CONSTANT BEAMWIDTH BEAMFORMER FOR DIFFERENCE FREQUENCY IN PARAMETRIC ARRAY

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ABSTRACT

The sound reproduction in air by using a parametric acoustic array [1] has been reported for a few decades. Two inaudible ultrasonic frequencies are produced from the parametric array. Due to the nonlinearity of air, it is possible to produce an audible frequency with its frequency equal to the difference in the two ultrasonic frequencies. However, there is not much work done in controlling the beam pattern of the difference frequency generated by the primary waves. In this paper, an algorithm is proposed to control the sidelobe level of the difference frequency directivity. By making use of array signal processing techniques, the algorithm is also capable of producing a constant beamwidth for broadband difference frequency.

1. INTRODUCTION

The development of the parametric acoustic array dated back as far as 1963, where Westervelt [1] described the difference frequency generation by using two well collimated primary sound beams. More works followed which improve on the quality and efficiency of the difference frequency generation by using the parametric array, such as Berktay's far-field solution [2], Yoneyama's Audio Spotlight [3]. Recently, there is an increasing awareness in producing difference-frequency by using two primary waves with difference frequencies. Several patents [4] have been granted and some commercial products based on the parametric array are also available.

However, most of the parametric arrays make use of perfectly collimated plane waves, which are radiated by a circular or rectangular piston source. In this paper, a weighted linear array of $2N+1$ equally spaced primary sources will be discussed, where N is a positive integer. An algorithm for producing a constant beamwidth directivity and a lower sidelobe level for the difference frequency is proposed in this paper.

This paper is organized in the following sections. In the next section, the theory for the parametric array using the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation will be presented. This is followed by section 3, which proposes an algorithm for designing a constant beamwidth beamformer for the difference frequency, which is produced by the primary frequencies. Section 4 shows some of the simulation results and comparisons between the conventional parametric array and the same array with the proposed algorithm, and section 5 concludes this paper.

2. THEORY

The KZK equation can be used to describe the nonlinearity, absorption due to viscosity and heat conduction, and diffraction effects in a parametric array accurately as shown:

$$\frac{\partial^2 p}{\partial z \partial \tau} = \frac{c_0}{2} \nabla_\perp^2 p + \frac{\delta}{2c_0^3} \frac{\partial^3 p}{\partial \tau^3} + \frac{\beta}{2\rho_0 c_0^3} \frac{\partial^2 p^2}{\partial \tau^2} \qquad (1)$$

where

p = acoustic pressure

z = coordinate along the axis of the beam

τ = retarded time

c_0 = small-signal sound speed

$\nabla_\perp^2$ = transverse Laplacian operator

δ = sound diffusivity

β = coefficient of nonlinearity

ρ_0 = ambient density

By using quasilinear theory, the nonlinear KZK equation can be reduced into two linear equations with the solution of the form:

$$p = p_1 + p_2 \qquad (2)$$

where:

$$p_1(r,z,\tau) = \frac{1}{2j}\Big[q_{1a}(r,z)e^{j\omega_a \tau} + q_{1b}(r,z)e^{j\omega_b \tau} \Big] + c.c. \qquad (3)$$

$$p_2(r,z,\tau) = \frac{1}{2j}\Big[q_{2a}(r,z)e^{j2\omega_a \tau} + q_{2b}(r,z)e^{j2\omega_b \tau} + q_+(r,z)e^{j\omega_+ \tau} + q_-(r,z)e^{j\omega_- \tau} \Big] + c.c. \qquad (4)$$

and q_{1a}, q_{1b}, q_{2a}, q_{2b}, q_+, q_- are complex pressure amplitudes for primary frequency ω_a, primary frequency

ω_b, second harmonic $2\omega_a$, second harmonic $2\omega_b$, sum frequency ω_+, and difference frequency ω_- respectively. *c.c.* denotes the complex conjugate of preceding terms.

In this paper, we are only interested in the difference frequency. The solution for q_- is:

$$q_-(r,z) = -\frac{\pi\beta k_-}{\rho_0 c_0^2}\int_0^z\int_0^\infty q_{1a}(r',z')q_{1b}^*(r',z')$$
$$G_-(r,z|r',z')r'dr'dz' \qquad (5)$$

where $k_- = \omega_-/c_0$ and $G_-(r,z|r',z')$ is given by:

$$G_-(r,z|r',z') = \frac{jk_-}{2\pi(z-z')}J_0\left(\frac{k_-rr'}{z-z'}\right)$$
$$\exp\left[-\alpha_-(z-z') - \frac{jk_-(r^2+r'^2)}{2(z-z')}\right] \qquad (6)$$

and $\alpha_- = \delta\omega_-^2/2c_0^3$.

For discussion, let us consider the difference frequency generation by a bi-frequency Gaussian source, where q_{1a} and q_{1b} are defined by:

$$q_{1a}(r,0) = p_{0a}\exp\left[-(r/a)^2\right]$$
$$q_{1b}(r,0) = p_{0b}\exp\left[-(r/b)^2\right] \qquad (7)$$

and p_{0a} and p_{0b} are the peak source pressure, a and b are the effective source radius.

By ignoring absorption and substitute Equations (7) and (6) into Equation (5), $q_1(r,z)$ becomes:

$$q_-(r,z) = j\frac{P_-}{f_-}\exp\left(-\frac{k_-^2 r^2}{2f_-}\right)$$
$$\left[E_1\left(\frac{k_a k_b(z_{0a}+z_{0b})^2 k_-^2 r^2}{2f_-(g_-+jf_-k_-z)}\right)\right.$$
$$\left. -E_1\left(\frac{k_a k_b(z_{0a}+z_{0b})^2 k_-^2 r^2}{2f_-g_-}\right)\right] \qquad (8)$$

where

$$P_- = \frac{\beta k_-^2 z_{0a}z_{0b}p_{0a}p_{0b}}{2\rho_0 c_0^2}$$
$$f_-(z) = k_a z_{0a} + k_b z_{0b} - jk_-z \qquad (9)$$
$$g_-(z) = k_-^2 z_{0a}z_{0b} - j(k_b z_{0a} + k_a z_{0b})k_-z$$

and $z_{0a} = \frac{1}{2}k_a a^2$, $z_{0b} = \frac{1}{2}k_b b^2$, $k_a = \frac{\omega_a}{c_0}$ and $k_b = \frac{\omega_b}{c_0}$.

If only far-field is considered [5], the directivity of the different frequency is given by the product of the primary beam directivities, i.e.:

$$D_-(\theta) = D_{1a}(\theta)D_{1b}(\theta) \qquad (10)$$

Now, consider a group of $M = 2N+1$ weighted primary sources, which are equally spaced. Due to the fact that the solution of a primary source by using quasilinear theory is linear [5]:

$$\frac{\partial q_1}{\partial z} + \frac{j}{2k}\nabla_\perp^2 q_1 + \alpha_1 q_1 = 0 \qquad (11)$$

the far-field directivity of the weighted primary sources array for frequency ω_a, $D_{1a}(\theta)$ can be written as:

$$D_{1a}(\theta) = D_1(k_a,\theta)H(k_a,\theta) \qquad (12)$$

where $D_1(k_a,\theta)$ is the aperture directivity for frequency ω_a, and

$$H(k_a,\theta) = \sum_{n=1}^M w_{an}e^{-jn\omega_a\tau_0\sin\theta} \qquad (13)$$

is the far-field array response, where w_{an} are the tap weights for frequency ω_a for $n = 1,2,\cdots,M$. $\tau_0 = d/c$ is the element spacing divided by the speed of sound.

Similarly, the far-field directivity at primary frequency ω_b, $D_{1b}(\theta)$ can be written as:

$$D_{1b}(\theta) = D_1(k_b,\theta)H(k_b,\theta) \qquad (14)$$

where:

$$H(k_b,\theta) = \sum_{n=1}^M w_{bn}e^{-jn\omega_b\tau_0\sin\theta} \qquad (15)$$

and w_{bn} are the tap weights for frequency ω_b for $n = 1,2,\cdots,M$.

By substituting Equations (12) and (14) into Equation (10), we have:

$$D_-(\theta) = D_1(k_a,\theta)D_1(k_b,\theta)H(k_a,\theta)H(k_b,\theta) \quad (16)$$

This shows that the far-field difference frequency directivity can be roughly estimated by the product of four terms: the aperture directivity of a single transducer for frequency ω_a, the aperture directivity of a single transducer for frequency ω_b, the far-field array response of weighting w_{an} and the far-field array response of weighting w_{bn}. It is therefore possible that we choose the weighting sets w_{an} and w_{bn}, such that the nulls of the response $H(k_b,\theta)$ appears at the maximum locations of the sidelobes of the response $H(k_a,\theta)$.

3. CONSTANT BEAMWIDTH BEAMFORMER FOR A BROADBAND DIFFERENCE FREQUENCY

Although there are many possible algorithms that can be derived to minimize the sidelobes of the difference frequency directivity in Equation (16). The following describes a technique which can be used to design a constant beamwidth beamformer [6] for broadband frequencies ω_{1b} with far-field directivity $D_{1b}(\theta)$. One

property of this method is that the first null location of the far-field directivity of the broadband $D_{1b}(\theta)$ is located at the peak location of the first sidelobe of the frequency ω_a far-field directivity, $D_{1a}(\theta)$.

Consider an array of $M = 2N+1$ parametric array, which is able to produce an audible frequency ($20Hz \sim 20kHz$) by using upper side band modulation with a $40kHz$ carrier frequency as shown in figure 1.
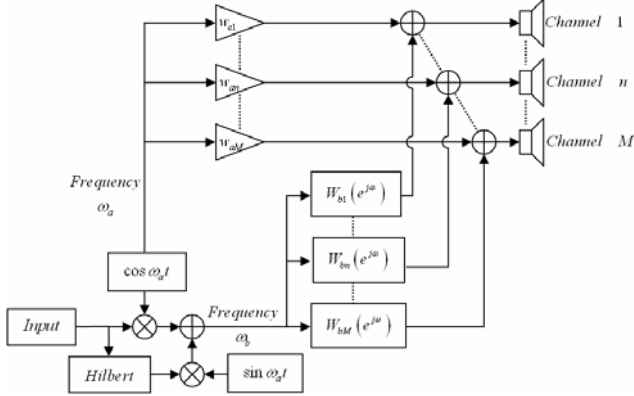

Figure 1: M weighted parametric array with weighting w_{an} and weighting response $W_{bn}(e^{j\omega})$.

The procedure for designing the weighting w_{an} and weighting response $W_{bn}(e^{j\omega})$ for a constant beamwidth beamformer of a broadband difference frequency is as follows:

Step 1: Determine the beamwidth required for the difference frequency, θ_{D-}.

Step 2: Calculate the required sidelobe attenuation (dB) for $H(k_a, \theta)$, R:

$$R = 20\log\left(T_{2N}\left(\frac{\cos\left(\dfrac{\pi}{4N}\right)}{\cos\left(\dfrac{kd\sin(\theta_{D-}/2)}{2}\right)} \right) \right) \quad (17)$$

where $T_{2N}(x) = \cos(2N\cos^{-1}x)$ and $k = k_a$.

Step 3: Use the number of sources M and sidelobe attenuation R to design a Chebyshev weighting w_{an} [7].

Step 4: Determine the peak location of the first sidelobe of $H(k_a, \theta)$, θ_{peak}. The beamwidth to be designed for $H(k_b, \theta)$, θ_{1b}:

$$\theta_{1b} = 2\theta_{peak} \quad (18)$$

Step 5: Again, using Equation (17) to determine the required sidelobe attenuation for $H(k_b, \theta)$ by replacing θ_{D-} and k with θ_{1b} and k_b respectivly.

Step 6: Use the number of sources M and sidelobe attenuation R to design a Chebyshev weighting w_{bn}. Repeat Step 5 and 6 for different frequency ω_b.

Step 7: Collect the weightings for broadband frequency ω_b, w_{bn} and form weighting response $W_{bn}(e^{j\omega})$.

4. SIMULATION RESULTS

For the simulations, the carrier frequency of the upper side band modulation is set at $40kHz$ ($f_{1a} = \omega_a/2\pi = 40kHz$). The input of the system accepts a broadband frequency of $20Hz \sim 20kHz$. A total of $M = 7$ parametric array is formed with inter element spacing, $d = 4.9mm$. The effective source radius is set at $a = b = 3.85mm$ and the speed of sound, $c = 330.7ms^{-1}$.

In this section, three different algorithm blocksets are simulated and compared. The first algorithm is the conventional parametric array, which is shown in Figure 2 with $w_{an} = 1$ for all n. The second algorithm is similar to conventional method, except that Chebyshev weighting, w_{an} is added to the carrier frequency together with its upper sideband frequencies. The last algorithm is the proposed algorithm, which is shown in Figure 1. Chebyshev weighting, w_{an} are added to the carrier frequency and a weighting response for different frequencies, $W_{bn}(e^{j\omega})$ are added to the upper side band frequencies.
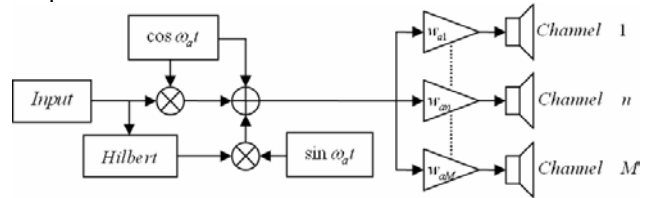

Figure 2: M weighted parametric array with a single weighting w_{an}.

By ignoring the distortion, the conventional array response for the difference frequency is shown in Figure 3. Although the response does have a narrow beamwidth, the sidelobes of the array response is very high with an average highest sidelobes of $-29.99dB$. Furthermore, the sidelobes is uncontrollable with the same hardware configurations.

In Figure 4, the simulation configuration is shown in Figure 2. A weighting w_{an} ($\theta_{D-} = 40°$) is added to channel n, which shows that the sidelobes can be attenuated in the expense of its beamwidth. The average highest sidelobes is found to be $-52.53dB$.

Figure 5 shows the array response for the difference frequency of the proposed algorithm (Figure 1). The required weighting response for broadband frequencies

ω_b for different channel, $W_{bn}(e^{j\omega})$ are shown in Figure 6. With the additional weighting response $W_{bn}(e^{j\omega})$, Figure 5 shows that the sidelobes can be further attenuated compared to Figure 4 with its average highest sidelobes equal to $-70.20 dB$.

5. CONCLUSION

By using array signal processing, the new algorithm proposed is capable of controlling the sidelobes of the parametric array response for the broadband difference frequency generated by the two primary waves. By introducing additional weighting response for broadband frequencies ω_b, $W_{bn}(e^{j\omega})$ it can be shown that the sidelobes of the broadband difference frequency response can be further attenuated. In addition, the proposed method is able to create a constant beamwidth for the broadband difference frequency.
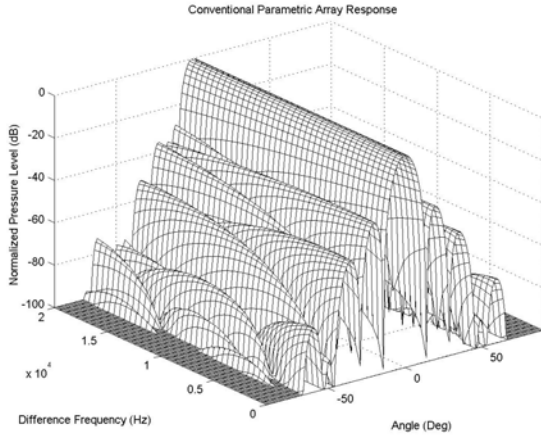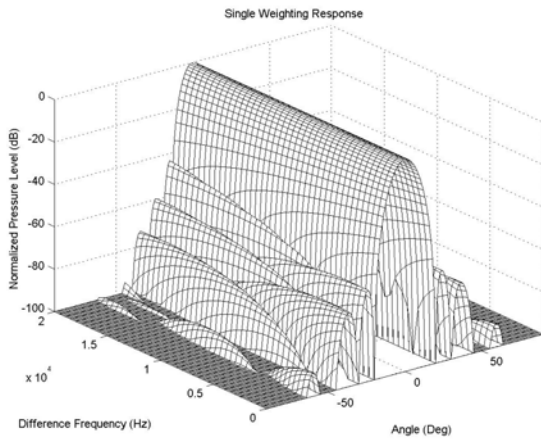

Figure 3: Conventional Parametric Array Response

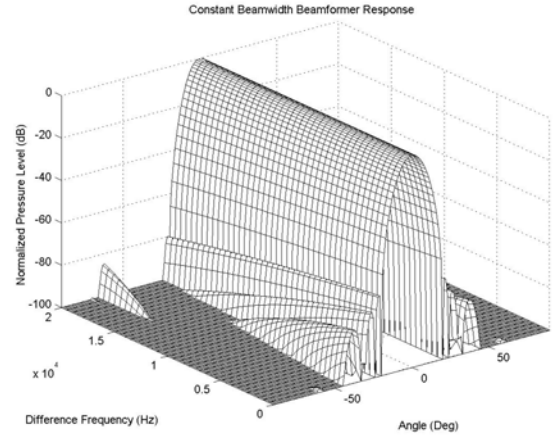Figure 4: Parametric Array Response with w_{an}

Figure 5: Parametric Array Response with w_{an} and $W_{bn}(e^{j\omega})$
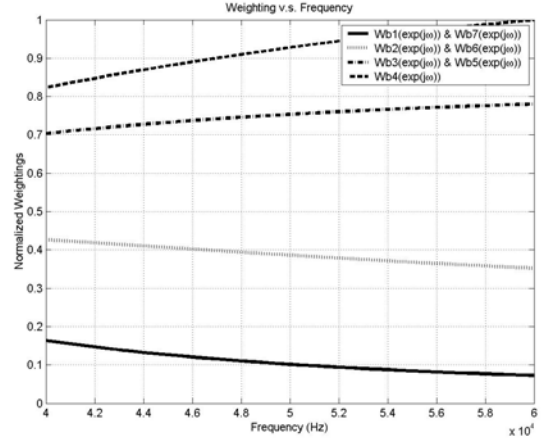

Figure 6: Chebyshev Weighting Response $W_{bn}(e^{j\omega})$

6. REFERENCES

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