DENOISING OF SEISMIC SIGNALS WITH OVERSAMPLED FILTER BANKS

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ABSTRACT

In several applications such as denoising, when signal expansion is not crucial, oversampled filter banks may outperform critically decimated filter banks. We study the performance of the recently proposed GLPBT (Generalized Lapped Pseudo-Biorthogonal Transform), a class of oversampled filter banks, on noise removal in seismic data, using controlled redundancy. We also investigate heuristics for the choice of an optimal threshold, which appears to depend non-trivially on the noise variance. Tests indicate that carefully designed oversampled filter banks are able to outperform critically sampled ones.

1. INTRODUCTION

The purpose of this paper is to study noise removal in seismic signals by thresholding output coefficients of oversampled filter banks.

The theory of filter banks has been primarily confined to the critically sampled case, for instance orthonormal or biorthonormal bases in $l_2(\mathbf{Z})$ [1]. The main advantage of this case is the non-expansive number of coefficients, which is often useful for compression purposes. In many other situations, there is no clear evidence that a projection, onto an ortho- or a biorthonormal basis, is the best suited signal representation. Such an observation has been made for instance for noise reduction in [2]. Following the non-linear noise reduction techniques proposed by D. Donoho in [3], M. Lang *et al.* proposed to make the wavelet transform invariant to integer shifts [4], resulting in greater noise robustness. They also report the successful application of the former scheme to geophysical data, as in [5]. Beyond traditional wavelets, it was shown in a limited scope in [6] that maximally decimated paraunitary M-channel filter banks were capable of outperforming discrete wavelets for seismic data compression but also for denoising purposes.

The technique in [2] was later extended to undecimated wavelet packets [7]. The penalty was paid by increasing computational complexity and memory requirements. Discrete wavelets form a subset of the filter bank theory. The theory of more general oversampled filter banks was recently extensively explored, for instance in [8]. Figure 1 shows an M-band filter bank uniformly decimated by a factor of N. The limit $M = N$ yields the critically sampled case. Oversampling occurs when the number of bands M is greater then the decimation factor N. In other words, the filter bank outputs in average M/N more coefficients than are present in the input signal (disregarding the usual extention at the end of the signal, see e.g. [9]). Cvetković *et al.* prove in [8] that perfect reconstruction oversampled filter banks are equivalent to a particular class of frames in $l_2(\mathbf{Z})$. Some work has also been performed on the existence on uniform filter banks with rational oversampling [10]. Oversampled filter banks generally enjoy some improvements over critically sampled ones. They become closer to shift invariant as the sampling ratio approaches unity. Associated with appropriate coefficient selection techniques, they should generally be more effective for noise removal, less prone to spurious artifacts such as pseudo-Gibbs phenomena in the vicinity of discontinuities. They also allow a more flexible design of the filter coefficients, for instance in the frequency selectivity.

Unfortunately, denoising results with filter banks for sample signals or natural images don't often exhibit improvements comparable to those obtained from decimated to undecimated wavelet shrinkage. We investigate here noise removal in seismic data using the recently proposed (by one of the authors, [11]) lattice structure based design for the "Generalized Lapped Pseudo-Biorthogonal Transform", or GLPBT. GLBPT is a class of oversampled filter banks. We first briefly review the main properties of the GLPBT and the settings of the filter banks used in this study in Section 2. The denoising method, based on classical shrinkage in the transform domain, is explained in Section 3. Results on seismic signals are detailed in Section 4, with an emphasis on heuristics for the optimal threshold choice.

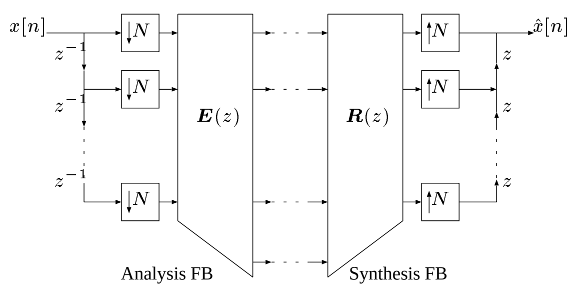
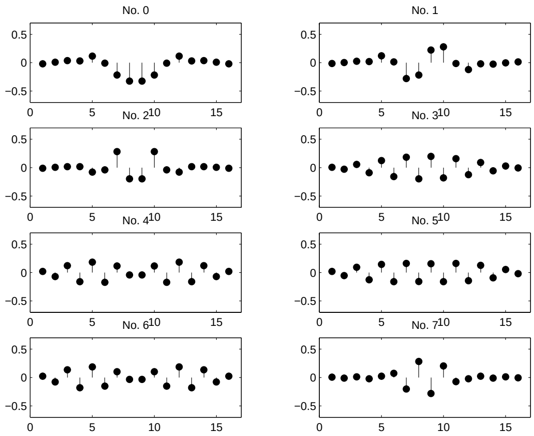
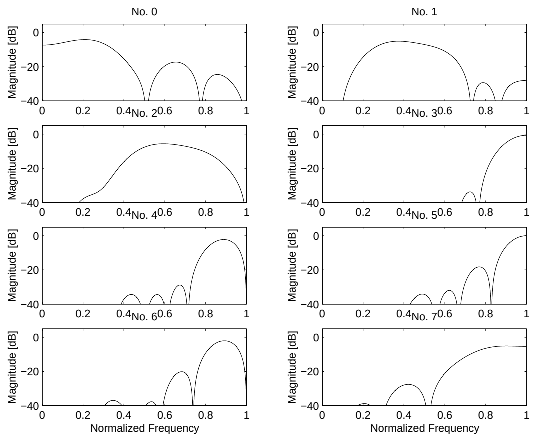


Fig. 1. Block diagram of the polyphase matrices of a processing system based on a M-band filter bank with N sampling ratio.



(a) Impulse responses for the analysis GLPBT1 FB.



(b) Frequency responses for the analysis GLPBT1 FB.

Fig. 2. Design exemple for a coding gain optimized 8-ch., 16-length, 2-OSF GLPBT, GLPBT1.

2. THEORY AND FILTER SETTINGS

2.1. GLPBT STRUCTURES AND PROPERTIES

Let us consider a M-channel, N decimation factor system of filter banks. Let $\mathbf{E}(z)$ (of size $M \times N$) and $\mathbf{R}(z)$ (of size $N \times M$) be the polyphase matrices of the analysis and the sysnthesis banks respectively. The system provides perfect reconstruction with zero delay if and only if:

$$\mathbf{R}(z)\mathbf{E}(z) = \mathbf{I}_N,$$

where $\mathbf{I}_N$ is the identity matrix. The system is called pseudo-biorthogonal when $\mathbf{R}(z) = \mathbf{E}^T(z^{-1})$, pseudo-orthogonal otherwise.

The Generalized Lapped Pseudo-Biorthogonal class of filter banks (GLPBT) used in this study also satisfy the linear phase properties. We refer to [1] for explanations on polyphase matrices, perfect reconstruction and linear phase and to [8, 11] for details on oversampled filterbanks.

The key properties that are of potential interest for denoising are:

- the GLPBT covers a very wide range of filter banks including maximally decimated (critical sampled) paraunitary FB, PR FB, and oversampled pseudo-orthogonal FB. It is a natural extension of the conventional FB with lattice structures.
- it is represented by lattice structures.The only parameters to be determined are plain rotation angles and positive diagonal entries. The lattice structure results in a fast implementation and objective driven (i.e. coding gain, frequency attenuation, ...) optimized filter coefficients. It should be noted that coding gain for design of oversampled filter banks is slightly different from that of critically decimated filter banks, when $M < N$.
- though the proposed structure is not complete, it contains the minimal number of delays.
- the noise robust GLPBT proposed in [11] possesses a noise suppression function for additive noise in the transform domain. This property will not be addressed throughout this work.

There are some advantages in choosing the more general M-channel FB class, since there is generally more freedom in the design parameters. The GLPBT are used here in a similar fashion as proposed in [2] for undecimated discrete wavelet tranforms, as show in Section 3.

2.2. FILTER BANK SETTINGS

The filter banks used in this study are defined as follows:

- LOT8x2 is a Malvar's LOT (Lapped Orthogonal Tranform, see [12]) with 8 channels and 16 coefficients. It is a critically sampled lapped orthogonal transform. It is represented in Figures by a simple line, with no particular decoration.
- GLPBT1 and GLPBT2 represent generalized lapped pseudo-biorthogonal transforms, optimized for coding gain and frequency attenuation respectively. They also possess 8 channels and 16 coefficients but are perfect reconstruction FB with a sampling rate of 4 (instead of 8). Their performances are represented in figures by x- and o-decorated lines respectively.

In legends, the subscript refers to the oversampling factor (OSF), given with respect to the number of channels, used for denoising. For instance, a OSF subscript of 8, as in LOT8x2_8, corresponds to a non-decimated FB. The impulse filter and frequency responses of the GLPBT1, optimized for coding gain are represented in Fig. 2.

3. METHOD

The noisy signal is transformed with each filter bank, the OSF (oversampling factor) being taken from the set $\{8, 4, 2\}$. The later choice ensures a perfect reconstruction of the data in the absence of processing between the analysis and the synthesis filter bank. Since we may choose the OSF in the divisors of the number of channels (e.g. 8), the general filter bank setting allows some more degrees of freedom over the wavelet case, where the OSF choice is limited to 1 or 2.

The proposed denoising method relies heavily on Donoho's thresholding methods. The two most frequently used methods are:

- *soft-thresholding*: it takes the coefficient c and shrinks it toward zero with respect to the threshold t, according to the function

$$\mathrm{ST}(c) = \mathrm{sign}(c) \cdot \max\left(|c| - t, 0\right),$$

- *hard-thresholding*: it discards every coefficient smaller than the threshold t,

$$\mathrm{HT}(c) = c \cdot \mathbf{1}\{|c| \leq t\},$$

  where $\mathbf{1}\{\cdot\}$ is the set characteristic function.

The results exposed in this works are based on the soft-threshold rule for theorical near-optimality reasons [3] as well as the relatively smooth nature of seismic signals, as compared to natural images.

In general, the associated synthesis filter bank is not unique. In this study, the thresholded coefficients are transforms back to the time domain using the most straightforward synthesis filter bank, based on a pseudo-inverse matrix.

4. RESULTS

The test data has been obtained from a reflection seismic survey in Louisana. The signal originates typically from an accelerometer sensor recording the vibration from a distance of a vibrating or explosive source. Seismic events crudely correspond to the arrival of reflected waves in subsurface layers. Seismic signals are generally non-stationary.

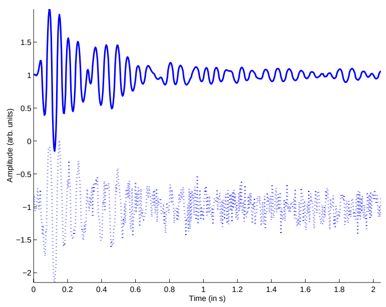


Fig. 3. Example of seismic trace and its noisy version.

In the first experiment, the original signal-noise ratio (SNR) is set to 25.7 dB. The noise variance σ^2 is assumed known. It is the case in seismic under the additive noise assumption: seismic signals generally have low activity (i.e. no signal) at both ends. Since we don't have estimates of an optimal threshold, the selected threshold t is first reported as a "linear" function of σ: $t = \alpha_\sigma \cdot \sigma$. The term α_σ is refered to the "threshold factor" in the following. We would like to investigate its behavior, depending on the chosen filter bank, the sampling ratio and the noise variance.

Figure 5 represents the threshold factor α_σ on the x-axis, with the resulting SNR after inverse transform on the y-axis. Each graph is related to a different OSF (from top to bottom, 8, i.e. undecimated, 4 and 2). From Fig. 5, we see that GLPBT1 exhibit a behavior similar to that of the LOT, with in average +0.3 dB SNR improvement.

The optimal α_σ is defined as:

$$\alpha_\sigma^{\mathrm{opt}} = \mathrm{argmax}_\sigma \{\mathrm{SNR}(t(\sigma))\}.$$

The optimum threshold factor (the one which yields maximum SNR) varies for each filter bank, making the optimal threshold selection difficult. Nevertheless, at the chosen SNR, the two GLPBT maxima exceed the LOT maximum. As a result, we may expect some improvements using structurally enforced oversampled filter banks.

The three graphs also show that the denoising performance increases with the oversampling factor: more redundancy induces better noise attenuation for all transforms.

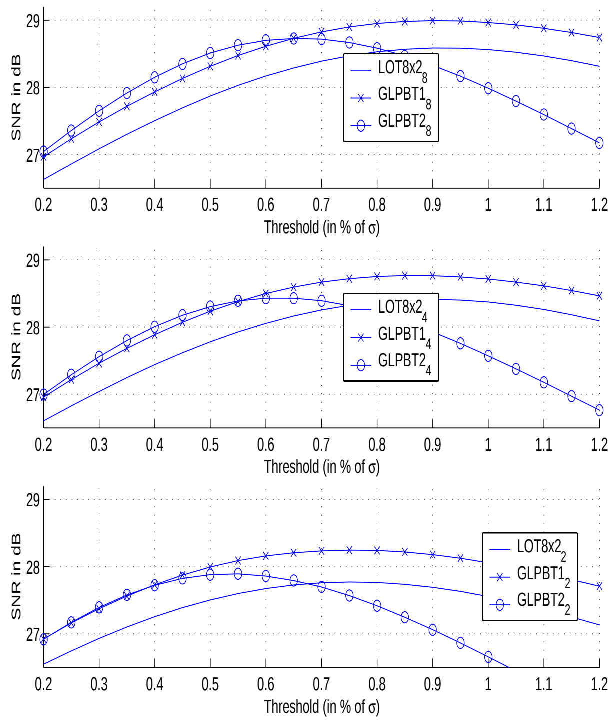


Fig. 4. Threshold selection depending on the oversampling factor, for an initial SNR of 26.7 dB.

We now focus on the choice of the threshold. Figure 5 represents $\alpha_\sigma^{\mathrm{opt}}$ as a function of the initial noise variance. The main purpose was to try to derive some heuristics for optimum threshold selection. We note that the LOT and GLPBT1 still exhibits similar behavior. Hence, a good threshold for GLPBT1 could be derived from that for the LOT. Unfortunately, the threshold factor behavior does not show except that it globally increases with the noise variance, while we expected a constant function, in similarity to D. Donoho universal threshold [3]. This point will be subject to further investigations.

Figure 6 provides empirical evidence that, once the optimal threshold is found, GLPBT are capable of global SNR improvements (in dB) over the LOT, on a wide range of initial SNRs. Given the initial SNR, Figure 6 represents the amout of optimal improvement we can get from oversampled filter banks. The Coding Gain optimized GLPBT clearly performs better at higher SNR (above 20 dB initial SNR), with about +0.3 dB in average. The Frequency Attenuation optimized GLPBT is superior for lower SNRs (below 20 dB).

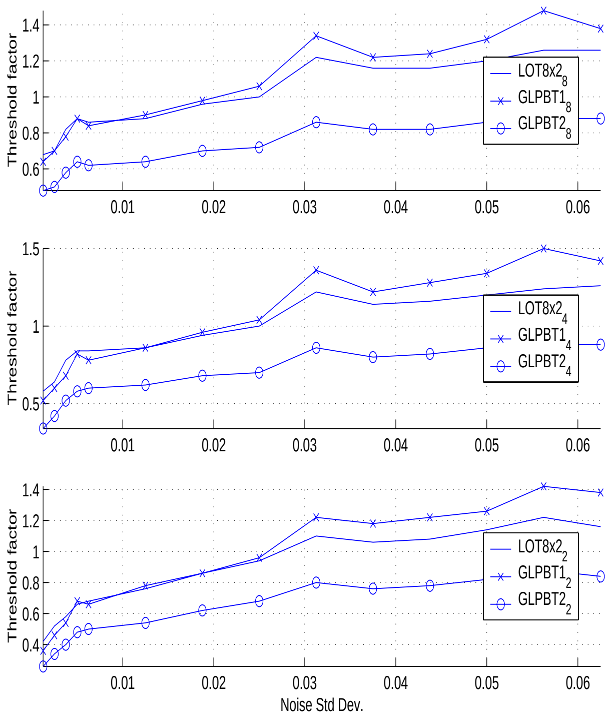


Fig. 5. Behavior of the optimum threshold factor as a function of the noise standard deviation σ.

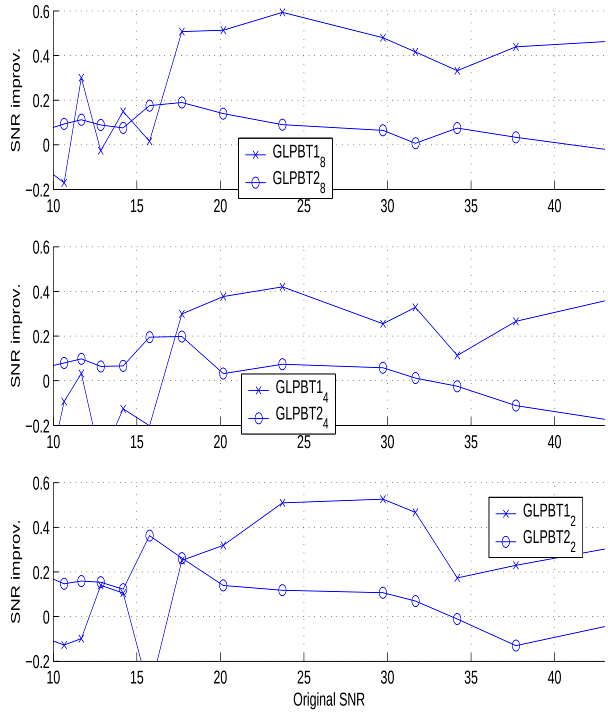


Fig. 6. SNR improvement of the GLPBT over Malvar's LOT.

5. CONCLUSIONS

We have performed noise removal in seismic signals using oversampled Generalized Lapped Pseudo-Biothorgonal transforms. The results are limited in scope, since we did not delt with thresholding coefficients in the transform domain with scale-band dependent thresholds and suggested in [7]. More involved powerful statistical techniques (e.g. based on Hidden Markov Models) also deserve further investigations. We nevertheless have shown that controlled redundancy is desirable, and that GLPBT are able to outperform traditional maximally decimated filter banks, and that controlled redundancy. Future work will deal with other designs as well as applications to natural images.

6. REFERENCES

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