OPTIMAL BIT-RATE ALLOCATION AND SYNTHESIS FILTER BANK DESIGN FOR MULTIRATE SUBBAND CODING SYSTEMS

Huan Zhou, Lihua Xie, Cishen Zhang

School of Electrical and Electronic Engineering
Nanyang Technological University, Singapore 639798

ABSTRACT

In this paper, a joint optimal design method for multirate subband coding systems is presented. For a multirate subband coding system with given analysis filters and average bit number of data quantizers, the design is to simultaneously find the synthesis filters and bit number allocation of the subband quantizers such that the variance of the reconstruction error of the system is minimized. We propose an iterative approach for obtaining the synthesis filters and the bit number allocation that minimize the reconstruction error of the system. Our simulation example demonstrate the favorable performance of the proposed method as compared with existing methods.

1. INTRODUCTION

Multirate subband coding systems are essential in information and signal coding. Two most essential components of a multirate subband coding system are a multirate filter bank system and a data quantization device. The multirate subband coding system is a quite complicated system involving periodic and nonlinear operations and signals at different sampling rates and with different data formats. At this stage, the available design methods are to design the multirate filter bank and the data quantization of the multirate subband coding systems separately. For example, some research results have been on the orthogonal (paraunitary) filter banks [4, 7] where orthogonal analysis/synthesis filters are first designed to obtain perfect signal reconstruction without considering the quantization and channel noises. The quantizers are then optimized to achieve the maximal coding gain based on the designed synthesis filter bank. Since the synthesis filter bank and the bit number of each subband quantizer jointly affect the system reconstruction error, this design approach may not be able to provide an overall optimal signal reconstruction and coding gain performance in the presence of quantization and channel noises. In fact, it has been known that a better coding gain may be achieved by a nonunitary filter bank [2].

In this paper, we consider an M-channel multirate subband coding system with a sequential multiple sampling scheme [9, 10]. The filters of the filter banks are FIR and the data quantizers are in the pdf-optimized quantizer model [3]. Assume that the FIR analysis filter bank is given which can provide desirable subband signals. Then the proposed joint optimal design is to design an FIR synthesis filter bank and allocate bit numbers to the subband data quantizers to minimize the variance of the overall reconstruction error caused by the filtering and multirate sampling distortions and subband data quantization errors. We will simplify this complicated joint optimization problem by using an iterative optimization approach, which can provide a better signal reconstruction and a higher coding gain as compared to traditional design methods such as the paraunitary filter bank.

2. STRUCTURE AND FORMULATION OF SUBBAND CODING SYSTEMS

Sequentially sampled multirate filter bank has been studied in [9, 10] where its computational and implementary advantages have been analyzed and demonstrated. Its equivalent periodic multirate system is shown in Figure 1, where decimators and interpolators operate periodically and sequentially with $\downarrow_i M$ and $\uparrow^i M$, $i = 1, 2 \cdots, M$, denoting, respectively, their operating at $n = kM + i$, and $h(n, m)$ and $f(n, m)$ are two SISO M-periodic filters. It has been shown [9] that the above sequentially sampled multirate filter bank is equivalent to a standard M-channel simultaneously sampled multirate filter bank with analysis and synthesis filters $h_i(n)$ and $f_i(n)$, $n = 1, 2, \cdots, M$ from an input and view point of view. It is easy to verify that the output signal $y(n)$

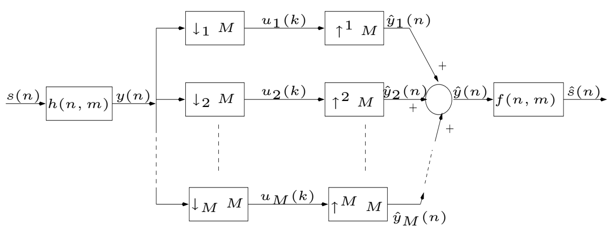

Fig. 1. Sequentially operated LPTV filter structure

of the periodic filter $h(n,m)$ and the input signal $\hat{y}(n)$ of the periodic filter $f(n,m)$ satisfy $\hat{y}(n) = y(n)$.

In this paper, we use the pdf-optimized quantizer model [5] to represent the quantizers Q_i, $i = 1, 2, \cdots, M$, of the subband coding system. Assume that the input $u_i(k)$ to the quantizer Q_i is a wide sense stationary random process with variance $\sigma^2_{u_i}$. Let $\sigma^2_{q_i}$ be the variances of the quantization error $q_i(k)$. It follows from the rate distortion theory [3] that the quantization error variance satisfies

$$\sigma^2_{q_i} = \beta(r_i) 2^{-2r_i} \sigma^2_{u_i}. \tag{2.1}$$

For simplicity, the quantizer performance factor $\beta(r_i)$ can be approximated by a constant c. Thus the quantization error can be rewritten as

$$\sigma^2_{q_i} = c 2^{-2r_i} \sigma^2_{u_i}. \tag{2.2}$$

In this expression, the error variance $\sigma^2_{q_i}$ is dependent on both the quantizer input signal and bits assigned to Q_i. Following from the Figure 1, the sequentially sampled subband coding system with quantizers is equivalent to the system in Figure 2, where $q(n)$ is the quantization error which is defined as $q(n) = q_i(k)$ when $n = kM + i$, where $q_i(k)$ is the quantization error of the i-th subband. Obviously, $\sigma^2_q(n)$, the variance of $q(n)$, is M-periodic, i.e. $\sigma^2_q(n + M) = \sigma^2_q(n)$. It is clear from Figure 2 that

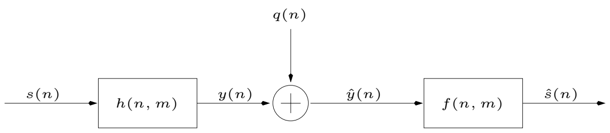

Fig. 2. An equivalent periodic filter structure of sequentially sampled subband coding system.

$$\hat{s}(n) = f(n,m) * h(n,m) * s(n) + f(n,m) * q(n). \tag{2.3}$$

3. JOINT OPTIMIZATION PROBLEM

In this paper, we assume that the M-channel multirate analysis filter bank has been designed to meet some subband decomposition specifications. Further assume that the available average bit number for the data quantizers of the M-channels is fixed, which is

$$R = \frac{1}{M} \sum_{i=1}^{M} r_i. \tag{3.1}$$

Let the reconstruction error be the difference between the reconstructed signal $\hat{s}(n)$ and the system input signal shifted by d steps, i.e.

$$e(n) = \hat{s}(n) - s(n-d). \tag{3.2}$$

It follows from (2.3) that the reconstruction error can be expressed as

$$e(n) = (f(n,m)*h(n,m) - \delta(n-d)) * s(n) + f(n,m) * q(n), \tag{3.3}$$

where $\delta(n-d)$ is the impulse response of the d-shift operator. We make an assumption that the input signal $s(n)$ is a WSS process and there exists a stable whitening filter $w(n)$ in terms of impulse response and a white noise process $s_w(n)$ with unit variance such that $s(n) = w(n) * s_w(n)$. Since $w(n)$, the whitening filter, and $h_i(n)$, $i = 1, 2, \cdots, M$, of the analysis filter bank are known, the quantization noise variance $\sigma^2_q(n) = \sigma^2_q(kM + i) = \sigma^2_{qi}$ can be easily calculated from (2.2). It follows from (3.3) that the error system $\mathcal{E}:\ (s_w, q_w) \to e$ is given by

$$\begin{aligned} e(n) &= (f(n,m) * h(n,m) - \delta(n-d)) * w(n) * s_w(n) \\ &\quad + f(n,m) * (\sigma_q(n) q_w(n)) \\ &= \mathcal{E}_w s_w(n) + \mathcal{E}_q q_w(n), \end{aligned} \tag{3.4}$$

whose inputs are the uncorrelated white noises $s_w(n)$ and $q_w(n)$ both with unit variance and the output is the reconstruction error $e(n)$.

We aim to formulate the reconstruction error variance and optimize it using the linear matrix inequality (LMI) approach. This requires that the error equation (3.4) be written into a state equation form.

Lemma 3.1: *A state space representation for the subband coding error system (3.4) is*

$$(\mathcal{E}): \quad \begin{aligned} \tilde{x}(n+1) &= \tilde{A}_n x(n) + \tilde{B}_n \tilde{s}(n), \\ e(n) &= \tilde{C}_n x(n) + \tilde{D}_n \tilde{s}(n), \end{aligned} \tag{3.5}$$

where

$$\tilde{A}_n = \begin{bmatrix} \hat{A} & \hat{B}C_n & 0 & \hat{B}D_n\bar{\bar{C}} \\ 0 & A & 0 & B\bar{\bar{C}} \\ 0 & 0 & \bar{A} & \bar{B}\bar{\bar{C}} \\ 0 & 0 & 0 & \bar{\bar{A}} \end{bmatrix} \in \mathcal{R}^{\tilde{N}\times\tilde{N}},$$

$$\tilde{B}_n = \begin{bmatrix} \hat{B}D_n\bar{\bar{D}} & \hat{B}\hat{b}_n \\ B\bar{\bar{D}} & 0 \\ \bar{B}\bar{\bar{D}} & 0 \\ \bar{\bar{B}} & 0 \end{bmatrix} \in \mathcal{R}^{\tilde{N}\times 2},$$

$$\tilde{C}_n = \begin{bmatrix} \hat{C}_n & \hat{D}_n C_n & -\bar{C} & \hat{D}_n D_n \bar{\bar{C}} \end{bmatrix} \in \mathcal{R}^{1\times\tilde{N}},$$

$$\tilde{D}_n = \begin{bmatrix} \hat{D}_n D_n \bar{\bar{D}} & \hat{D}_n \hat{b}_n \end{bmatrix} \in \mathcal{R}^{1\times 2}. \tag{3.6}$$

while (A, B, C_n, D_n), $(\hat{A}, \hat{B}, \hat{C}_n, \hat{D}_n)$, $(\bar{A}, \bar{B}, \bar{C}, \bar{D})$ and $(\bar{\bar{A}}, \bar{\bar{B}}, \bar{\bar{C}}, \bar{\bar{D}})$ are respectively the state-space representations of analysis filter $h(n,m)$ with order N, the synthesis filter $f(n,m)$ with order $\hat{N}$, the d-shift system and the whitening filter $w(n)$ with order N_w. Note that since the synthesis filters are FIR, $\hat{A}$ and $\hat{B}$ can be known matrices [10].

Since the reconstruction error signal $e(n)$ is the output of the periodic system (3.5)-(3.6) with wide sense stationary white noise inputs $s_w(n)$ and $q_w(n)$, it is a wide sense cyclostationary process. Then, $\sigma^2_{e,n}$, the variance of the reconstruction signal $e(n)$, satisfies $\sigma^2_e(n) = \sigma^2_e(n+M)$. Let $\bar{\sigma}^2_e$ be the average variance of $e(n)$ defined as $\bar{\sigma}^2_e = \frac{1}{M}\sum_{i=1}^{M}\sigma^2_e(i)$. Then, it is known [10] that $\bar{\sigma}^2_e = \|\mathcal{E}\|^2_2$.

It follows from the analysis of the preceding subsections that the H_2 norm of the reconstruction error system is dependent on both the synthesis filter bank system and the quantizer bit numbers of each subband channel. Thus we propose the joint optimization problem for the design of the subband coding system.

4. JOINT OPTIMAL DESIGN

4.1. Optimization of synthesis filter bank system with fixed bit numbers

Theorem 4.1 *[10] Given the analysis filter bank and the bit number allocation $r_i,\ i = 1, 2\cdots, M$, the optimal synthesis FIR filter bank can be designed by the following convex optimization:*

$$\min_{S_n, Q_n, \hat{C}_n, \hat{D}_n} \|\mathcal{E}\|^2_2 = \frac{1}{M}\min_{S_n, Q_n, \hat{C}_n, \hat{D}_n}\sum_{n=1}^{M} trace(S_{n+1}), \tag{4.1}$$

subject to

$$L_1 = \begin{bmatrix} -S_{n+1} & \tilde{B}_n^T Q_{n+1} & \tilde{D}_n^T \\ Q_{n+1}\tilde{B}_n & -Q_{n+1} & 0 \\ \tilde{D}_n & 0 & -I \end{bmatrix} < 0, \tag{4.2}$$

$$L_2 = \begin{bmatrix} -Q_n & \tilde{A}_n^T Q_{n+1} & \tilde{C}_n^T \\ Q_{n+1}\tilde{A}_n & -Q_{n+1} & 0 \\ \tilde{C}_n & 0 & -I \end{bmatrix} < 0, \tag{4.3}$$

for $n = 1, 2, \cdots, M$, simultaneously, where S_n and Q_n are symmetric positive definite matrices with $S_1 = S_{M+1},\ Q_1 = Q_{M+1}$, and $\tilde{A}_n,\ \tilde{B}_n,\ \tilde{C}_n$ and $\tilde{D}_n$ are periodic matrices as defined in (3.6).

Remark 4.1 *The above optimization is a convex optimization and can be solved by employing the Matlab LMI Toolbox [1].*

4.2. Optimization of bits allocation with fixed synthesis filter bank

Assume that the synthesis filter bank is known. The bit number allocation problem can be stated as: *with the known analysis and synthesis filter bank parameters $(C_n,\ D_n, \hat{C}_n, \hat{D}_n)$, find integer M-periodic bits allocation r_n, $n = 1, 2\cdots, M$, satisfying the bit constraints (3.1) and $r_n \in Z^+$, such that the H_2 norm of the error system (3.5), $\|\mathcal{E}\|_2$, is minimized.*

Observe from (3.4) that $\mathcal{E}_w$ is independent of the bit rates r_n. Further, since $s_w(n)$ and $q_w(n)$ are assumed to be independent, the optimization of $\|\mathcal{E}\|_2$ is then equivalent to that of minimizing

$$J = \|\mathcal{E}_q\|^2_2 = \frac{1}{M}\sum_{n=1}^{M} trace(\hat{D}_n^T\hat{D}_n + \hat{B}^T\bar{Q}_n\hat{B})\sigma^2_q(n) \tag{4.4}$$

subject to bit constraint (3.1) and $\bar{r} \subset Z^+$, where $\bar{Q}_n$ is the solution of the periodic Lyapunov equation:

$$\hat{A}_n^T\bar{Q}_n\hat{A}_n - \bar{Q}_n + \hat{C}_n^T\hat{C}_n = 0. \tag{4.5}$$

If we relax the integer constraint first, the problem can be solved by applying the Lagrange multiplier method. Introduce Lagrangian function $L(\bar{r}, \lambda)$ as

$$L(\bar{r}, \lambda) = J - \lambda(\sum_{n=1}^{M} r_n - MR), \tag{4.6}$$

which is a function of $r_1,\ r_2, \cdots, r_M$ and λ. Then an optimal solution is obtained only if the following M equations hold:

$$\frac{\partial L(\bar{r}, \lambda)}{\partial r_n} = 0, \quad n = 1, 2\cdots, M \tag{4.7}$$

together with constraint (3.1).

4.3. Iterative approach to the joint design of the synthesis filter bank and bit numbers

In the previous section, we have shown that for the given quantizer bit numbers, the optimal synthesis filter bank can be designed using the LMI approach. Conversely, for the given synthesis filter bank, a relaxed bit number solution can be derived by using Lagrange multiplier method. We are now in the position to present an iterative design procedure as follows:

- Step 1. Given an initial set of bit numbers satisfying the constraint (3.1), design a synthesis filter bank by applying Theorem 4.1 to minimize the H_2 norm of the reconstruction error system, i.e. $\min_{\hat{C}_n, \hat{D}_n} \|\mathcal{E}\|_2$.

- Step 2. Fix the synthesis filter bank as derived in the last step. Find a relaxed optimal bit number solution by minimizing the cost J using the Lagrange multiplier approach. Note that the optimal solution $\bar{r}$ may not necessarily be an integer solution at this stage.

- Step 3. Fix the derived bit numbers obtained in the last step. Redesign the synthesis filter bank to minimize the H_2 norm of the reconstruction error system.

- Step 4. Repeat Steps 2 and 3 until the difference of the H_2 norm of the reconstruction error system between the two consecutive iterations is less than a prespecified tolerance ζ. Denote the derived optimal bit allocation by $\bar{r}_{opt}$.

Now the derived $\bar{r}_{opt}$ may not be integers. To obtain a set of optimal integer bit numbers, we apply the Branch and bound method [6] in integer programming.

5. EXAMPLE AND PERFORMANCE ANALYSIS

We adopt a 2-channel analysis filter bank given in [8] to make a comparison between the proposed approach and the paraunitary filter bank (PUFB) design approach. The frequency response of the analysis filter bank is shown in Figure 3. By the PUFB approach, bits allocation is calculated

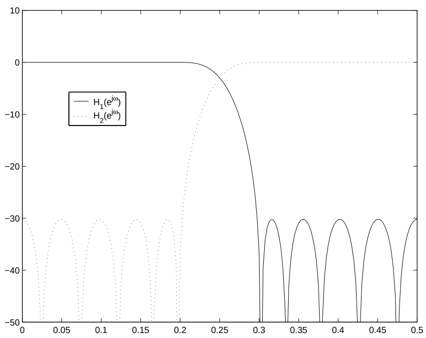

Fig. 3. Magnitude responses of a 2-band paraunitary filter bank with filter length 19 each

by [7]

$$r_i = R + 0.5log_2 \frac{\sigma_{u_i}^2}{(\Pi_{i=1}^{M} \sigma_{u_i}^2)^{1/M}}$$

together with the Branch and bound method for obtaining an integer solution. The comparison between the PUFB and the filter bank designed by the proposed approach is tabulated in Tables 1 and 2, where the input is a Gaussian-Markov 1 process with auto-correlation coefficient of 0.9 and SNR stands for the signal (input signal)-to-noise (reconstruction error) ratio. The results show that the proposed

Table 1. Reconstruction results comparison between PUFB and the proposed FB, for different bit budgets

R	PUFB design		proposed approach	
	int. bits	SNR_{PUFB}	joint bits	SNR_{joint}
2	[3 1]	14.7852	[3 1]	19.0265
3	[4 2]	20.8058	[4 2]	24.0851
4	[5 3]	26.8264	[5 3]	29.7126

approach not only has a better reconstruction performance in terms of SNR, but also gives higher coding gains. These also testify that nonunitary filter bank can have a larger coding gain than a unitary one [2].

Table 2. Coding gain comparison

R	G_{PUFB}	G_{joint}
2	5.0789	19.3798
3	5.0789	12.7891
4	5.0789	11.0273

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