A NEW VARIABLE STEP SIZE LMS ALGORITHM WITH APPLICATION TO ACTIVE NOISE CONTROL

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ABSTRACT

A new adaptive step size adjustment least mean square (LMS) algorithm is presented in this paper. The proposed algorithm modified the existing LMS using the estimated output error as an important component for the modification of the step size. Experiment results demonstrate that application of the new algorithm leads to a significant gain in *SNR* (signal-to-noise ratio), thus visibly reduces the output error and greatly improves the convergence speed in the context of the adaptive active noise canceling.

1. INTRODUCTION

The LMS algorithm [1] is the most widely used adaptive algorithm for applications in nonstationary noise environment, due to its simplicity and robustness. The step size μ of LMS, which is a very important parameter in the adaptive filtering and together with the reference signal power, govern the stability, convergence speed and the fluctuation of the LMS adaptation process. Besides, a fixed μ cannot respond to time-varying channel parameters, thus leading to poor performance. To deal with this problem, in the last decades, a lot of variable step-size LMS algorithms have been developed. In 1986, Harris proposed the Variable-Step-Size LMS [2]. Shan and Kailath proposed the Correlation LMS algorithm (CLMS) [3] in 1988, and so on.

An important variable step-size algorithm to optimize the speed of convergence, at the same time, maintaining the desired stability of the algorithm, is the Normalized LMS (NLMS)[4]. The advantages of the NLMS are its fast convergence speed, stability and good performance. But the existing approaches, including the NLMS, the step-size, which is considered as time-variable, are implemented based on the power estimate of input signals. Therefore when the input signal is stationary, the step sizes of a filter are time-variable only on the early stages of adaptive process. In general, when the signal is nonstationry, then a larger μ is used and when the signal is stationary and convergence rate is not important, a smaller μ is used. For this consideration, in this paper, a modification of NLMS algorithm, named as VS-NLMS, is presented in which the step size is decided not only by the estimate of input signals but also the estimate of error signals, so that the proposed new algorithm visibly reduces output error, keep continuously track of the time-variable statistics of signals and achieve a faster convergence rate while maintaining the stability of the NLMS.

2. The NORMALIZED LMS (NLMS) ALGORITHM FOR ADAPTIVE FILTER

A general form of the adaptive filter is illustrated in Figure 1, where *d(n)* is a desired response (or primary input signal), *y(n)* is the actual output of a programmable digital filter driven by a reference input signal *x(n)*, and the error *e(n)* is the difference between *d(n)* and *y(n)*. The function of the adaptive algorithm is to adjust the digital filter coefficients to minimize the mean-square value of *e(n)*.

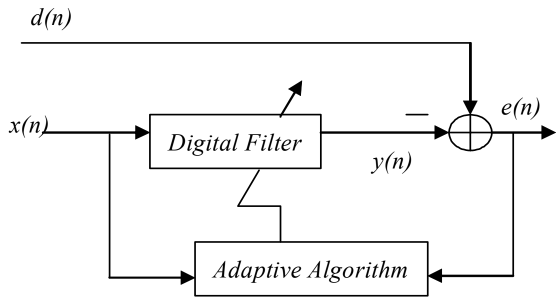

Fig1.Basic concept of adaptive filter

A technique to adjust the convergence speed is the Normalized LMS (NLMS) algorithm. The NLMS is shown as follows:

$$w\ (n+1) = w\ (n) + \mu(n)x(n)e(n) \qquad (1)$$

μ(n) is adaptive step size which is computed as

$$\mu(n) = \frac{\alpha}{L\hat{P}_i(n)}, \qquad 0 < \alpha < 2 \qquad (2)$$

$\hat{P}_i(n)$ is the estimated power of *x(n)* at time n , *L* is order of the filter, and α is the normalized step size. An exponential window is used to estimate the power of *x(n)*

$$\hat{P}_i(n) = (1-\beta)\hat{P}_i(n-1) + \beta x^2(n) \qquad (3)$$

where β is a smoothing parameter, which is in terms of its equivalent (exponential) window length

$$M \equiv \frac{1}{\beta} \qquad (4)$$

3. THE PROPOSED NEW ALGORITHM

The principle of the proposed VS-NLMS is to estimate output error so as to update the step size, and it significantly improves the output SNR, greatly reduces the output error and achieves a faster convergence speed.

Based on the NLMS algorithm, the coefficient of the step size is given by (2) the numerator α is the same to each coefficient and the denominator of (2) is updated by the power estimate $\hat{P}_i(n)$

In the method above, only the denominator is the time-varying. In fact, if the estimate of the power of output error has a significant variation, the numerator is also needed to be changed. The new time-varying step size is summarized as follows:

$$\mu(n) = \alpha + \beta^2 \sum_{i=n-K+1}^{n} \varepsilon_i e(i)^2 ,$$

$$\left(\sum_{i=n-K+1}^{n} \varepsilon_i = 1, \quad \beta = \frac{1}{M}, \quad 0 < \alpha < 2 \right) \qquad (5)$$

$$\mu(n+1) = \begin{cases} \mu(n), & \text{if } \mu(n) \in (\mu_{\min}, \mu_{\max}) \\ \mu_{\max}, & \text{if } \mu(n) \geq \mu_{\max} \\ \mu_{\min}, & \text{if } \mu(n) \leq \mu_{\min} \end{cases} \qquad (6)$$

so the step size of the coefficient is

$$\mu_i(n) = \frac{\mu(n)}{\theta + L\hat{P}_i(n)}, \quad 0 < \theta << 1 \qquad (7)$$

The purpose for adding θ to the denominator is to ensure that $\mu_i(n)$ is bounded if $L\hat{P}_i(n)$ is small when input signal is absent for a long time.

The weight vector in the VS-NLMS is:

$$w_i(n+1) = w_i(n) + \frac{\mu(n)}{\theta + L\hat{P}_i(n)} x(n)e(n) \qquad (8)$$

The process of the VS-NLMS is shown as follows: the step size is proportional to the error signals. At the beginning of the iterations, output error has big value, and then according to (5), the step size will has big value, so that the convergence speed is increased. When the adaptive filter is in the steady state, the output error has a relatively small value, thus the step size is decreased. The new algorithm keep continuously track of signals, so it reduces the output error and optimizes the speed of convergence while maintaining the desired steady-state performance.

In this paper, a filter with $L=40$, $\theta=0.06$, $K=10$ is adopted. The parameter β, which is inverse proportion to M, is an important value for the whole iteration process. According to the NLMS, if the signal is stationary, a long M is used. On the other hand, a relatively short M is used to track power changes for a nonstationary signal. In the proposed VS-NLMS, if the signal is stationary, a long M is used, so β is very small. According to (5), the changes of μ are very small too. And in the nonstationary environment, a smaller M and larger μ can track the changes of the signals.

The computational complexity of NLMS and VS-NLMS is presented in Table 1. N is the length of the input signals. The computational cost of the new proposed algorithm is comparable with that of NLMS.

Algorithm	ADD. / SUB.	MULT. / DIV.
NLMS	3N	6N+1
VS-NLMS	3N+12	6N+8

Table.1 Comparison of Computational Complexity of NLMS and VS-NLMS

4. EXPERIMENT RESULTES

The VS-NLMS algorithm presented in this paper has been compared with CLMS, NLMS. Results of three experiments are given to demonstrate the performance of VS-NLMS algorithm. The criterion used for evaluating the performance of the proposed VS-NLMS algorithm is informal listening tests and SNR. In experiment one, the speech signal is sinusoidal signals. The noise is the stationary Gaussian white noise. In experiment two and three; the noise signals are the sound of a factory and a diesel locomotive respectively. The testing system of three experiments is shown in Fig.2.

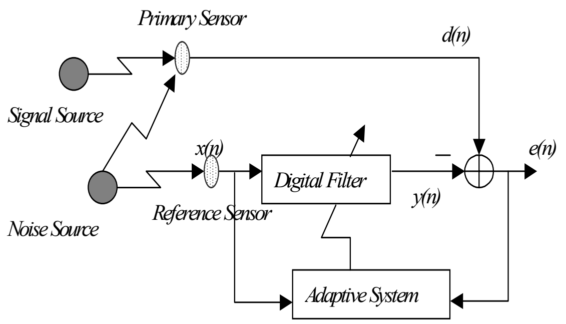

Fig.2. Block diagram of system identification of adaptive filtering

	Experiment 1			Experiment 2			Experiment 3		
	Input SNR (dB)	Output SNR (dB)	△SNR (dB)	Input SNR (dB)	Output SNR (dB)	△SNR (dB)	Input SNR (dB)	Output SNR (dB)	△SNR (dB)
CLMS	-0.03	13.15	13.18	-0.05	7.45	7.50	-0.21	10.47	10.68
NLMS		17.12	17.15		10.27	10.32		10.32	10.53
VS-NLMS		26.74	26.77		22.46	22.51		10.80	11.01
CLMS	-0.46	5.23	5.69	-0.38	3.65	4.03	-0.38	6.33	6.71
NLMS		7.11	7.57		4.12	4.50		6.20	6.58
VS-NLMS		11.99	12.45		7.35	7.73		8.11	8.49
CLMS	-2.23	2.46	4.69	-1.12	2.67	3.79	-0.60	6.15	6.75
NLMS		4.52	6.75		2.78	3.90		5.98	6.58
VS-NLMS		6.44	8.67		4.89	6.01		7.51	7.80

Table.2 Input and output SNR comparison of algorithms

Table 2 summaries the results of three experiments. The performances of the algorithms are compared under a very low *SNR* of below *0 dB*. As shown in the table.2, the VS-NLMS obtains the highest *SNR* gain between the input and the output *SNR*.

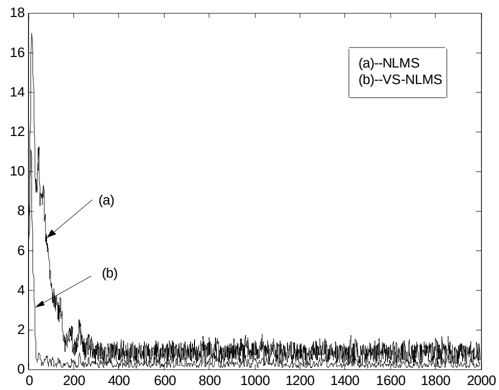

Fig.3.Performance of NLMS and VS-NLMS

In Fig.3, the behaviors of NLMS and VS-NLMS are presented. As shown in Fig.3, using the estimate of output error for updating the step-size has significantly reduced the output error and obtains a faster convergence speed.

5. CONCLUSIONS

In this paper, The VS-NLMS algorithm with a time-varing step size is presented. The improvement of the VS-NLMS lies in the introduction of the estimate of error signals for adjusting step size. Experiment results shows that the output *SNR* and the convergence speed have been significantly improved, the computational cost of the VS-NLMS and the NLMS is comparable, and the output error is greatly reduced.

6. REFERENCES

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