A NEW ADAPTIVE IIR ALGORITHM FOR ACTIVE NOISE CONTROL*

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ABSTRACT

In some situations of active noise control, IIR filters are more suitable than FIR filters owing to the poles in the transfer function. A number of algorithms have been derived for applying IIR filters in active noise control (ANC), however, most of them use the direct form IIR filter structure, which faces the difficulties of checking stability and relatively slow convergence speed for noise composed of narrow band components with large power disparity. To overcome these difficulties along with using the direct form IIR filters, a new adaptive algorithm is proposed in this paper, which uses and updates the lattice form adaptive IIR filter in an active noise control system. The comparison between the proposed algorithm and the commonly used filtered-u LMS and filtered-v LMS algorithms shows the superiority of the proposed algorithm.

1. INTRODUCTION

Although the algorithms using adaptive IIR filters for active noise control have been proposed for many years, they still have not been widely used in the application of the actives noise control system due to the following disadvantages. First, IIR filters are not unconditionally stable due to the possibility that some poles of the filters might move outside of the unit circle during the weights update. Second, the existing adaptive algorithms have a lower convergence speed and may converge to a local minimum. Therefore, it is recommended that whenever possible, adaptive FIR filters should be used [1].

The adaptive IIR filters used in active noise control are usually in the direct form, for example, the filtered-u LMS (FULMS) algorithm [2] and filtered-v LMS (FVLMS) algorithm [3]. All these adaptive algorithms use the direct form IIR filter, hence having the same problems of possible instability and slow convergence. The lattice structure is an alternative form of a digital filter, which possesses the advantages of inherent stability and greatly reduced sensitivity to the eigenvalue spread of the reference signal [4]. The idea of using lattice filters in active noise control is not new. However, it is usually used as a preprocessor followed by a FIR filter [5-6]. This paper will propose a new adaptive algorithm for using the lattice form adaptive IIR filter in active noise control, which makes full use of the othorgonalization property of the lattice structure and avoids the problem of slow convergence and possible instability while holding the benefits of adaptive IIR filter.

2. THE LATTICE GRADIENT DECENT ALGORITHM FOR ANC

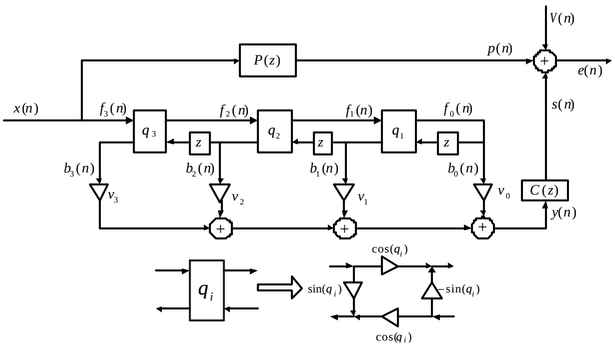

Fig. 1 Tapped state normalized lattice filter for active noise control for M=3

Fig. 1 shows the flowgraph of the tapped state normalized lattice form IIR filter for active noise control for the case in which the filter order *M* is set to 3. In this figure, the primary path transfer function *P*(*z*) represents the transfer function from the noise source to error sensor, the cancellation path transfer function *C*(*z*) represents the acoustic path from the secondary source to error sensor. {*z*(·)} is some kind of noise which is statistically independent of the reference signal {*x*(·)}. The filter parameters are the rotation angles $\{q_1,\ldots,q_M\}$ plus the tap parameters $\{n_0,\ldots n_M\}$. The cascade structure in the lattice

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filter propagates a forward signal $f_k(n)$ and a backward signal $b_k(n)$ at time n and section number k. By adapting $\{\theta_k\}$ in such a way that $|\sin\theta_k| < 1$, the stability of the lattice filter is ensured [4].

The output of the lattice filter is

$$y(n) = \sum_{k=0}^{M} b_k(n) u_k \tag{1}$$

$\{b_k(n)\}$ for $k = M, M\text{-}1, \ldots, 1$ are obtained by the Schur recursion

$$\begin{bmatrix} f_{k-1}(n) \\ b_k(n) \end{bmatrix} = \begin{bmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{bmatrix} \begin{bmatrix} f_k(n) \\ b_{k-1}(n-1) \end{bmatrix} \tag{2}$$

where $f_M(n) = x(n)$ and $b_0(n) = f_0(n)$.

As with the direct form algorithm, the output error signal is

$$\begin{aligned} e(n) &= p(n) + s(n) + z(n) \\ &= [P(z) + W(z)C(z)]x(n) + z(n) \end{aligned} \tag{3}$$

where $W(z)$ is the transfer function of the lattice filter. The parametric derivatives of this error signal are given by

$$\begin{aligned} \frac{\partial e(n)}{\partial u_k} &= \frac{\partial s(n)}{\partial u_k} = \frac{\partial W(z)}{\partial u_k} C(z)x(n) \\ \frac{\partial e(n)}{\partial \theta_k} &= \frac{\partial s(n)}{\partial \theta_k} = \frac{\partial W(z)}{\partial \theta_k} C(z)x(n) \end{aligned} \tag{4}$$

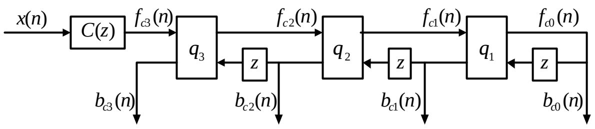

Fig. 2 Filtered regressor signals for the tap parameters

The derivative with respect to the tap parameters $\{n_k\}$ is straightforward. In the lattice form, there is

$$W(z) = \sum_{k=0}^{M} u_k B_k(z) \tag{5}$$

so that

$$\frac{\partial e(n)}{\partial u_k} = B_k(z)C(z)x(n) \tag{6}$$

where $B_k(z)$ is the transfer function of the lattice filter corresponding to the kth backward signal. The signals obtained from Eq. (6) are called filtered regressor signals as they are formed by filtering the input with the cancellation path transfer function and the lattice filter. The signals can be obtained with an auxiliary lattice filter, as shown in Fig. 2 for the case $M = 3$, where the filtered regressor signals for the tap parameters $\{n_k\}$ is $\{b_{ck}(n)\}$. The instantaneous estimate of the gradient signal of the cost function $E[e^2(n)]$ corresponding to the tap parameter n_k is

$$\nabla_{u_k}(n) = 2e(n)\frac{\partial e(n)}{\partial u_k} = 2e(n)b_{ck}(n) \tag{7}$$

Obtaining derivative signals with respect to the rotation angles $\{\theta_k\}$ is more complicated. Via Eq. (3) and (5), there is

$$\frac{\partial e(n)}{\partial \theta_k} = \frac{\partial W(z)}{\partial \theta_k} C(z)x(n) = \sum_{l=0}^{M} u_l \frac{\partial B_l(z)}{\partial \theta_k} C(z)x(n) \tag{8}$$

which requires obtaining the sensitivity function $\partial B_l(z)/\partial\theta_k$.

Set now $\dfrac{\partial B_l(z)}{\partial \theta_k} \triangleq B_{\theta k,l}(z)$, $\quad \dfrac{\partial F_l(z)}{\partial \theta_k} \triangleq F_{\theta k,l}(z)$

By applying differential operator $\partial/\partial\theta_k$ to the z-transform of Eq. (2), we have

$$\begin{bmatrix} F_{\theta k,l-1}(z) \\ B_{\theta k,l}(z) \end{bmatrix} = \begin{bmatrix} \cos\theta_l & -\sin\theta_l \\ \sin\theta_l & \cos\theta_l \end{bmatrix} \begin{bmatrix} F_{\theta k,l}(z) \\ zB_{\theta k,l-1}(z) \end{bmatrix} \quad \text{if } k \neq l \tag{9}$$

And

$$\begin{aligned} \begin{bmatrix} F_{\theta k,l-1}(z) \\ B_{\theta k,l}(z) \end{bmatrix} &= \begin{bmatrix} \cos\theta_l & -\sin\theta_l \\ \sin\theta_l & \cos\theta_l \end{bmatrix} \begin{bmatrix} F_{\theta k,l}(z) \\ zB_{\theta k,l-1}(z) \end{bmatrix} \\ &+ \begin{bmatrix} -\sin\theta_l & -\cos\theta_l \\ \cos\theta_l & -\sin\theta_l \end{bmatrix} \begin{bmatrix} F_l(z) \\ zB_{l-1}(z) \end{bmatrix} \quad \text{if } k = l \end{aligned} \tag{10}$$

By using

$$\begin{aligned} F_{l-1}(z) &= \cos\theta_l \cdot F_l(z) - \sin\theta \cdot zB_{l-1}(z) \\ B_l(z) &= \sin\theta_l \cdot F_l(z) + \cos\theta \cdot zB_{l-1}(z) \end{aligned} \tag{11}$$

For $k = l$, there is

$$\begin{bmatrix} F_{\theta k,l-1}(z) \\ B_{\theta k,l}(z) \end{bmatrix} = \begin{bmatrix} \cos\theta_l & -\sin\theta_l \\ \sin\theta_l & \cos\theta_l \end{bmatrix} \begin{bmatrix} F_{\theta k,l}(z) \\ zB_{\theta k,l-1}(z) \end{bmatrix} + \begin{bmatrix} -B_l(z) \\ F_{l-1}(z) \end{bmatrix} \tag{12}$$

Because $F_M(z) = 1$ and $B_0(z) = F_0(z)$ for all $\{\theta_k\}$, the boundary conditions for completing the recursion are

$$F_{\theta k,M}(z) = 0, \quad B_{\theta k,0}(z) = F_{\theta k,0}(z) \tag{13}$$

For illustration purposes, Fig. 3 shows the filtered regressor signal corresponding to the rotation parameter θ_2 for the filter order of $M = 3$.

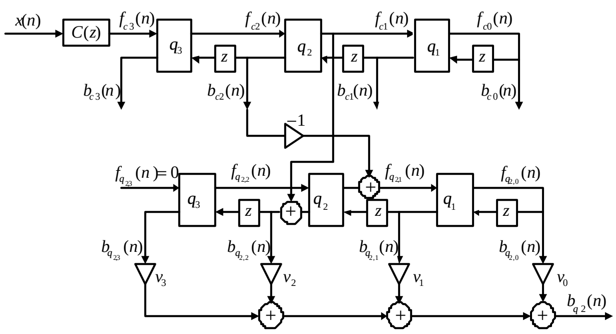

Fig. 3 Filtered regressor signal for the rotation parameter

The instantaneous estimate of the gradient signal of the cost function $E[e^2(n)]$ corresponding to the rotation parameter q_k is

$$\nabla_{q_k}(n) = 2e(n)\frac{\partial e(n)}{\partial q_k} = 2e(n)b_{q\,k}(n) \qquad (14)$$

An overall algorithm listing is given in Table I. Note that the "Test" step in the algorithm not only guarantees the stability of the adaptive process but also ensures the uniqueness of the mapping from the transfer function space to the parameter space.[9]

3. THE SIMPLIFIED GRADIENT LATTICE ALGORITHM FOR ANC

It should be noted that M additional lattice filters are required to obtain the filtered regressor signals $\{-\tilde{N}q_k(n)\}$ corresponding to the rotation parameters. Thus the complexity is of the order M^2, both for computation and storage. Considering that the normally used direct form IIR filters such as filtered-u [2] algorithm and simplified filtered-v algorithm [3] required only order M computation and storage, the increased complexity of the lattice form is an obvious disadvantage. A simplified gradient lattice algorithm with a computation complexity order of M is described below. The derivation is quite complex, and the details are omitted for brevity. However a similar deviation of adaptive lattice algorithm that used in system identification can be found in reference [4], which does not take into account the cancellation path as in an ANC system.

For illustration purposes, Fig. 3 shows the filtered regressor signal corresponding to the rotation parameter q_2 for the filter order of $M = 3$.

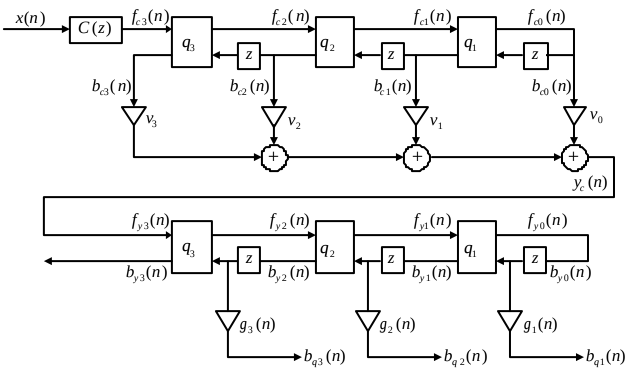

Fig. 4 Generation of filtered regressor signals in simplified gradient algorithm

The resulting algorithm would appear as

$$v_k(n+1) = v_k(n) - me(n)\cdot B_k(z)C(z)x(n), \quad k = 0,1,...,M$$
$$q_k(n+1) = q_k(n) + me(n)\cdot g_k z B_{k-1}(z)W(z)C(z)x(n), \quad k = 1,2,...,M \qquad (15)$$

where $\quad g_k = \prod_{l=k+1}^{M} \cos q_l, \quad g_M = 1 \qquad (16)$

4. SIMULATIONS

In this section, several illustrative results are presented on comparisons between the proposed algorithm and commonly used FULMS algorithm [2] and FVLMS algorithm [3]. Only the simplified gradient lattice algorithm will be used for simulations, which will be called LFRLMS (Lattice Filtered Reference LMS) algorithm in the following context. All the simulations were conducted by using the acoustic transfer functions in a single input and output active noise control system measured in the anechoic room with a sampling rate of 8000Hz. Fig. 5 shows a schematic diagram of the system, where two identical loudspeakers were placed 20cm from each other, one acted as the noise source and the other acted as the control source. An error microphone was placed 1.3m away from the center of two loudspeakers. The impulse responses corresponding to the primary path and the cancellation path are shown in Fig. 6. In all the simulations the filter order was 64 and the stepsize parameters of the adaptive algorithms were adjusted to the extent that any increase to the parameter would cause the control process unstable.

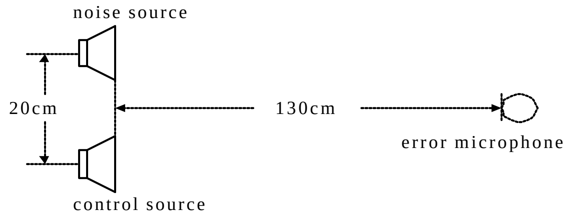

Fig.5 Schematic diagram of the simulation system

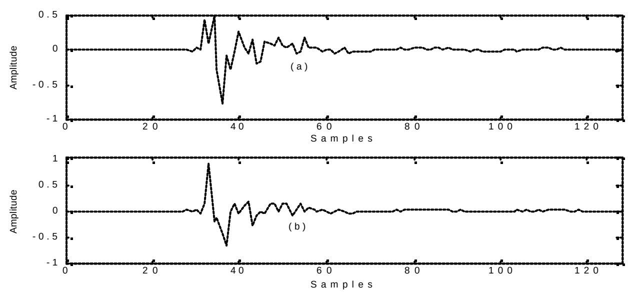

Fig. 6 Impulse responses used for primary and secondary path with (a)primary path and (b)secondary path

The white Gaussian noise generated by the computer was used as the noise source first, and the convergence speed of the three algorithms are shown in Fig. 7. It can be seen that there is not dramatic difference among these three algorithms as far as for attenuating white noise. FVLMS algorithm and FULMS algorithm perform almost the same and proposed LFRLMS algorithm gives slightly better performance than the other two algorithms. Theoretically the convergence performance of the lattice form and direct form IIR filters should be similar for the noise signal with quite flat power spectrum. However, because lattice form adaptive IIR filters are more stable, the convergence coefficient may be set a little larger, resulting in faster convergence speed.

In an actual ANC system, the noise signal to be controlled sometimes contains narrow band components with large power disparity such as fan noise and babble noise. This leads to a great obstacle for normal active control strategy because the convergence rate of normal LMS algorithm decreases much with the large eigenvalue spread of input autocorrelation matrix, which is always caused by great power disparity of the input signal [7]. To test the efficiency of different algorithms for the controlling of more "real" noise, the summation of 100 sinusoid signals with random frequency between 0Hz and 3000Hz were used as the noise for the simulations and all the sinusoid components have random amplitudes between 0 and 0.5 and random initialization phases between 0 and 360 degrees. The learning curves shown in Fig. 8-9 were obtained from an ensemble of 20 trials. Comparing Fig. 8 with Fig. 7, it can be seen that the convergence rate of LFRLMS algorithms only decrease slightly while both FVLMS and FULMS algorithm converge much slower than that for controlling white noise signal; and from the learning curve of 50,000 iterations in Fig. 9, it can be seen that LFRLMS algorithm converges on a level approximately 2dB below the other two algorithms.

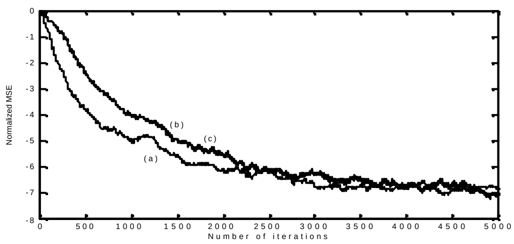

Fig. 7 Comparisons between different algorithms with (a) LFRLMS algorithm (b) FULMS algorithm and (c) FVLMS algorithm

5. CONCLUSIONS

In this paper, the gradient IIR lattice algorithms for ANC were proposed. Then the simplified gradient IIR lattice algorithm was tested by using the measured transfer functions from an active noise control system. The simulation results demonstrated the proposed lattice form adaptive IIR filtering algorithm not only converges faster than the commonly used FULMS and FVLMS algorithms when the noise source consists of sinusoid components with wide power disparity, but also converges to a smaller mean squared error. It also showed that the proposed algorithm is far less sensitive to the cancellation path modeling error, which possibly results in a more robust system in practice. Theoretical analysis of the stability of the proposed algorithm and the implementation of the algorithm in a real time DSP ANC system are under-going.

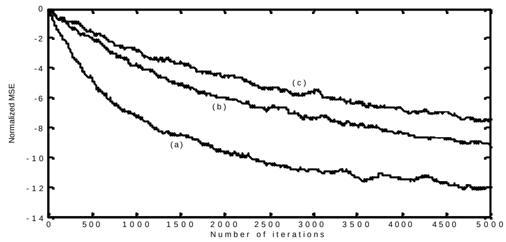

Figure 8 Learning curves for different algorithms of 5,000 iterations with noise source of large power disparity. (a) LFRLMS algorithm (b) FULMS algorithm and (c) FVLMS algorithm

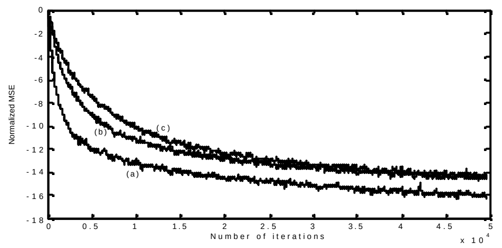

Figure 9 Learning curves for different algorithms of 50,000 iterations with noise source of large power disparity. (a) LFRLMS algorithm (b) FULMS algorithm and (c) FVLMS algorithm

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