INSTANTANEOUS FREQUENCY ESTIMATION BY USING WIGNER DISTRIBUTION AND VITERBI ALGORITHM

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ABSTRACT

Estimation of the instantaneous frequency (IF) in a high noise environment, by using the Wigner distribution (WD) and the Viterbi algorithm, is considered. The proposed algorithm combines the nonparametric IF estimation based on the WD maxima with minimization of the IF variations between consecutive points. Algorithm realization is performed recursively by using the (modified) Viterbi algorithm. Performances are compared with the IF estimation based on the WD maxima.

1. INTRODUCTION

The instantaneous frequency (IF) estimation is an important research field. For known signal model there are well defined parametric methods [1, 2]. When the IF exhibits abrupt changes or parametric model is unknown, time-frequency (TF) representations can be useful analysis tools [3, 4]. There are several methods for non-parametric IF estimations based on the TF representations. Almost all of them use position of the TF representation maxima in the initial stage of algorithms [3]-[6]. The non-parametric methods for the IF estimation are more sensitive to the noise influence than their parametric counterparts. An analysis of the high noise generated error to the IF estimation is done in [7]. It causes that TF distributions have maxima outside the signal auto-term. This kind of error, when it appears, dominates over other sources of error. Therefore, our goal is to create a non-parametric algorithm for the IF estimation that can perform accurately for high noise environments. Monocomponent FM signal with constant amplitude, in an additive, white, Gaussian noise (AWGN) with independent real and imaginary parts is considered. The key criteria used in the algorithm are: the IF should pass through as many as possible points of the WD with highest magnitudes, while the IF variation between two consecutive points should not be too fast. Algorithm can be realized recursively as an instance of the (generalized) Viterbi algorithm [8].

The paper is organized as follows. The WD based IF estimator is described in Section II. The algorithm for the IF estimation in a high noise environment with numerical examples and statistical study is presented in Section III. Conclusions are given in Section IV.

2. WD AS AN IF ESTIMATOR

Consider a signal $f(t) = Ae^{j\phi(t)}$ corrupted by the AWGN $\nu(t)$ with variance $2\sigma^2$ (variance of the real and imaginary part is σ^2). Noisy signal $x(t)$ is of the form $x(t) = f(t) + \nu(t) = Ae^{j\phi(t)} + \nu(t)$. The IF is defined as the first derivative of the phase: $\omega(t) = \phi'(t)$. The WD of a discrete-time signal is given by

$$WD_x(t,\omega) = \sum_k w_h(kT)x(t+kT)x^*(t-kT)e^{-j2\omega kT}, \quad (1)$$

where the window function $w_h(kT)$ has the width h, and T is the sampling interval. The WD is highly concentrated around the signal's IF. Therefore, the IF estimation can be performed based on the position of the WD maxima [3, 4]:

$$\hat{\omega}(t) = \arg\max_{\omega} WD_x(t,\omega). \quad (2)$$

In order to explain our motivation for development of the new algorithm, consider a linear FM signal with the IF concentrated along the discrete frequency grid on $\omega(t) = at+b$. Let the window $w_h(nT)$, with $h = NT$, be rectangular and wide enough so that the auto-term for each instant is concentrated in a single point. The remaining error in the system is due to the influence of high noise only. The WD mean value is given by $E\{WD_x(t,\omega)\} = WD_f(t,\omega) + 2\sigma^2$, while the WD variance is $\sigma_{WD}^2 = 4N\sigma^2(A^2+\sigma^2)$ [9]. Since there is a large number of terms in the sum (1), we can assume that the central limit theorem may be applied to the WD values. Thus, these values are Gaussian in nature, with $\mathcal{N}(0,\sigma_{WD})$ outside the IF, and $\mathcal{N}(NA^2,\sigma_{WD})$ at the IF. The constant $2\sigma^2$ in the mean value is omitted. The pdf for the WD values along the IF is then:

$$p(\xi) = \frac{1}{\sqrt{2\pi}\sigma_{WD}} e^{-(\xi-NA^2)^2/2\sigma_{WD}^2}. \quad (3)$$

The WD, outside the IF position, assumes a value greater than Ξ with the probability:

$$Q(\Xi) = \frac{1}{\sqrt{2\pi}\sigma_{WD}} \int_{\Xi}^{\infty} e^{-(\xi^2/2\sigma_{WD}^2)} d\xi$$
$$= 0.5\text{erfc}\left(\Xi/(\sqrt{2}\sigma_{WD})\right). \quad (4)$$

Probability that any of $N-1$ WD values outside the IF is greater than Ξ is

$$G(\Xi) = 1 - (1 - Q(\Xi))^{N-1}. \quad (5)$$

σ/A	0.25	0.50	0.75
P_E	$1.90{\cdot}10^{-138}$	$5.55{\cdot}10^{-22}$	$1.98{\cdot}10^{-7}$
σ/A	1.00	1.25	1.50
P_E	$4.71{\cdot}10^{-3}$	$1.37{\cdot}10^{-1}$	$4.49{\cdot}10^{-1}$
σ/A	1.75	2.00	2.25
P_E	$6.97{\cdot}10^{-1}$	$8.33{\cdot}10^{-1}$	$9.02{\cdot}10^{-1}$

Table 1: Error probability of the IF estimation.

When a WD value outside the IF surpasses its value at the IF, then a large estimation error occurs. Probability of this error occurrence is [7]:

$$P_E = \int_{-\infty}^{\infty} G(\xi)p(\xi)d\xi$$

$$= 1 - \int_{-\infty}^{\infty} \left(1 - 0.5\text{erfc}\left(\xi/\sqrt{2}\sigma_{WD}\right)\right)^{N-1} p(\xi)d\xi. \quad (6)$$

Probabilities of the IF error, for various σ/A, are given in Table I. It can be seen that P_E is a rapidly increasing function with respect to σ/A . For values $\sigma/A > 2$ it approaches 100%. The probability of error is uniformly distributed over $N-1$ samples outside the IF. Thus, we can write the IF estimation as:

$$\hat{\omega}(t) = \begin{cases} \omega(t) & \text{with probability } 1 - P_E \\ \omega \in Q_\omega, \omega \neq \omega(t) & \text{with probability } P_E. \end{cases} \quad (7)$$

3. ALGORITHM FOR THE IF ESTIMATION

The IF itself is usually a slow varying function. In the case of a high noise the estimation errors are dominantly of impulse nature. This kind of error can be reduced by applying the median filter directly to the estimated IF [11]. From filter theory it is known that the median filter can eliminate impulses whose occurrence frequency is up to 50%. However, in our experiments we could not get the expected results by using the median filter. Namely, the errors in the IF estimation are not statistically independent. If a large error occurs at the considered point, there is a high probability that the error exists in the neighboring points. This fact significantly reduces the median filter efficiency. It clearly shows that for the IF estimation in the high noise environment we need a more accurate tool. It can be seen from Table I that, for $\sigma/A > 1.7$, the P_E is higher than 70% and the median filter cannot be successfully used. In order to develop a more sophisticated algorithm for the IF estimation in a high noise environment, we will assume the following: (1) If the WD maximum at the considered instant is not at the IF point, there is a high probability that the IF is at a point having one of the largest WD values; (2) Assume that the IF variation between two consecutive points is not large. According to these two assumptions, we can define the algorithm. The basic idea for this algorithm comes from the problem of edge-following in digital image processing [10]. Roughly speaking, our algorithm is similar to following one: connect points at the map such that the path length and the altitude variations become as small as possible.

Consider a time interval $n \in [n_1, n_2]$. Let all paths between n_1 and n_2 belong to a set $\mathbf{K}$. Assume that all paths from the set $\mathbf{K}$ can take only discrete frequency values which belong to the set Q_ω. We form the IF estimate as a path that minimizes the expression:

$$\hat{\omega}(n) = \arg\min_{k(n)\in\mathbf{K}} \left[\sum_{n=n_1}^{n_2-1} g(k(n), k(n+1)) + \sum_{n=n_1}^{n_2} f(WD(n,k(n)))\right] = \arg\min_{k(n)\in\mathbf{K}} p(k(n); n_1, n_2), \quad (8)$$

where $p(k(n); n_1, n_2)$ is a sum of the penalty functions along the line $k(n)$, from the instant n_1 to n_2. The function $g(x,y) = g(|x-y|)$ is nonincreasing with respect to $|x-y|$ (between the IF values in the consecutive points $x = k(n)$ and $y = k(n-1)$), and $f(x)$ is nondecreasing in $x = WD(n,k(n))$. In this way the larger values of the WD become important candidates for the position of the IF at the considered instant. For a considered n, the function $f(x)$ can be formed as follows. The WD values, $WD(n,\omega)$, $\omega \in Q_\omega$, are sorted into the nonincreasing sequence:

$$WD(n,\omega_1) \geq WD(n,\omega_2) \geq .... \geq WD(n,\omega_j) \geq$$
$$... \geq WD(n,\omega_M),\ \omega_j \in Q_\omega,\ j \in [1,M], \quad (9)$$

where $j = 1, 2, ..., M$, is the position within this sequence. Then, the function $f(x)$ is formed as:

$$f(WD(n,\omega_j)) = j - 1. \quad (10)$$

Thus, we have a function which realizes our idea that the points with large WD values are important candidates for the IF estimates. The function $f(x)$ is not formed directly by using values of the WD, since the signal and noise parameters can be time-varying. It means that a particular distribution value at the considered instant may highly probably belong to the signal term, while in other points it can be influenced by noise. For $g(x,y)$ =const., the IF estimation (8) is reduced to the position of the WD maxima. In this paper we will use the penalty function:

$$g(x,y) = \begin{cases} 0 & |x-y| \leq \Delta_1 \\ c(|x-y| - \Delta_1) & \Delta_2 \geq |x-y| > \Delta_1 \\ c(\Delta_2 - \Delta_1) & |x-y| > \Delta_2. \end{cases} \quad (11)$$

The reasonable choice for Δ_1 would be the maximal expected value of the IF variation between consecutive points. It means that there is no additional penalty due to this function for small IF variation (within Δ_1 points, for two consecutive instants). In order to track abrupt changes in the IF function we limited penalty function $g(x,y)$ to the value $c(\Delta_2 - \Delta_1)$.

3.1. Implementation

Let the TF plane contains M frequencies and Q instants, $\mathbf{T} = \{(n_i, \omega_j) | i \in [1,Q], j = [1,M]\}$. There are M^Q paths between the two end instants. This fact makes a direct search for the optimal path impossible. Fortunately, the algorithm can be realized recursively, as an instance of the generalized Viterbi algorithm [8] with the following steps.

(a) Let optimal paths, connecting the instant n_1 and all points to the instant n_i, are determined. Those paths, denoted as $\pi_i(n;\omega_j)$, $n \in [n_1, n_i]$ for $j \in [1, M]$, can be written as:

$$\pi_i(n;\omega_j) = \arg \min_{k(n)\in \mathbf{K}_{ij}} p(k(n); n_1, (n_i, \omega_j)), \ \ j \in [1, M], \tag{12}$$

where the set $\mathbf{K}_{ij}$ contains all paths between the instant n_1 and the point (n_i, ω_j), while $p(k(n); n_1, (n_i, \omega_j))$ is a sum of the path penalty functions for the line $k(n)$. In the Viterbi algorithm terminology, paths (12) are known as the partial best paths. Current IF estimate, within the interval $[n_1, n_i]$, can be written as:

$$\hat{\omega}_{(i)}(n) = \arg \min_{\pi_i(n;\omega_j), j\in[1,M]} p(\pi_i(n;\omega_j); n_1, (n_i, \omega_j)), \tag{13}$$

for the interval $[n_1, n_i]$.

(b) The partial best paths at the next instant n_{i+1} can be represented as concatenation of (12) with the points at the new instant $\pi_{i+1}(n;\omega_j) = [\pi_i(n;\omega_l),(n_{i+1},\omega_j)]$, $j \in [1, M]$, for $l \in [1, M]$, that produce the minimal value of $p(\pi_i(n;\omega_l); n_1, (n_i, \omega_l)) + g(\omega_l, \omega_j) + f(WD(n_{i+1}, \omega_j))$, for each ω_j, $j \in [1, M]$. Note that $f(WD(n_{i+1}, \omega_j))$ is constant for the considered partial best path. For the considered point it is necessary to search over M paths, M^2 for the entire instant, and QM^2 for the entire plane. However, in order to eliminate some of non-optimal paths we used the fact that $g(\omega_l, \omega_j)$ is an increasing function of the distance $|\omega_j - \omega_l|$. We started from the points with the smallest distance, and proceed toward the larger ones. The search is stopped when minimal penalty function in the scanned part is smaller than the minimal function in the remaining part. In this way, we got significant calculation savings. They can be even greater than 50%. The current estimate is the partial best path producing the smallest penalty function. Step (b) should be repeated for each point.

3.2. Examples

We will consider two signals that can be relatively difficult for other non-parametric methods as well as for parametric methods where the signal model is not known. They are: sinusoidally modulated IF, $f_1(t) = \exp(j32\sin(\pi t))$, and signal with abrupt change in the IF, $f_2(t) = \exp(j64\pi|t|)$. Both signals are sampled with $N = 256$ samples within $t \in [-0.5, 0.5]$. They are corrupted by the AWGN with SNR=0 dB. The IF estimates using the WD with Hanning window of the width 128 samples are shown in Figs. 1a and 1b. In both cases thin lines represent true value, dotted lines show the estimates performed with maxima of the WD, and thick lines are the IF estimates performed by the proposed algorithm. It can be seen that the proposed algorithm is accurate in both cases. The parameters were set to $c = 2.5$, $\Delta_1 = 3$ and $\Delta_2 = 30$. Statistical study is performed, as well. Considered range of SNR was $[-10, 10]$ dB. For all values from this range 50 trials is considered. The MSE, obtained with the WD maxima and the proposed algorithm are presented in Fig. 2. The proposed algorithm behaves significantly better than the estimation based on the WD maxima.

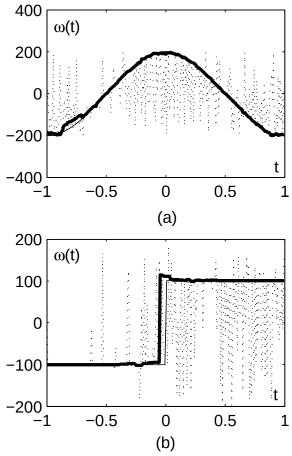

Figure 1: IF estimation: (a) Signal $f_1(n)$; (b) Signal $f_2(n)$. Dotted line - WD maxima; Solid line - Exact value; Thick line - Proposed algorithm.

3.3. Multicomponent Signals

The WD has emphatic cross-terms in the case of multicomponent signals. They can make the IF estimation impossible. This is the reason why distributions with reduced interferences should be used [12, 13]. Here, the weighted pseudo-WD, known as the S-method [13]-[15], will be used. It can produce the auto-terms close to those in the WD, but with significantly reduced cross-terms. Algorithm for the IF estimation of the multicomponent signals can be summarized as follows: (a) Estimate the IF ,$\hat{\omega}^{(0)}(n)$, by using the proposed algorithm and set $i = 0$. This IF corresponds to the highest signal component; (b) Form a new TF representation by taking zero-values in the region around IF estimate $[\hat{\omega}^{(i)}(n) - \delta, \hat{\omega}^{(i)}(n) + \delta]$; (c) Repeat the algorithm for this TF representation, and obtain the next IF estimate $i = i + 1$, $\hat{\omega}^{(i)}(n)$. Steps (b) and (c) should be repeated for each component.

The algorithm will be demonstrated on a two-component signal with well separated IFs:

$$f(t) = \exp(j16\pi|t| - j56\pi t) + \exp(j32\sin(\pi t) + j56\pi t). \tag{14}$$

Signal is sampled with the rate $\Delta t = 1/128$. The IF estimation performed by the proposed algorithm, with parameters $c = 5$, $\Delta_1 = 2$, $\Delta_2 = 40$ and $\delta = 2$ for noise-free signal and the AWGN with SNR=-3 dB, is shown in Figs. 3a and 3b, respectively.

4. CONCLUSION

We have presented an approach for the IF estimation based on the WD and the Viterbi algorithm in a high noise environment. It uses two assumptions: the IF is placed at the position of the TF representation with high magnitude; and the IF is a slow-varying function. The algorithm can be applied not only on the WD, but also on any other TF distribution. It can also be used for the IF estimation of

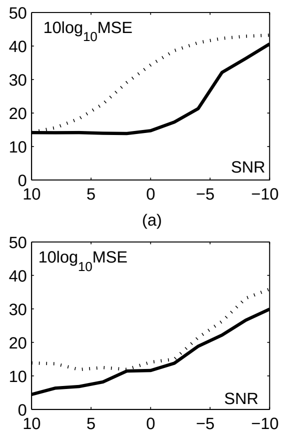

Figure 2: MSE as a function of the SNR: (a) Signal $f_1(n)$; (b) Signal $f_2(n)$. Thick line - Proposed algorithm; Dotted line - WD maxima.

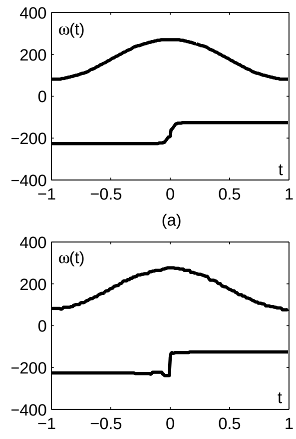

Figure 3: IF estimation of multicomponent signals: (a) Noise–free signal; (b) SNR=−3dB.

signal components in multicomponent signals. An alternative penalty function with more detailed algorithm analysis is presented in [16].

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5. REFERENCES

[1] P. M. Djurić and S. M. Kay, "Parameter estimation of chirp signals," IEEE Trans. ASSP, Vol. 38, No.12, Dec. 1990, pp. 2118-2126.

[2] S. Peleg and B. Porat, "Estimation and classification of polynomial-phase signals," IEEE Trans. Inf. Th., Vol. 42, No.2, Mar. 1991, pp. 422-430.

[3] B. Boashash: "Estimating and interpreting the instantaneous frequency of a signal - Part 1: Fundamentals," Proc. IEEE, Vol. 80, No.4, Apr. 1992, pp. 519-538.

[4] P. Rao and F.J. Taylor: "Estimation of the instantaneous frequency using the discrete Wigner distribution," El. Let., Vol. 26, 1990, pp. 246-248.

[5] C. De Luigi and E. Moreau: "An iterative algorithm for estimation of linear frequency modulated signal parameters," IEEE Sig. Proc. Let., Vol. 9, No.4, Apr. 2002, pp. 127-129.

[6] V. Katkovnik and LJ. Stanković: "Instantaneous frequency estimation using the Wigner distribution with varying and data-driven window length," IEEE Trans. Sig. Proc., Vol. 46, No.9, Sept. 1998, pp. 2315-2326.

[7] I. Djurović and LJ. Stanković: "Influence of high noise on the instantaneous frequency estimation using time-frequency distributions," IEEE Sig. Proc. Let., Vol. 7, No.11, Nov. 2000, pp. 317-319.

[8] G. D. Forney: "The Viterbi algorithm," Proc. IEEE, Vol.61, No.3, Mar. 1973, pp. 268-278.

[9] LJ. Stanković and S. Stanković: "On the Wigner distribution of the discrete-time noisy signals with application to the study of quantization effects," IEEE Trans. Sig. Proc., Vol. 42, No.7, July 1994, pp. 1863-1867.

[10] A. Martell: "Edge detection using heuristic search methods," Computer Graphics and Image Processing, Vol. 1, No.2, Aug. 1982, pp. 169–182.

[11] I. Djurović and LJ. Stanković: "Robust Wigner distribution with application to the instantaneoues frequency estimation," IEEE Trans. Sig. Proc., Vol. 49, No.12, Dec. 2001, pp. 2985-2993.

[12] J. Jeong and W.J. Williams: "Kernel design for reduced interference distributions," IEEE Trans. Sig. Proc., Vol. 40, No.2, Feb. 1992, pp. 402-412.

[13] LJ. Stanković: "A method for time-frequency signal analysis," IEEE Trans. Sig. Proc., Vol. 42, No.1, Jan. 1994, pp. 225-229.

[14] B. Boashash and B. Ristić: "Polynomial time-frequency distributions and time-varying higher order spectra: Applications to analysis of multicomponent FM signals and to treatment of multiplicative noise," Sig. Proc., Vol. 67, No.1, May 1998, pp. 1-23.

[15] L.L. Scharf and B. Friedlander: "Toeplitz and Hankel kernels for estimating time-varying spectra of discrete-time random process," IEEE Trans. Sig. Proc., Vol. 49, No.1, Jan. 2001, pp. 179-189.

[16] I. Djurović and LJ. Stanković: "An algorithm for the Wigner distribution based instantaneous frequency estimation in a high noise environment," IEEE Trans. Sig. Proc., revised.