RATE-LIMITED EAFRP: A NEW IMPROVED MODEL FOR HIGH-SPEED NETWORK TRAFFIC

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ABSTRACT

The EAFRP model has been recently proposed for modeling the self-similar and impulsive traffic of high-speed networks. For mathematical simplicity, it assumes that the available transmission bandwidth in the network is infinite. We here propose a modification of the model that takes into account the fact that the network has a limit, R, on the total traffic rate through it, and in addition, each user's traffic rate is often independently limited to a value, L, which is significantly lower than the network's limit ($L < R$). We show that the existence of these two rate limits, L and R, results in the distinctive two slope behavior of the loglog survival function of the overall traffic, a fact that has not been explained so far by existing models. We further show that it achieves a closer approximation of the observed reality than the EAFRP model.

1. INTRODUCTION

Over the past decade, and via extensive high-definition measurements, it has been established that high-speed network traffic is bursty on many or all time scales. Such behavior is distinctly different from that of traditional circuit switched voice traffic, and from a modeling point of view, is usually associated with self-similar and impulsive processes. Statistical modeling of traffic is very important in network engineering, and a substantial body of literature has been devoted to it. However, models developed for traditional teletraffic no longer apply to data traffic. In high-speed networks, the packets are communicated in a *packet train* fashion; furthermore, the length of the packet train is heavy-tail distributed. This observation led to the well-known On/Off model [7] also called Alternating Fractal Renewal Process (AFRP) [4]. While the AFRP model provides insight on the causes of self-similarity of traffic, its Gaussian aggregated results is inconsistent with real traffic data, which depart greatly from Gaussianity. An extension of the AFRP, namely the EAFRP, that captures heavy-tailness as well as self-similarity of traffic has been proposed in [6].

In this paper, we propose a modification of the EAFRP model that enables a closer match to real traffic. The modification is motivated by the distinctive two-slope appearance of the log-log complementary distribution (or survival) function (LLCD) of real traffic data, and the nature of the true bounds on the user transmission rates in real networks. Limits on the sender's and the receiver's TCP window sizes, TCP congestion avoidance strategies, and bandwidth bottlenecks within the end-systems are among many of the reasons that lead to an independent limit on each individual user's transmission rate [9]. In reality, therefore, if R is the peak rate of the link on to which traffic from multiple users is multiplexed, the sum of the user transmission rates is bounded by R and each user's transmission rate is bounded by an even smaller quantity, L ($L < R$). In our modified EAFRP model, i.e., the rate-limited EAFRP, we capture this reality by modeling the transmission rate during the On states by a cut-off Pareto random variable, while the On/Off durations are distributed as in the EAFRP model. We show that the existence of these two rate limits, L and R, results in the distinctive two slope behavior of the LLCD of the overall traffic, a fact that has not been explained so far by existing models. We validate our theoretical findings based on real traffic measurements and provide queuing analysis of the proposed model.

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2. MATHEMATICAL PRELIMINARIES

The Pareto distribution is defined in terms of its complementary distribution, or, survival function as:

$$\bar{F}(x;\alpha,K) = P(X \geq x) = \begin{cases} (\frac{K}{x})^{\alpha}, & x \geq K, \\ 1, & x < K, \end{cases} \quad (1)$$

where K is positive constant and $0 < \alpha < 2$ and especially the mean exists if $1 < \alpha < 2$. When plotted is log-log scale, as x increase, $\bar{F}(x;\alpha,K)$ appears as a straight line with slope $-\alpha$.

In this paper we will be using a variation of the Pareto distribution, namely the cut-off Pareto, defined in terms of density function equals:

$$f_L(x;\alpha,K) = f(x;\alpha,K)(1-u(x-L)) + (\frac{K}{L})^{\alpha}\delta(x-L) \quad (2)$$

where $f(.)$ denotes the Pareto density function, $u(.)$ is the unit step function, $\delta(.)$ is the Dirac function, and L represents a limit imposed to the random variable. It can be easily verified that the integral of $f_L(x;\alpha,K)$ taken for x between $-\infty$ to ∞ is one.

3. THE PROPOSED MODEL

The EAFRP model was proposed in [6] as an extension to the AFRP [4]. It yields traffic that is impulsive and long-range dependent in the generalized codifference sense at both single and multi-user case. However, for mathematical tractability, it relies on the assumption of infinite bandwidth be available during the On state, an assumption that is not met in a real network. When used to synthesize traffic, although it matches well a significant portion of the log-log complementary distribution (LLCD), it does not capture the distinctive two-slope appearance of the LLCD. This traffic behavior, although never commented on, can also be seen in the figures of [2],[3] and other papers.

To illustrate the latter point, let us analyze some real data which was collected at the 100Mbps network of the Department of the Electrical and Computer Engineering at Drexel University within three consecutive days (from 10 am Sep 23 to 10 am Sep 26 in 2001). The LLCD graph corresponding to a two-hour ($N = 720000$) segment of the data is shown in Fig. 1. The Pareto model, hypothesized by the EAFRP model, would attempt to fit a line to the graph of Fig. 1 (see dashed line in Fig. 1). However, a straight line passed through most points of the graph would not be able to match the rightmost part of the graph, and would overestimate the probability of the rate exceeding very large values, compared with proposed model. As it will be shown in the sequel, due to the finite bandwidth available to the users, the latter probability is smaller than the value suggested by the Pareto model.

Proposition 1 *Let us consider an On/Off process, $s(t)$,defined as:*

(A1) *The On periods $\{X_n, n \in Z\}$ and the Off periods $\{Y_n, n \in Z\}$ are i.i.d., independent of each other with distributions $\bar{F}_1 = \bar{F}(x;\alpha_1, K_1)$ and $\bar{F}_0 = \bar{F}(x;\alpha_0, K_0)$, with $\alpha_0, \alpha_1 > 1$, thus have finite means μ_1 and μ_0 respectively;*

(A2) *The rates during the On states are random variables cut-off Pareto distributed with probability density function $f_L(x;\alpha, K)$, and independent of X_n, Y_n.*

Then, $s(t)$ is distributed according to:

$$f_s(x) = \frac{\mu_0}{(\mu_1+\mu_0)}\delta(x) + \frac{\mu_1}{(\mu_1+\mu_0)}f_L(x;\alpha, K) \qquad (3)$$

Proof: See Appendix A.

Let us now consider M independent i.i.d. On/Off process, $s_i(t), i = 1, ..., M$, each $s_i(t)$ constructed according to assumptions (A1),(A2). Let us form the process $S(t)$ as the superposition of the $s_i(t), i = 1, ..., M$, followed by a thresholding operation with threshold R. In the following we will provide some insight on the form of the pdf and the LLCD of the process $S(t)$.

For simplicity, let us first consider the case where $M = 2$. The pdf of $S(t)$ will be:

$$f_S(x) = (f_s(x) * f_s(x))(1 - u(x - R)) \qquad (4)$$

where $f_s(x)$ denotes the pdf of each $s_i(t)$ given in (3), and '*' denotes convolution. To simplified the expression, we use $f_L(x)$ to replace $f_L(x;\alpha, K)$. Based on (2),(3) and(4) we get:

$$f_S(x) = \begin{cases} (\frac{\mu_0}{(\mu_1+\mu_0)})^2 & x = 0, \\ 2\frac{\mu_0\mu_1}{(\mu_1+\mu_0)^2}f_L(x) & K < x < 2K, \\ (\frac{\mu_1}{(\mu_1+\mu_0)})^2 f_L(x) * f_L(x) + 2\frac{\mu_0\mu_1}{(\mu_1+\mu_0)^2}f_L(x) & 2K < x < L - K, \\ (\frac{\mu_1}{(\mu_1+\mu_0)})^2 f_L(x) * f_L(x) & L - K < x < L, \\ f_s(x)(1-u(x-L)) * f_s(x)(1-u(x-L)) & L < x < min(2L, R) \\ 0 & x > min(2L, R) \end{cases} \qquad (5)$$

Let us consider the interval $2K < x < L - K$. Over that interval, $f_L(x;\alpha, K)$ is identical to the Pareto pdf, $f(x;\alpha, K)$. Thus, $f_S(x)$ for $2K < x < L - K$ is the sum of two terms, a Pareto pdf and a convolution of Pareto pdf's. The second term, if L is large enough, will also be a Pareto pdf with parameters α and $2K$ [1]. In fact the latter result also holds for the case where $s_i(t)$ are non-i.i.d., expect that the resulting parameters would be $min\{\alpha_1, \alpha_2\}$ and $K_1 + K_2$, respectively. For $L >> K$ and for x approaching $L - K$, $f_S(x)$ will behave like a Pareto pdf. Thus, if L is large enough, a linear segment will appear in the LLCD of $S(t)$ within the range $[2K, L - K]$. Over the interval between L and R, $f_s(x)$ is hard to express in closed form, however, numerical evaluation suggests that the LLCD exhibits behavior corresponding to a Gaussian process. We should note here that at R the LLCD should be $-\infty$.

It is simple to extend the above result to any M. Still in that case the LLCD will exhibit a linear trend over the interval $[MK, L]$, followed by a Gaussian-type decay, i.e. the tail decays with the manner corresponding to a Gaussian process in LLCD. As expected by the Central Limit Theorem, as M increases, the resulting process will become Gaussian. This is in agreement by the discussion above, since for M very large, the linear segment will occur over increasingly smaller range and will eventually disappear. Of course, the larger the L, the larger M it will take for the result to become Gaussian.

Since it is hard to derive the pdf of the superposition $S(t)$ in closed form, we approximate it over the interval $[MK, R]$ by the following mixture:

$$\begin{aligned} f_S(x) &\sim (\frac{K_{A_2}}{R})^{\alpha_{A_2}}\delta(x - R) + f(x; A_1, K_1)[1 - u(x - L)] \\ &+ f(x; A_2, K_2)[u(x - L) - u(x - R)] \end{aligned} \qquad (6)$$

where

- $A_1 = \alpha$ if the $\{s_i(t), i = 1, ...M\}$ are i.i.d., otherwise, $A_1 = min\{\alpha_1, ..., \alpha_M\}$.
- $K_1 = KM$ if the $\{s_i(t), i = 1, ...M\}$ are i.i.d., otherwise, $K_1 = \sum_{i=1}^{M} K_i$
- α_{A_2} can be any positive value.
- $$K_{A_2} = exp\{\frac{1}{\alpha_{A_2}}ln[K_{A_1}^{\alpha_{A_1}} L^{(\alpha_{A_2}-\alpha_{A_1})}]\} \qquad (7)$$

The corresponding survival function is:

$$\bar{F}_S(x) = \begin{cases} 1 & 0 \le x < K_{A_1} \\ (\frac{K_{A_1}}{x})^{\alpha_{A_1}} & K_{A_1} \le x < L \\ (\frac{K_{A_2}}{x})^{\alpha_{A_2}} & L \le x < R \\ 0 & x > R \end{cases} \quad (8)$$

Proposition 2 *The process defined through (A1),(A2) , and also the supposition of M such processes are long-range dependent*

Proof: See Appendix B.

In the proof of Proposition 2, we assume that $ML < R$. If $ML > R$, there is a non-zero probability of traffic congestion at the multiplexing point. The total transmitted traffic rate, in the presence of congestion, is not a mere supposition of the processes that describe the transmission rates of individual users, and therefore, it is not clear that it will still be long-range dependent. While a detailed queuing-theoretic analysis is necessary to obtain the characteristics of the process that describes the total traffic in the presence of such congestion, initial results in [10] obtained through simulation suggest that at least the degree of long-range dependence in the total traffic will reduce in comparison to that of the process describing the individual user transmission rates.

To provide some insight on the queuing characteristics of traffic synthesized by the proposed model, we assume that the total network traffic can be represented by a single On/Off process defined as in (A1) and (A2). Let $Q(t),\ t \ge 0$ denote the buffer content at time t, and let

$$Q(\infty) \stackrel{\text{d}}{=} \lim_{t\to\infty} Q(t) \quad (9)$$

where $\stackrel{\text{d}}{=}$ represents equality in distribution. It can be shown that stationary queue length $Q(\infty)$ is heavy-tail distributed with tail index $\alpha_1 - 1$. The proof is omitted due to lack of space and can be found in [8].

4. RESULTS

In this section, we provide simulations to support the claim that the proposed modification of the On/Off process results in total traffic that exhibits a two-slope LLCD. We provide evidence that suggests that real traffic and traffic synthesized as superposition of the proposed On/Off processes look very similar in terms of their LLCDs and autocorrelations.

We consider again the Drexel traffic data (see Fig.1). We next show how one can constructively generate this traffic based on the model of (6), and also as a supposition of single user traffic traces. The proposed mixture model (see (6)) was fitted to the traffic as follows. Let [5]:

$$U^2 = n \int_{-\infty}^{+\infty} \left\{ \bar{F}_{S,n}(x) - \bar{F}_S(x) \right\}^2 d\bar{F}_S(x) \quad (10)$$

where $\bar{F}_S(x)$ is the survival function of test model, and $\bar{F}_{S,n}(x)$ is the empirical distribution function (EDF)defined as;

$$\bar{F}_{S,n}(x) = \frac{\text{number of observations} > x}{n} \quad (11)$$

where n is the total number of available samples. Also, let the model parameters be (in vector form): $\mathbf{q} = [\alpha_{A_1}\ K_{A_1}\ \alpha_{A_2}\ L]$. We computed

$$\mathbf{q}_0 = arg \min_{\mathbf{q}\in Q} \left\{ n \int_{-\infty}^{+\infty} \left\{ \bar{F}_{S,n}(x) - \bar{F}_S(x) \right\}^2 d\bar{F}_S(x) \right\} \quad (12)$$

where $Q = \big[[0,2]\ 10^{[0,5]}\ [2,10]\ 10^{[0,5]}\big]$. The above range was used based on the experience that we has gathered working with the real data. The parameters were found to be: $\alpha_{A_1} = 1.13$, $K_{A_1} = 10^{1.78}$, $\alpha_{A_2} = 7$, $L = 10^{4.5}$, which yields $K_{A_2} = 10^{4.04}$ via (7). For a 100Mbps LAN, $R = 1.25 * 10^5 Byte/$time unit where we choose time unit $= 0.01s$. Based on these numbers we synthesized traffic following the pdf of (6) and show its LLCD in Fig. 2(a), along with its autocorrelation. The autocorrelation shown is the mean$\pm$ standard deviation computed over 50 independent traffic realizations. For easy reference, the LLCD and autocorelation of the real traffic is superimposed on these figures (see dashed line in 2).

We next synthesized 50 i.i.d. cut-off Pareto processes, distributed according to $f_{10^{4.5}}(x; 1.13, 10^{1.78})$. The LLCD of their supposition is shown in Fig. 2(b), along with the corresponding autocorrelation. We should note here that the number of users was taken 50, which was approximately the number of users on the system when the traffic data was collected. For comparison convenience, the corresponding LLCD and autocorrelation of the real date is superimposed in Fig. 2, which suggests that the synthesized traffic matches the real traffic both based on outlook and statistics.

5. CONCLUSION

We have presented a constructive model for high-speed network traffic that achieves a close approximation to real traffic than previously known constructive models. The modeling of traffic generated by a single user was performed along the lines of the EAFRP [6], with the primary difference being the introduction of a maximum rate limit (L). The total traffic rate through the network also experiences a limit (R) in real networks. We have shown that the existence of these two limits, L and R, lead to a total traffic whose LLCD and autocorrelation match those of real traffic very closely. In particular, this provides insight for the first time into the two-slope appearance of the LLCD of real traffic. We have further shown that the supposition of the proposed rate-limited EAFRP processes is long-range dependent. Thus, our model preserves the long-range dependence of the total traffic in the traffic in the absence of congestion. The correlation structure of the total traffic when $ML > R$, i.e. when there is a non-zero probability of congestion, remains to be investigated.

The proposed model has finite variance and thus, as the number of users increases, the total traffic will eventually become Gaussian. This is consistent with what can be observed in data that are collected at gateways. However, we were able to show that as L increases, it will take a larger number of users for the traffic to become Gaussian. This implies that in modern networks, when L is large, the traffic will be non-Gaussian. In that case, the proposed model can be a useful tool in making certain design choices in the network infrastructure.

6. APPENDIX A

$s(t)$, can be expressed as $s(t) = A(t)V(t)$ where $V(t)$ is an AFRP, and $A(t)$ corresponds the transmission rate, distributed according to assumption (A2). The probability density function of $s(t)$ is: $f_s(x) = P[V(t) = 0]\delta(x) + P[V(t) = 1]f_L(x)$ where $\delta(x)$ is the Dirac function, taking value of 1 at $x = 0$ point only. Since $P[V(t) = 1] = 1 - P[V(t) = 0] = \frac{\mu_1}{(\mu_1+\mu_0)}$ for $x > 0$, $f_s(x)$ is a scaled version of $f_L(x)$. Thus, $s(t)$ is a cut-off Pareto, exhibiting a power-law survival function for values less that L.

7. APPENDIX B

Due to the cut-off nature of $f_L(x)$, the proposed model will have finite moments. Therefore, long-range dependence is here examined in terms of its covariance. The joint characteristic function of an On/Off process with arbitrarily distributed On durations can be found in [6]. Based on that expression, the covariance of $s(t)$ can be found as the second-order derivative of the characteristic function, $\phi_s(s_1, s_2; \tau)$, evaluated as 0 (see also [8]).

The overall traffic $S(t)$ is the supposition of M independent and identical distributed as proposed process $s_m(t)$, $(m = 1, 2, ..., M)$, i.e. $S(t) = \sum_{m=1}^{M} S_m(t)$ then,

$$\phi_S(s_1, s_2; \tau) = \prod_{m=1}^{M} \phi_{s_m}(s_1, s_2; \tau) \tag{13}$$

The covariance function of $S(t)$ equals:

$$\begin{aligned} c_S(\tau) &= -\frac{\partial^2 \phi_S(s_1, s_2; \tau)}{\partial s_1 \partial s_2}\Big|_{s_1=0, s_2=0} \\ &= \sum_{m=1}^{M} \Big\{ -\frac{\partial^2 \phi_{S_m}(s_1, s_2; \tau)}{\partial s_1 \partial s_2} \Big\}\Big|_{s_1=0, s_2=0} \\ &\times \prod_{n=1, n\neq m}^{M} \big\{ \phi_{S_n}(s_1, s_t; \tau) \big\}\Big|_{s_1=0, s_2=0} \end{aligned} \tag{14}$$

It can be easily derived that:

$$c_S(\tau) \sim M c_{\rho_1}^{M-1} c_{\rho_2} \tau^{M(1-\alpha_i)} \tag{15}$$

where $\alpha_i = min(\alpha_1, \alpha_0)$.

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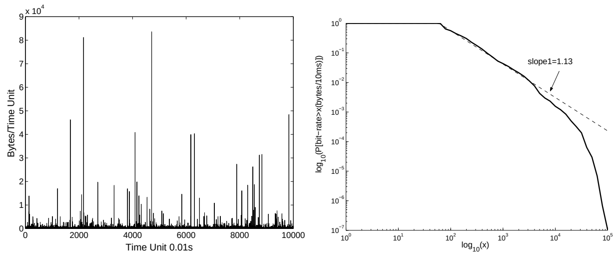
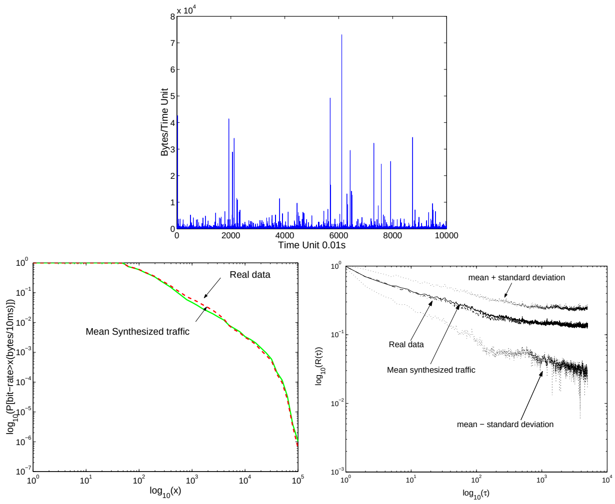
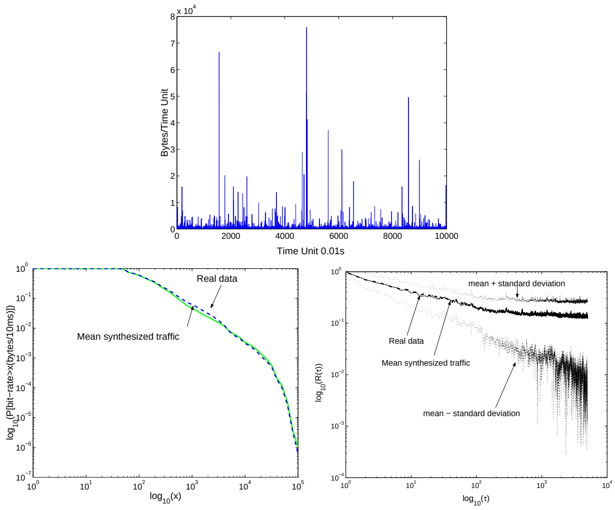


Figure 1: The Drexel traffic trace: LLCD of real traffic (solid line) and that of traffic generated based on the EAFRP model of [6].



(a) Synthesized traffic based on the mixture model of (6).



(b) Synthesized traffic as a supposition of 50 i.i.d. On/Off processes with cut-off Pareto rates.

Figure 2: Model validation based on the Drexel traffic trace