to their true values within the interval (4s, 25s) with PE property of $s(k)$. However, the canceling can be attained by the proposed method in the both intervals independent of the PE property of the source noise.



(a) no control (b) the proposed method

Fig. 2. Numerical simulation: Canceling error $e_c(k)$

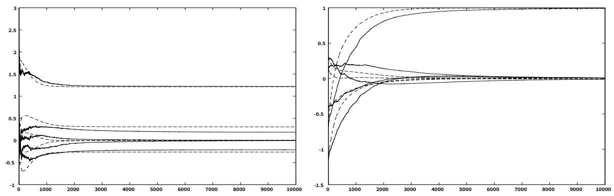


Parameter $c_{12}(k)$ Parameter $c_{16}(k)$

Fig. 3. Numerical simulation: $c_{12}(k)$ and $c_{16}(k)$ by proposed method in numerical simulation

4.2. Convergence behavior of adaptive algorithm

Local convergence can be investigated by the aid of the averaging method [9] in which the mean profiles of the parameter convergence can be calculated. Fig.4 shows the calculated profiles obtained by the proposed algorithm (16)



time [ms] time [ms]

(a) $\{\hat{\theta}_{K,i}(k)\}$ and $\{\bar{\theta}_{K,i}(k)\}$, (b) $\{\hat{\theta}_{C,i}(k)\}$ and $\{\bar{\theta}_{C,i}(k)\}$

Fig. 4. Mean behavior of adaptive parameters.

in which the step gains αs are chosen small constants. Several parameters in $\hat{\boldsymbol{\theta}}_K(k)$ and $\hat{\boldsymbol{\theta}}_C(k)$ are plotted by the solid lines and the corresponding averaged trajectories of $\bar{\boldsymbol{\theta}}_K(k)$ and $\bar{\boldsymbol{\theta}}_C(k)$ are given by the dotted lines, then the both are almost the same.

4.3. Experimental results in air duct

Fig.5 depicts an experimental setup for a noise suppression system in an air duct. The primary source noise $s(k)$ is generated from a loudspeaker by passing white noises through a lowpass filter with a passband of 400 Hz. The sampling frequency is chosen 1kHz. The source noise is detected by the reference microphones placed at B and B', while the error microphones are placed at A, A' and A". By switching the two reference microphones by the mixer, we can simulate unknown changes of the path dynamics $G_2(z)$ and $G_3(z)$. Similarly by switching the three error microphones, we can also simulate changes of the path dynamics $G_1(z)$ and $G_4(z)$. The order of FIR filters $\hat{C}(z,k)$, $\hat{K}(z,k)$ and $\hat{D}(z,k)$ is 80.

The reference microphone was fixed at the position B, and the location of the error microphone was changed as A"$\rightarrow$ A $\rightarrow$ A" $\rightarrow$ A', that simulates uncertain changes of $G_1(z)$ and $G_4(z)$. The error $e_c(k)$ is plotted in Fig.6, which gives that the filtered-x algorithm cannot be robust to changes in $G_3(z)$ and $G_4(z)$ as shown in Fig.6(b), while the proposed method could give very stable performance to any changes in $G_4(z)$ as shown in Fig.6(c).

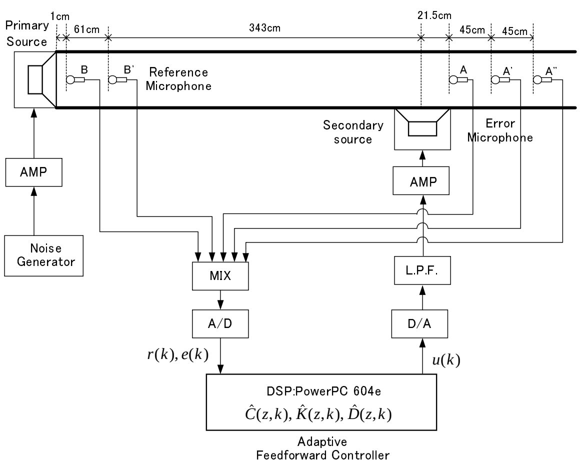
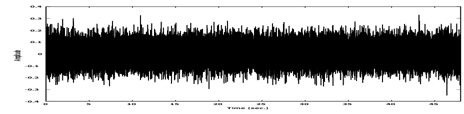
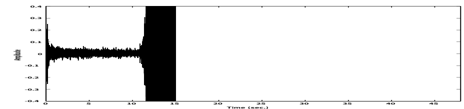


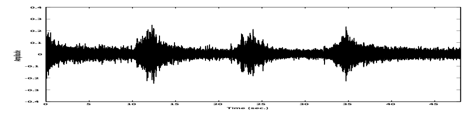
Fig. 5. Experimental setup using air duct



(a) $e_c(k)$ in case without control.



(b) $e_c(k)$ by the modified filtered-x algorithm [6].



(c) $e_c(k)$ by the proposed direct adaptive algorithm.

Fig. 6. Experimental results for changes of secondary path

5. CONCLUSION

The proposed fully direct adaptive control approach is applicable even when all of the primary and secondary path dynamics are uncertainly changeable. To reduce the canceling error, two virtual errors are introduced and are forced into zero by adjusting the three adaptive FIR filters in an on-line manner, which enables the noise cancellation at the objective point. Unlike the previous methods, dither signals or the PE property of the source noise are not required.

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