SIMPLIFIED LMS ALGORITHMS IN THE CASE OF NON-UNIFORMLY SAMPLED SIGNALS

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ABSTRACT

In a previous paper we introduced an adaptive ARMA estimation method for time series with missing samples [1]. Due to the non-linearity of the optimization criterion in the case of missing observations, the proposed method has led to an LMS-like algorithm with a higher computational complexity than the standard LMS. As many applications require very low complexity algorithms [2], the purpose of the present paper is to introduce simplified versions of the LMS adapted to the non-uniform sampling context. Both waveform reconstruction performances and computational costs are evaluated as a function of probability density of the sampling process. Stationary and non-stationary contexts are considered.

1. INTRODUCTION

In many physical systems, signals are sampled in a deliberately non-uniform manner. Such cases are found, for example, for power saving or, more generally for data compression purpose. In these cases, samples are supposed to be on a periodic frame, some of them being missing. Random pattern of loss may be various depending on the application: for instance, Bernouilli law (constant probability of loss), or Markov law for packet-mode loss distribution. An ARMA model being suitable for a large class of signals, an ARMA identification process has been proposed in [1] for time domain reconstruction or spectral estimation of such non-uniformly sampled signals. In the present paper we adopt a classical approximation approach in order to drastically reduce the computational burden of the standard LMS for adaptive imbedded algorithms. This approach is applied to the previous real-time ARMA identification method in the case of missing observations. The outline of the paper is as follows. In Sec. 2.1 a brief overview of the simplified variants of the LMS frequently used in adaptive filtering applications, namely the signed-error and the sign-sign LMS, is given. In Sec. 3 the algorithms derivation for non-uniformly sampled signals (NUSS) is carried out and in Sec. 4 the resulting algorithms are applied to a synthetic ARMA signal and then to a speech signal for a validation of our low computational cost adaptive predictor.

2. BACKGROUND

2.1. Simplified LMS algorithms

For an ARMA(N_a, N_b) adaptive predictor, the predicted signal $\hat{y}_n$ is classically given by (eq. 1):

$$\hat{y}_n = \boldsymbol{\theta}^\top \boldsymbol{h}_n \tag{1}$$

with: $\boldsymbol{\theta}^\top = [\boldsymbol{\theta}_a^\top \quad \boldsymbol{\theta}_b^\top]$, $\boldsymbol{h}_n^\top = [\boldsymbol{y}_n^\top \quad \boldsymbol{e}_n^\top]$, and:

$$\begin{cases} \boldsymbol{y}_n^\top = [y_{n-1}, \ldots, y_{n-N_a}] & \\ \boldsymbol{e}_n^\top = [e_{n-1}, \ldots, e_{n-N_b}] & \\ \boldsymbol{\theta}_a^\top = [a_1, \ldots, a_{N_a}] & \text{AR parameters} \\ \boldsymbol{\theta}_b^\top = [b_1, \ldots, b_{N_b}] & \text{MA parameters} \end{cases} \tag{2}$$

The standard LMS algorithm is:

$$\begin{cases} \boldsymbol{\theta}_{a,n+1} = \boldsymbol{\theta}_{a,n} + \mu_a e_n \boldsymbol{y}_n \\ \boldsymbol{\theta}_{b,n+1} = \boldsymbol{\theta}_{b,n} + \mu_b e_n \boldsymbol{e}_n \end{cases} \tag{3}$$

Although standard LMS is computationally quite simple, there are always applications for which even $(N_a + N_b)$ multiplications are too many. The introduction of sign operators respectively on the prediction error and / or on the information vector in the typical adaptive parameter update kernel (eq. 3) leads to a drastic reduction in the computational load. This gives respectively the signed-error (eq. 4) and the sign-sign LMS (eq. 5) algorithms [2][3][4]:

$$\begin{cases} \boldsymbol{\theta}_{a,n+1} = (1-\alpha)\,\boldsymbol{\theta}_{a,n} + \mu_a sign\,(e_n)\,\boldsymbol{y}_n \\ \boldsymbol{\theta}_{b,n+1} = (1-\beta)\,\boldsymbol{\theta}_{b,n} + \mu_b sign\,(e_n)\,\boldsymbol{e}_n \end{cases} \tag{4}$$

$$\begin{cases} \boldsymbol{\theta}_{a,n+1} = (1-\alpha)\,\boldsymbol{\theta}_{a,n} + \mu_a sign\,(e_n)\,sign\,(\boldsymbol{y}_n) \\ \boldsymbol{\theta}_{b,n+1} = (1-\beta)\,\boldsymbol{\theta}_{b,n} + \mu_b sign\,(e_n)\,sign\,(\boldsymbol{e}_n) \end{cases} \tag{5}$$

Where the α and β leakage factors are introduced to prevent the instability of these algorithms in ideal use due to the inclusion of sign operators [4].

2.2. NUSS LMS

In the case of missing samples, the problem may be written as follows with a vector built with the L last samples in order to approximate the mean square error using a sliding window of L samples, this approximation being well suited for a large number of applications [1]:

$$\hat{\boldsymbol{y}}_{n+1,L} = \boldsymbol{H}_{n+1,L}^{\top}\boldsymbol{\theta}, \tag{6}$$

where:

$$\boldsymbol{H}_{n+1,L} = \begin{bmatrix} h_n & \cdots & h_{n-L} \\ \vdots & \vdots & \vdots \\ h_{n-N_a+1} & \cdots & h_{n-L-N_a+1} \\ h_n - \boldsymbol{\theta}^{\top}\boldsymbol{h}_n & \cdots & h_{n-L} - \boldsymbol{\theta}^{\top}\boldsymbol{h}_{n-L} \\ \vdots & \vdots & \vdots \\ h_{n-N_b+1} & & h_{n-L-N_a+1} \\ -\boldsymbol{\theta}^{\top}\boldsymbol{h}_{n-N_b+1} & \cdots & -\boldsymbol{\theta}^{\top}\boldsymbol{h}_{n-L-N_b+1} \end{bmatrix},$$

$$h_{n-j} = \begin{cases} y_{n-j} & \text{if sample } n-j \text{ is known,} \\ \hat{y}_{n-j} & \text{if sample } n-j \text{ is lost,} \end{cases}$$

$$\boldsymbol{h}_n = \begin{bmatrix} h_{n-1} \\ \vdots \\ h_{n-N_a} \\ h_{n-1} - \boldsymbol{\theta}^{\top}\boldsymbol{h}_{n-1} \\ \vdots \\ h_{n-N_b} - \boldsymbol{\theta}^{\top}\boldsymbol{h}_{n-N_b} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_{a,n} \\ \boldsymbol{h}_{b,n} \end{bmatrix}.$$

An LMS-like algorithm is built with a gradient optimization formula to update the parameters when the sample is available [1]:

$$\begin{cases} \boldsymbol{\theta}_{a,n+1} = \boldsymbol{\theta}_{a,n} - \mu_a \dfrac{\partial J_{n+1,L}}{\partial \boldsymbol{\theta}_a} \\ \boldsymbol{\theta}_{b,n+1} = \boldsymbol{\theta}_{b,n} - \mu_b \dfrac{\partial J_{n+1,L}}{\partial \boldsymbol{\theta}_b} \end{cases} \tag{7}$$

$$J_{n+1,L} = \frac{1}{L}\left(\boldsymbol{y}_{n+1,L} - \hat{\boldsymbol{y}}_{n+1,L}\right)^{\top}\left(\boldsymbol{y}_{n+1,L} - \hat{\boldsymbol{y}}_{n+1,L}\right) \tag{8}$$

the involved matrices and derivatives being built in a recursive way.

The main drawback of the NUSS LMS is its relatively high complexity. Actually, the equations giving the gradient lead to the following complexity per sample:

$$\begin{cases} 2(N)^3 + 2(N)^2 & \text{missing sample} \\ 3(N)^3 + 4(N)^2 + N & \text{available sample} \end{cases} \tag{9}$$

Where: $N = N_a + N_b$

3. NUSS SIMPLIFIED LMS ALGORITHMS

3.1. NUSS simplified LMS

The first step to reduce NUSS LMS complexity is to neglect the dependence on $\boldsymbol{\theta}$ of the $\boldsymbol{H}$ matrix in the computation of the derivatives. This approximation is justified in non stationary context where the evolution of the $\boldsymbol{\theta}$ parameters leads to an inaccuracy in the contribution of the second order terms in $\boldsymbol{\theta}$. Its application leads to the following formula in place of the recursive derivation of the equations 7 and 8:

$$\begin{cases} \boldsymbol{\theta}_{a,n+1} = \boldsymbol{\theta}_{a,n} + \mu_a \boldsymbol{h}_{b,n+1}(1)\,\boldsymbol{h}_{a,n} \\ \boldsymbol{\theta}_{b,n+1} = \boldsymbol{\theta}_{b,n} + \mu_b \boldsymbol{h}_{b,n+1}(1)\,\boldsymbol{h}_{b,n} \end{cases} \tag{10}$$

The adopted approach allows a significant complexity reduction:

$$\begin{cases} N_a + N_b & \text{missing sample} \\ 2(N_a + N_b) & \text{available sample} \end{cases} \tag{11}$$

3.2. NUSS sign family LMS

The second step to complete the previous complexity reduction, is to introduce the same sign operators as in the periodic sampling case. The resulting adaptive update kernels are given in equations (12) and (13) in which the use of sign operators requires only bits sliding.

3.2.1. NUSS sign-error LMS

$$\begin{cases} \boldsymbol{\theta}_{a,n+1} = (1-\alpha)\,\boldsymbol{\theta}_{a,n} + \mu_a sign\left(\boldsymbol{h}_{b,n+1}(1)\right)\boldsymbol{h}_{a,n} \\ \boldsymbol{\theta}_{b,n+1} = (1-\beta)\,\boldsymbol{\theta}_{b,n} + \mu_b sign\left(\boldsymbol{h}_{b,n+1}(1)\right)\boldsymbol{h}_{b,n} \end{cases} \tag{12}$$

3.2.2. NUSS sign-sign LMS

$$\begin{cases} \boldsymbol{\theta}_{a,n+1} = (1-\alpha)\,\boldsymbol{\theta}_{a,n} + \mu_a sign\left(\boldsymbol{h}_{b,n+1}(1)\right) sign\left(\boldsymbol{h}_{a,n}\right) \\ \boldsymbol{\theta}_{b,n+1} = (1-\beta)\,\boldsymbol{\theta}_{b,n} + \mu_b sign\left(\boldsymbol{h}_{b,n+1}(1)\right) sign\left(\boldsymbol{h}_{b,n}\right) \end{cases} \tag{13}$$

These approaches lead to very low complexity algorithms the behavior of which is evaluated in the next section.

4. SIMULATION RESULTS

4.1. Tests signals

In order to evaluate and to compare the performances of the proposed algorithms with the complete NUSS LMS algorithm [1] in a stationary context, a low-pass ARMA $(2, 2)$ signal is considered. This signal is generated as the output of an elliptic filter (passband ripple $R_p = 4dB$, stopband attenuation $R_s = 15dB$ and cut-off frequency: $\nu_c = 0.15$). The sampling scheme is defined as follows: each sample of the uniformly sampled signal has the probability $q = 1 - p$ of being lost (Bernouilli law). All simulations are carried out with adaptation steps $\underline{\mu}$ satisfying the classical stability conditions [1]:

$$0 < \mu_a N_a \sigma_y^2 + \mu_b N_b \sigma_e^2 < 1 \tag{14}$$

where : $\underline{\mu} = [\mu_a \quad \mu_b]^\top$, σ_y^2 being the signal variance and σ_e^2 the prediction error variance. The performances are also evaluated in a non-stationary context with various speech signals.

4.2. Algorithms convergence

Preliminary studies of the algorithms convergence has shown that the NUSS sign-sign LMS algorithm converges significantly more slowly than the others NUSS simplified LMS algorithms. This may be seen with the $\boldsymbol{\theta}_a(1)$ parameter in figure 1 where the same μ_a has been used for all the algorithms. However, it may be noticed in figure 2 a very similar behavior for all the algorithms when an adapted value for μ_a is used. Moreover, the speed of convergence for

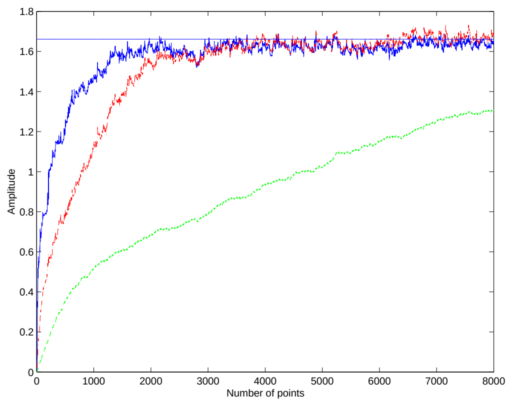

Fig. 1. *Original (—) and estimated parameters: NUSS simplified LMS (blue)(—), sign-error LMS (red)(-.-.-), sign-sign LMS (green)(- - -) for* $p = 1$

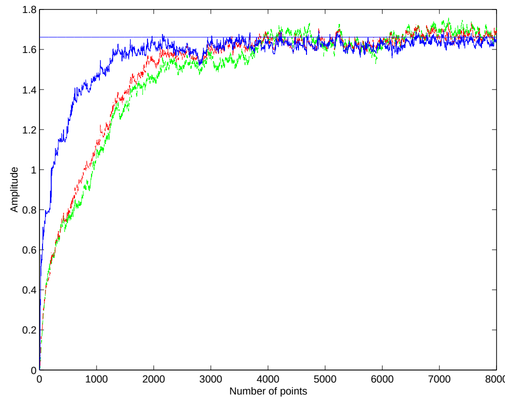

Fig. 2. *Original (—) and estimated parameters: NUSS simplified LMS (blue)(—), sign-error LMS (red)(-.-.-), sign-sign LMS (green)(- - -) for* $p = 1$

all algorithms is also depending on the probability of lost samples as it may be seen in figures 3 and 4 which give respectively the NUSS simplified LMS and the NUSS sign-sign LMS mean square prediction error for different values of the probability p.

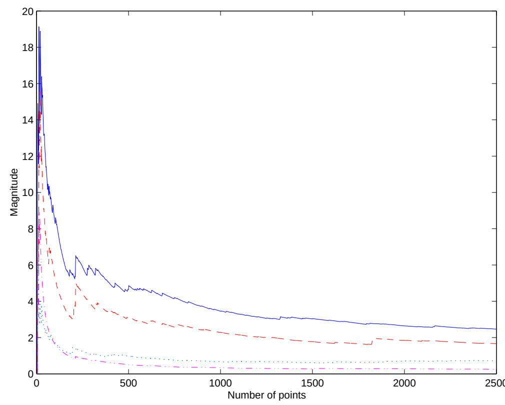

Fig. 3. *Mean square error for the NUSS simplified LMS:* $p = 0.9$ *(-.-.-),* $p = 0.8$ *(...),* $p = 0.7$ *(- - -),* $p = 0.6$ *(—)*

4.3. Time domain reconstruction

4.3.1. stationary context

Averaged values of signal to prediction noise ratios (evaluated after the convergence delay of the algorithms) are presented in table 1 for all algorithms and for different values of probability p. Very similar performances are obtained for the simplified algorithms. They are lightly below those of the NUSS LMS for small densities of lost samples. For higher densities, opposite results may be seen. This can be explained by the increase of lost samples density which

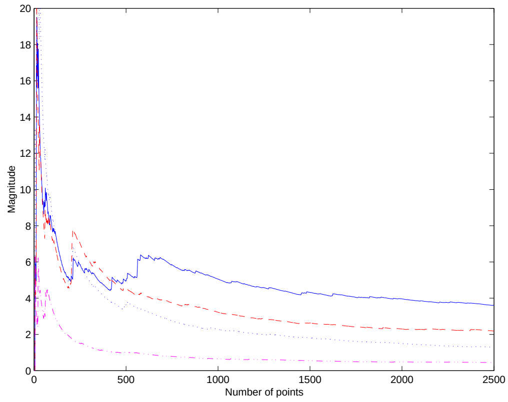

Fig. 4. *Mean square error for the NUSS sign-sign LMS: $p = 0.9$ (-.-.-), $p = 0.8$ (...), $p = 0.7$ (- - -), $p = 0.6$ (—)*

leads to an increase of cumulative errors in the iterative process to compute the derivative of the H matrix.

p	0.9	0.8	0.7	0.6
NUSS LMS	21	16.3	12.6	9.4
NUSS simplified LMS	20.5	16	12.6	10
NUSS sign-error LMS	20.6	16	12.9	10.1
NUSS sign-sign LMS	20.5	16.1	12.6	10.1

Table 1: Signal-to-prediction noise ratio (dB).

The performances of the waveform reconstruction are also very similar for the simplified algorithms. An example of the quality of the signal reconstruction is given in figure 5 for the NUSS sign-sign LMS in the case where 20% of the samples are lost.

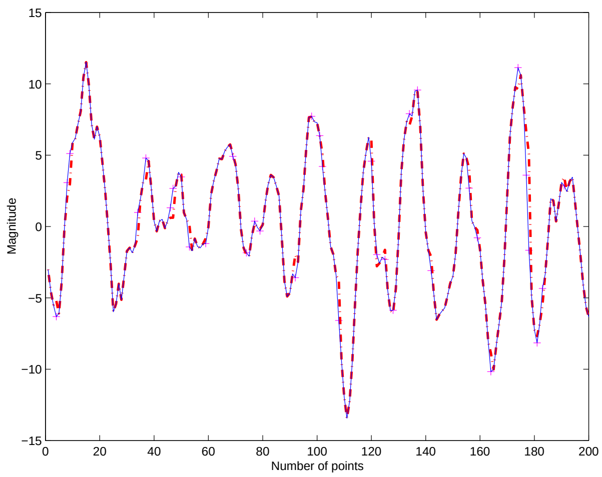

Fig. 5. *Original (—) and reconstructed (-.-.-) signal, missing samples (++) with the NUSS sign-sign LMS*

4.3.2. non-stationary context

Number of simulations have been carried out to evaluate the performances of the proposed algorithms for speech signals. The results reported here come from the sentence: "Mary had a little lamb its fleece was white as snow". As mentioned above, it may be seen in table 2, that in non-stationary context, the performances of the NUSS LMS aren't anymore above those of the NUSS simplified LMS. It may be noticed that at least both the NUSS simplified LMS and the NUSS sign-error LMS are well suited for the ARMA identification of non uniformly sampled speech signals.

p	0.9	0.8	0.7	0.6
NUSS LMS	19.2	14.6	11.6	9.1
NUSS simplified LMS	19.1	14.8	11.8	9.4
NUSS sign-error LMS	18.9	14.6	11.4	9
NUSS sign-sign LMS	17.9	13.7	10.5	8.4

Table 2: Signal-to-prediction noise ratio (dB).

5. CONCLUSION

A significant reduction of the NUSS LMS algorithm complexity, for similar performances, may be reached by the proposed simplified versions. These algorithms may be used for various applications as data compression. In a previous work [6], it was shown that such algorithms are required in ADPCM speech coders with adaptive transmission.

6. REFERENCES

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