A PILOTED ADAPTIVE NOTCH FILTER

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ABSTRACT

In this paper, we propose a new adaptive system where the main zero of the notch filter is sandwiched between two piloted zeros. The gradient of the cost function provides information on the direction that the main zero should be steered towards the input sinusoid frequency. The piloted zeros are used to provide an indication on the distance between the frequency of the sinusoid and the zero location of the main notch filter. This information is used to control the adaptive step-size, for fast convergence if the frequency of the sinusoid is far away from the main zero of the notch and for small misadjustment if that distance is small. The technique is very effective for locking on frequencies, which are neither very close to dc nor very close to half sampling frequency. The complexity of the piloted filter is compatible to the conventional normalized LMS (NLMS) notch filter.

1. INTRODUCTION

In the LMS based adaptive notch filter [1-3], the notch frequency is updated every sampling instance and eventually converges to the frequency of the input sinusoid. The convergent rate and the misadjustment are influenced by the adaptation step-size [2]. A larger adaptation step-size will result in a faster convergent rate but will yield a larger misadjustment. Ideally, a large step-size should be used when the distance between the zero of the notch and the frequency of the sinusoid is large in order to improve the convergent rate. A small step-size should be employed if the zero of the notch has converged to the vicinity of the frequency of the sinusoid. As a consequence, a variable step-size (VS) algorithm will outperform a fixed step-size algorithm if appropriate step-sizes are obtainable. Several techniques for obtaining good estimates of step-sizes have been reported in the literature to support the variable step-size LMS algorithms [4-10]. All these algorithms derive the step-size based on the prediction errors at several time instances (*i.e.* time domain averaging).

Harris *et al* [4] suggested a method, which makes use of the sign change between consecutive gradient estimates to determine the step-size. Mathews and Xie [5] proposed using the product of consecutive gradient estimates to update the step-size. The step-size may also be made a function of the gradient estimate. Let $\nabla(n)$ denotes the lowpass filtered value of the gradient estimate at time n. The step-size can be made proportional to the magnitude of $\nabla(n)$ or to an exponential function of the magnitude of $\nabla(n)$. This approach has been adopted by Karni [6] and Shan [7].

In this paper, we propose a brand-new adaptive system consisting of three notches, namely, a main notch and two piloted notches. The main notch frequency is sandwiched between the two piloted notch frequencies. The piloted notches are used to provide an indication on whether the distance between the frequency of the sinusoid and the zero of the main notch is large. If the frequency of the sinusoid is sandwiched between the zeros of the two piloted notches, *i.e.*, if it is lower than the frequency of the zero with the higher notch frequency but higher than that with the lower notch frequency, the signs of the gradient estimates for the two piloted notches will point in opposite directions. This situation can be detected by examining the signs of the gradient estimates of the three notches. If they are not all equal, it is taken as an indication that the frequency of the sinusoid is sandwiched between the zeros of the two piloted notches; a smaller step-size is preferred. If all the signs of the gradient estimates are the same, a larger step-size can be employed.

This paper is organized as follows. Presented in Section 2 is a very brief description of the adaptive notch filter for the purpose of defining notations. The concept of the piloted notches is presented in Sections 3. In Section 4, we discuss the details about the steer directions obtained by the notches when the input signal is a noise free pure sinusoid and how to use this information to derive an adaptive piloted normalized least mean square (P-NLMS) notch filtering algorithm. The effects on the steer directions when the input consists of a sinusoid contaminated with white noise are concisely discussed in Sections 5. Computer simulation results demonstrating the superiority of the piloted notch filter are shown in Section 6.

2. THE ADAPTIVE NOTCH FILTER

A second order direct form IIR adaptive notch filter consists of a pole followed by a zero, which is configured in Fig. 1. In Fig. 1, $W(n)$ is the adjustable weight of the notch filter at time n with limitation as $-2 < W(n) < 2$. $x(n)$ is the notch filter's state. $e(n)$ is the output of the notch filter. Pole radius r is a positive constant less than one. It indicates the bandwidth of the notch filter, also gives the information about the distance of the pole from the origin. To simplicity the presentation, $y(n)$ is considered as a single sinusoid of the form

$$y(n) = a\sin(w_0 n + \beta), \quad (1)$$

where w_0 is the frequency of the sinusoid input signal, β is a phase constant and a is the magnitude of the input. The transfer function of the notch filter, $H(z)$, is given by [2]

$$H(z) = \frac{1 + W(n)z^{-1} + z^{-2}}{1 + rW(n)z^{-1} + r^2 z^{-2}}, \quad (2)$$

The notch frequency $\theta(n)$ at time n is given by [2]

$$\theta(n) = \cos^{-1}\left(-W(n)/2\right). \quad (3)$$

Most popular adaptive algorithms for adapting $W(n)$ are called the least mean squared (LMS-type) algorithms, which can be shown as

$$W(n+1) = W(n) - 2\mu(n)x(n-1)e(n), \quad (4)$$

where $\mu(n)$ is the time-varying step-size at time n.

The main challenge in the LMS-type algorithm is to figure out the method to get proper $\mu(n)$, which have been studied by many researchers [4-7]. After the detailed studies about the formalisms suggested by the authors based on the system shown in Fig. 1, we found that it is lack of the information on how far away the notch frequency from the sinusoid frequency. In the Section 2, we will introduce a new system with the piloted notches to overcome the disadvantage.

3. THE PILOTED NOTCHES

We shall refer to the notch presented in Fig. 1. as the main notch. Two piloted notches will be introduced to form a new system as shown in Fig. 2. Here, we set one of the notch frequencies as ϕ_h and another is ϕ_l. Specifically, ϕ_h is higher than the main notch and ϕ_l is lower than the main notch. Furthermore, we also assume that ϕ_l is larger than zero and ϕ_h is less than half sampling frequency. Now, we construct two piloted notch filters have the following transfer function

$$H_h(z) = \frac{1+W_h(n)z^{-1}+z^{-2}}{1+rW(n)z^{-1}+r^2z^{-2}}, \quad (5)$$

$$H_l(z) = \frac{1+W_l(n)z^{-1}+z^{-2}}{1+rW(n)z^{-1}+r^2z^{-2}}. \quad (6)$$

Note that $H_h(z)$ and $H_l(z)$ use the same denominator as $H(z)$ in (2). This leads us to reduce the computational complexity. Moreover, we define

$$W_h(n) = W(n) - \alpha_h, \quad (7)$$

$$W_l(n) = W(n) + \alpha_l. \quad (8)$$

where α_h and α_l are positive values, which correspond to the effect of ϕ_h and ϕ_l on $W_h(n)$ and $W_l(n)$. The values of α_h and α_l can be pre-computed based on the piloted task required. It is interesting to find that α_h and α_l need not be precise. This is because the piloted notches are used only for estimating whether w_0 is sandwiched between those of the two piloted notches. In particular, α_h and α_l can be selected as a convenient power-of-2 so that $e_h(n)$ and $e_l(n)$ can be obtained from that of $H(z)$ by simple shift-add operations; in this case, the additional cost for obtaining $e_h(n)$ and $e_l(n)$ is minimal.

Applying (2), (7) and (8), $H_h(z)$ and $H_l(z)$ in (5) and (6) becomes

$$H_h(z) = H(z) - \frac{\alpha_h z^{-1}}{1+rW(n)z^{-1}+r^2z^{-2}}, \quad (9)$$

$$H_l(z) = H(z) + \frac{\alpha_l z^{-1}}{1+rW(n)z^{-1}+r^2z^{-2}}. \quad (10)$$

Let $e_h(n)$ and $e_l(n)$ be the time domain outputs of the notch filters as shown in Fig. 2. From (9) and (10), we can derive the following,

$$e_h(n) = e(n) - \alpha_h x(n-1), \quad (11)$$

$$e_l(n) = e(n) + \alpha_l x(n-1). \quad (12)$$

The implementation structure for obtaining $e_h(n)$ and $e_l(n)$ is also shown in Fig. 2.

4. THE ADAPTIVE PILOTED NOTCH FILTERING ALGORITHM

As discussed above, the function of the piloted notches is to provide an indication on whether w_0 is far away from or close to the main notch frequency. Now, let's have a close look at how the proposed new piloted system in Fig. 2 can provide the information on the relative position of the sinusoid frequency of $y(n)$ and the main notch.

To simplify the presentation, the input signal $y(n)$ is considered as a pure sinusoid given in (1). If we consider the notch filter as a linear filter, from Fig. 2, $x(n)$ is also a sinusoid signal with the same frequency but different amplitude and phase, which can be given as:

$$x(n) = A\cos(w_0 n + \gamma), \quad (13)$$

where

$$A^2 = a^2 |H(e^{jw_0})|^2 = \frac{a^2}{(1-r^2)^2 + r(2r\cos w_0 - W(n))(2\cos w_0 - rW(n))} \quad (14a)$$

$$\gamma = \beta + \tan^{-1}\left(\frac{rW(n)\sin w_0 + r^2\sin 2w_0}{1+rW(n)\cos w_0 + r^2\cos 2w_0}\right). \quad (14b)$$

From Fig. 2, the error output $e(n)$ can be expressed as:

$$e(n) = A\cos(w_0 n + \gamma) + W(n)\cos(w_0(n-1)+\gamma) + A\cos(w_0(n-2)+\gamma). \quad (15)$$

Hence, the product of $x(n-1)e(n)$ is given by

$$\begin{aligned} x(n-1)e(n) &= A^2\cos(w_0(n-1)+\gamma)[\cos(w_0 n+\gamma) \\ &\quad + W(n)\cos(w_0(n-1)+\gamma) + \cos(w_0(n-2)+\gamma)] \\ &= x^2(n-1)(2\cos w_0 + W(n)) \end{aligned} \quad (16)$$

Substituting (3) into (16), we have

$$x(n-1)e(n) = 2x^2(n-1)[\cos(w_0) - \cos(\theta(n))], \quad (17)$$

From (17), we can see that $x(n-1)e(n) > 0$ indicates $\cos(w_0) > \cos(\theta(n))$ and then $w_0 < \theta(n)$. Hence, $W(n)$ should be decreased in the next iteration to bring $\theta(n)$ closer to w_0. Inversely, $x(n-1)e(n) < 0$ indicates $\cos(w_0) < \cos(\theta(n))$ and then $w_0 > \theta(n)$. In this case, $W(n)$ should be incremented in the next iteration to bring $\theta(n)$ closer to w_0. As a result, the sign of $x(n-1)e(n)$ provides the steer direction (the direction in which $\theta(n)$ should be changed in order to bring it nearer to w_0) of the main notch. The steer direction represents the relative position of w_0 and $\theta(n)$.

Similarly, for the two notches, we get the following:

When $x(n-1)e_h(n) > 0$, it indicates $w_0 < \phi_h$,

When $x(n-1)e_h(n) < 0$, it indicates $w_0 > \phi_h$,

When $x(n-1)e_l(n) > 0$, it indicates $w_0 < \phi_l$,

When $x(n-1)e_l(n) < 0$, it indicates $w_0 > \phi_l$.

The results obtained above show that the signs of $x(n-1)e(n)$, $x(n-1)e_h(n)$ and $x(n-1)e_l(n)$ provide information on the relative positions of w_0, ϕ_h, $\theta(n)$, and ϕ_l. Next, let's evaluate some details:

Case 1: $x(n-1)e(n) > 0$, $x(n-1)e_h(n) > 0$ and $x(n-1)e_l(n) > 0$

In this case, we can verify that $w_0 < \phi_l < \theta(n) < \phi_h$. This means the frequency of the sinusoid signal is very far away from the main notch. The larger adaptation stepsize can be used to speed up the convergence rate.

Case 2: $x(n-1)e(n) < 0$, $x(n-1)e_h(n) < 0$ and $x(n-1)e_l(n) < 0$

In this case, it can be verified that $w_0 > \phi_h > \theta(n) > \phi_l$. This means the frequency of the sinusoid signal is also very far away from the main notch. The larger adaptation stepsize can be used to speed up the convergence rate.

Case 3: $x(n-1)e(n) < 0$ and $x(n-1)e_h(n) > 0$

In this case, it can be verified that $\phi_l < \theta(n) < w_0 < \phi_h$. This indicates the frequency of the sinusoid signal is sandwiched between the main notch and the higher notch. The smaller adaptation stepsize should be used to reduce the misadjustment.

Case 4: $x(n-1)e(n) > 0$ and $x(n-1)e_l(n) < 0$

In this case, it can be verified that $\phi_l < w_0 < \theta(n) < \phi_h$. This indicates the frequency of the sinusoid signal is sandwiched between the main notch and the lower notch. The smaller adaptation stepsize should be used to reduce the misadjustment.

From the above discussion, we can summary that when $\text{sign}(x(n-1)e(n))$ = $\text{sign}(x(n-1)e_h(n))$ = $\text{sign}(x(n-1)e_l(n))$, a larger value of $\mu(n)$ can be used to reduce the time required to steer the main notch frequency to the frequency of the sinusoid. Otherwise, $\mu(n+1)$ should be chosen as smaller value to reduce the misadjustment. Clearly, this new approach provides us an effective method to adapt the stepsize. The proposed piloted notch LMS (P-LMS) algorithm is formalized in Table 1.

Table 1. Piloted Normalized LMS (P-NLMS) Algorithm
Initialization: $W(3)=0$, $x(1)=x(2)=x(3)=0$;
Iteration from n=3…,length:
$x(n)=y(n)-rW(n)x(n-1)-r^2x(n-2)$;
$\sigma_x^2(n)=\lambda\sigma_x^2(n-1)+x(n)^2$ (λ is forgetting factor)
$e(n)=x(n)+x(n-1)W(n)+x(n-2)$;
$e_l(n)=e(n)+\alpha_l x(n-1)$; $e_h(n)=e(n)-\alpha_h x(n-1)$;
$ss=sign(e(n)x(n-1))+sign(e_h(n)x(n-1))+sign(e_l(n)x(n-1))$
if ss==3 then $u(n)=\mu_$large (larger stepsize)
else $u(n)=\mu\_small$ (smaller stepsize)
$W(n+1)=W(n)-\mu(n)x(n-1)e(n)/\sigma_x^2(n)$

5. EFFECTS OF THE GAUSSIAN NOISE

So far, we just discussed the input of the adaptive notch filter is a pure sinusoid signal. If the input is corrupted by a white Gaussian noise, the signs of $x(n-1)e_h(n)$ and $x(n-1)e_l(n)$ may not indicate correctly whether the frequency of the input sinusoid is higher than or lower than that of the piloted notch.

Considering the noise case, we have studied the following topics: (1) Steer direction under Gaussian noise; (2) Probability of producing the correct steer direction; (3) Effect of the input SNR on the steer direction; (4) Effect of the pole radius r on the steer direction.

Due to the space limitation, here we just present some conclusions drawn from the theoretical analysis and some simulation results (The details will present in another paper).

(1) Steer direction under Gaussian noise

The theoretical result shows that when $|W(n)|$ does not approach to zero or 2 (it means the notch frequency does not approach to d.c. or the half sampling frequency), the average effect due to Gaussian noise on the sign of $x(n-1)e(n)$ can be ignored.

(2) Probability of producing the correct steer direction

We study the probability that the sign of $x(n-1)e(n)$, $x(n-1)e_h(n)$ and $x(n-1)e_l(n)$ can provide the correct steer direction for the main notch. As one of the example, Fig. 4 shows that the probability that $\text{sign}(x(n-1)e(n))$ gives the correct steer direction as a function of notch position $\theta(n)$. Where, the frequency w_0 is chosen as 0.8π, the input SNR is 0 dB and the pole radius $r=0.9$. The solid line curves correspond to theoretical results. The arrays of symbols represent the results obtained from 100 independent simulation runs. It shows that the simulation results correspond to our theoretical analysis. The curves shown in Fig. 4 can be explained intuitively as follows. When the initial notch frequency is far away from the frequency of the sinusoid, the signal is swamped by noise. The probability of obtaining a correct steer direction is low. As the notch moves toward the sinusoid, the probability of producing a correct steer direction increases. However, when the notch position is very near to the frequency of the sinusoid, because of the parabolic nature of the performance surface of the objective function, the probability of producing a correct steer direction becomes low again. It is interesting to note that the probability of producing a correct steer direction is larger than 0.45, *i.e.*, it is always better than a random guess.

(3) Effect of the input SNR on the steer direction

By fixing the input sinusoid frequency w_0, the notch position $\theta(n)$ and the pole radius r, we find that when the input SNR is low, *i.e.*, the input signal is too noisy, the steer direction becomes less accurate as an indicator of the position of the frequency of the input sinusoid.

(4) Effect of the pole radius r on the steer direction

From the theoretical analysis and simulation results, we note that the closer are the poles of the notch filter to the unit circle, the lower is the probability that the steer direction points to the correct direction when $\theta(n) \neq w_0$. This is because, when $\theta(n) \neq w_0$, the noise component near the pole is greatly enhanced if the pole is very close to the unit circle. This implies that a smaller pole radius will result in a better estimate for the steer direction, which in turn will result in a faster convergence speed of the notch filter. However, it is known that smaller pole radius will cause bad steady state performance [3]. There is a compromise of the choice of the r to obtain the faster convergence as well as smaller misadjustment.

6. SIMULATION RESULTS

A simulation has been done to demonstrate the performance of the proposed piloted notch filter in tracking the frequency of a sinusoid contaminated by white Gaussian noise. The result is compared with a conventional notch filter with only the main notch algorithm using the LMS and the NLMS algorithms [2]. The NLMS algorithm can be taken as the time-varying stepsize algorithm. The input SNR is 3 dB, the input sinusoid frequency is abruptly switched every 3000 iterations. As an example, the input frequencies are set as $[0.3\pi, 0.6\pi, 0.3\pi] = [0.942, 1.885, 0.942]$, the pole radius r is 0.9; For the LMS algorithm [2], the stepsize used is 0.003; For the NLMS [2], λ is 0.95, the stepsize is $\mu' = 0.5$. For the P-NLMS algorithm, $\alpha_h = \alpha_l = 0.125 = 2^{-3}$, $\mu\_\text{large} = 3\mu'$ and $\mu\_small = \mu'$. The simulation results over 100 runs are shown in Fig. 5. To get the fair comparison, the convergence misadjustment for three algorithms reaches the same level. From Fig. 5., we can see that the initial convergence of the NLMS and P-NLMS algorithms are 200 and 130 iterations, respectively. Similarly, the tracking convergence of the NLMS and P-NLMS algorithms is 400 and 200 iterations, respectively. The P-LMS algorithm is much superior to the NLMS algorithm. Obviously, the P-NLMS is much faster than the conventional non-pilot LMS algorithm. It is expected that the larger stepsize can be used to improve the convergence speed if the steer direction by the pilot notches can be accurately indicated. It is noted that there is possibility that the pilot notches will indicate the wrong steer direction due to the noise even after the convergence. The incorrect indication of the steering direction will cause the increase of the misadjustment due to the possible use of the big stepsize. However, this problem can be solved by adding more pilot notches to guide the steering direction more precisely. Therefore, the performance of the P-NLMS algorithm and piloted notch filter could be further improved by using more step-sizes.

7. CONCLUSION

In this paper, a brand-new concept in designing the variable step-size adaptation algorithm has been introduced. Specifically, we proposed a piloted notch filter structure consisting of a main notch and two piloted notches. We have shown that the piloted notch filter gives a more accurate estimate on the steer direction of the notch filter. The piloted notches are very useful in providing information on whether the frequency of the input sinusoid is far away from the notch frequency of the main notch. This information can be used to design a variable step-size algorithm at very low price of additional computational complexity. The performance of the piloted notch filter can be further improved by using more piloted and more step sizes.

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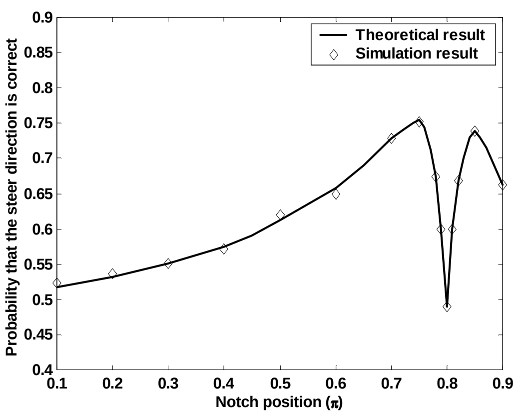

Fig. 4. Probability of $\text{sign}[x(n-1)e(n)]$ producing the correct estimate of the steer direction *vs* notch position plot.

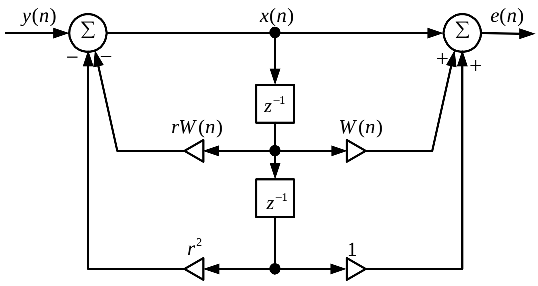

Fig. 1.The structure of an IIR adaptive notch filter

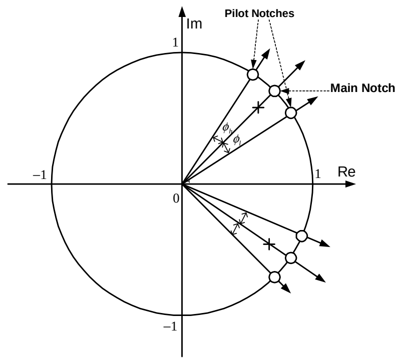

Fig. 2. The pole-zero positions of the piloted notch filter.

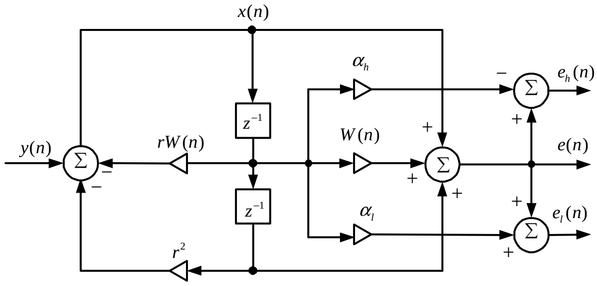

Fig. 3. Piloted notch filter. If α_h and α_l are integer power-of-two, the additional cost for obtaining $e_h(n)$ and $e_l(n)$ is minimal.

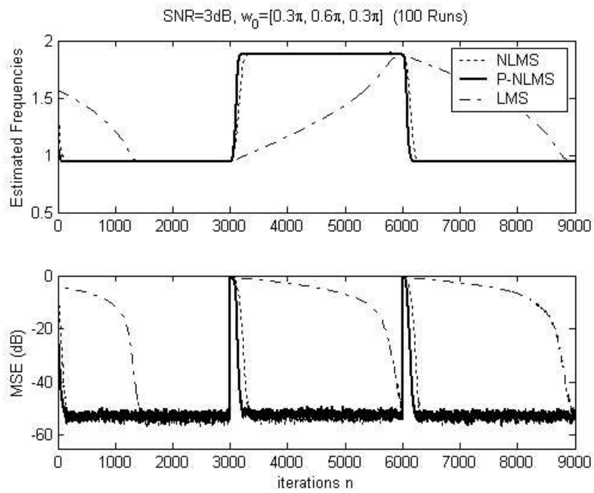

Fig. 5. Convergence performance of the estimated frequencies and the means squared error (MSE), which is defined as $MSE(n) = 10\log_{10}([\hat{f}(n) - f_i]^2)$