Space-Time Interpolation for Adaptive Arrays with Limited Training Data

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Abstract

This paper describes a method for improving the small sample support space-time adaptive processing (STAP) performance of distorted linear arrays. Receive arrays which deviate from a straight line may occur, for example, in conformal radar and towed sonar array applications. With limited training data, distorted linear arrays suffer greater signal to interference plus noise (SINR) loss due to inflation of the clutter covariance matrix rank. In this paper, Brennan's rule for the clutter covariance matrix rank is extended to 2-D arrays and used to motivate the development of a space-time interpolation (STINT) method for clutter rank reduction. By using a space-time transformation that minimizes the constrained mean-square-error between clutter at the distorted array and a virtual uniform line array, STINT processing lowers the clutter covariance rank and hence improves output SINR when training data is limited. Simulation results also indicate that STINT processing reduces the minimum detectable target velocity (MDV) achievable by finite sample support STAP.

1. INTRODUCTION

Given a sufficiently accurate estimate of the space-time clutter covariance matrix, STAP has a significant advantage over deterministic beamformer-Doppler weight design because clutter nulls are placed precisely where they are needed for the observed data and array calibration. When the clutter is inhomogeneous in range, however, STAP is often seriously degraded by a lack of snapshot support. Improving limited training data performance and also reducing the computational requirements for STAP has motivated the development of partially adaptive schemes [1] which utilize fewer adaptive degrees of freedom requiring fewer snapshots to estimate the interference. The effectiveness of partially adaptive schemes is primarily dictated by the rank of the underlying clutter covariance matrix of the received array data. In the case of a uniform linear array (ULA) aligned along the direction of platform motion, this rank ideally corresponds to the well-known Brennan's rule. However, array shape distortion or platform velocity misalignment generally results in: 1) increased clutter covariance matrix rank [1], and 2) range inhomogeneity which limits sample support [2]. In order to overcome the effects of clutter rank inflation, we have developed a partially adaptive approach derived by mapping the clutter returns from a distorted array to a virtual ULA via joint space-time interpolation. A separable space-time interpolation scheme has been developed previously [3] which "focuses" the array response vectors from adjacent range cells onto a given range cell. In this paper, we design *non-separable* space-time interpolation filters to minimize the interpolated *clutter* mean-square error between the distorted array and the virtual ULA, subject to a constraint maintaining the desired target response. The low space-time clutter rank facilitates mapping of clutter from all directions via a single linear transformation, whereas in [3] the mapping is different for subsets of directions and Doppler frequencies.

2. STAP MODEL

Consider a narrowband pulse-Doppler radar system using an N-element array and an M-pulse coherent waveform with pulse repetition interval T_r, traveling with a platform velocity v_a along the x-axis. The received clutter data at the n^{th} sensor located at $(x_n = (n-1)d, y_n)$, sampled at time $t_m = \tau + mT_r$ can be modeled as:

$$r_{nm} = \sum_{\theta_k} \alpha(\theta_k) e^{j\frac{2\pi}{\lambda}(\sin\theta_k \cdot x_n + \cos\theta_k \cdot y_n) + j2\pi m f_k T_r} \tag{1}$$

where $\alpha(\theta_k)$ is the complex Gaussian scatter amplitude from azimuth θ_k, $f_k = (2v_a/\lambda)\sin(\theta_k)$ is the clutter Doppler frequency and λ is the radar operating wavelength. We consider ranges where the variation in elevation angle ϕ is negligible. Without loss in generality it is assumed throughout that $\phi = 0$ and $d = \lambda/2$. The space-time received data snapshot across a single coherent processing interval (CPI) can be written as

$$\mathbf{x} = \alpha_t \mathbf{v}_t(\theta_t, f_t) + \mathbf{r} + \boldsymbol{\varepsilon}, \tag{2}$$

where $\mathbf{v}_t(\theta_t, f_t)$ is the $MN \times 1$ target space-time response, α_t is the complex target amplitude, $\mathbf{r}$ is the $MN \times 1$ received clutter from all directions and $\boldsymbol{\varepsilon}$ is $MN \times 1$ additive white noise with covariance $\sigma^2 \mathbf{I}_{MN}$. The space-time target response can be expressed in terms of the $M \times 1$ temporal response $\mathbf{b}(f_t)$ and the $N \times 1$ spatial response $\mathbf{a}(\theta_t)$ as

$$\mathbf{v}_t(\theta_t, f_t) = \mathbf{b}(f_t) \otimes \mathbf{a}(\theta_t) \quad (3)$$

where $\left[\mathbf{b}(f_t)\right]_n = e^{j2\pi(n-1)f_t T_r}$, $\left[\mathbf{a}(\theta_t)\right]_n = e^{j(2\pi/\lambda)(x_n \sin(\theta_t) + y_n \cos(\theta_t))}$, θ_t is the target location, f_t is the target Doppler, $\otimes$ denotes the Kronecker product and $[\mathbf{x}]_n$ is the n^{th} element of $\mathbf{x}$.

3. CLUTTER RANK

The approximate rank of the space-time clutter covariance matrix for a ULA with inter-element spacing d, is given by the well-known Brennan's rule [1].

$$\rho_{ULA} \cong \lfloor N + (M-1)\beta \rfloor, \quad (4)$$

where $\beta = 2v_a T_r / d$ is the number of half inter-element displacements traversed by the array moving along its axis, in one sampling interval and $\lfloor x \rfloor$ is the smallest integer $\geq x$.

The approximate clutter rank for a ULA with $\lambda/2$ spaced elements can be derived using the relationship between the "effective aperture" – spatial-bandwidth product and the clutter rank [4]. In this way, the spatio-temporal spectrum of the clutter is interpreted purely in terms of its "effective" 1D spatial spectrum. The "effective aperture" defined with respect to the maximum unambiguous phase change across the clutter space-time steering vector from a single clutter patch [1], is given by $[N-1+(M-1)\beta]\lambda/2$ while the maximal normalized spatial bandwidth is $2/\lambda$. Therefore, the approximate clutter rank is given by

$$\begin{aligned}\rho_{ULA} &= \left[(N-1)\lambda/2 + (M-1)\beta\lambda/2\right]2/\lambda + 1 \\ &= N + (M-1)\beta\end{aligned} \quad (5)$$

which is the same as Brennan's rule.

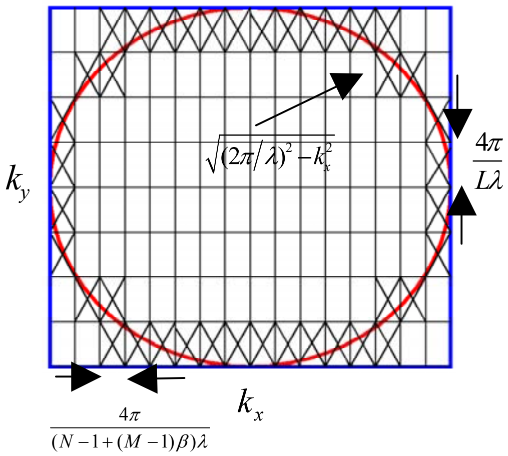

Fig. 1. 2D Clutter Spectrum

For an array traveling along the x-direction with a transverse distortion of $L\lambda/2$ and effective aperture along the x-direction of $[N-1+(M-1)\beta]\lambda/2$ the analogous 2-D Brennan's rule can be derived in a similar fashion. For a wave propagating with wavenumber $k = \sqrt{k_x^2 + k_y^2}$, the clutter spatial spectrum lies on a 2D ring as shown in Fig. 1. The 2-D Brennan's rule is given by the number of spatial frequency resolution cells spanned by the locus of the clutter wavenumber spectrum. This is found to be approximately

$$\rho_{DIS} \cong 2\lfloor N + L + (M-1)\beta \rfloor. \quad (6)$$

Clearly from (6), we observe that the clutter rank nearly doubles even for small transverse distortions.

4. SPACE-TIME INTERPOLATOR DESIGN

The objective here is to design a linear transformation that "interpolates" the clutter space-time steering vectors from a distorted array to fit those from a virtual ULA while simultaneously preserving the target's space-time response. Let

$$\mathbf{V}_d = \left[\mathbf{v}(\varpi_1) \ \cdots \ \mathbf{v}(\varpi_J)\right], \quad (7)$$

represent the $MN \times J$ matrix of clutter space-time steering vectors for the distorted array, across a fine grid in azimuth, where $\mathbf{v}(\varpi_k)$ is the space-time steering vector from a single clutter patch at θ_k, with normalized Doppler frequency $\varpi_k = f_k T_r$ and J is the number of clutter patches. Let $\tilde{\mathbf{V}}_l = \left[\tilde{\mathbf{v}}(\varpi_1) \ \cdots \ \tilde{\mathbf{v}}(\varpi_J)\right]$ denote the $MN \times J$ clutter steering matrix for a virtual ULA with inter-element spacing $d = \lambda/2$, traveling with the same velocity as the distorted array. Since the clutter steering matrices $\tilde{\mathbf{V}}_l$ and $\mathbf{V}_d$ span the clutter subspace for the ULA and the distorted array, their ranks are respectively given by (5) and (6). The goal is then to design an $MN \times MN$ rank reducing transformation $\mathbf{T} = [\mathbf{t}_1 \ \cdots \ \mathbf{t}_{MN}]$ so that $\mathbf{T}^H \mathbf{V}_d \approx \tilde{\mathbf{V}}_l$, while simultaneously preserving the target response. The problem can be formulated in terms of the columns of $\mathbf{T}$ as

$$\hat{\mathbf{t}}_i = \arg\min_{\mathbf{t}_i} \left\| \mathbf{V}_d{}^H \mathbf{t}_i - \mathbf{b}_i \right\|_2, \tag{8}$$

subject to the constraint

$$\mathbf{v}_t{}^H \mathbf{t}_i = \tilde{v}^i{}_t, \tag{9}$$

where $[\mathbf{b}_1 \ \cdots \ \mathbf{b}_{MN}] = \tilde{\mathbf{V}}_l{}^H, 1 \le i \le MN$, $\mathbf{v}_t$ is the target space-time steering vector for the distorted array and $v^i{}_t$ represents the conjugate of the target response for the i^{th} space-time element of the virtual ULA. The constraint preserving the target response in (9) is necessary since it would otherwise be distorted during clutter interpolation. In the current framework, a separate interpolation filter needs to be designed for each target Doppler and azimuth, however, the STINT technique can easily be extended to include a range of target directions/Doppler frequencies by incorporating multiple constraints into (9). In this paper, we shall evaluate only a single target constraint.

The optimization in (8) can be reformulated as an unconstrained least squares problem in the subspace orthogonal to the constraint subspace. Let

$$\mathbf{Q}\tilde{\mathbf{r}} = \mathbf{v}_t, \tilde{\mathbf{r}} = [r_1 \quad \mathbf{0}_{1\times MN-1}]^H \tag{10}$$

represent the QR decomposition of the constraint vector where $\tilde{\mathbf{r}}$ is an $MN \times 1$ vector and $\mathbf{Q}$ is an $MN \times MN$ orthonormal matrix. The equivalent unconstrained least squares problem is then to solve for $\mathbf{t}_i = \mathbf{Q}\begin{bmatrix} \mathrm{y}_i \\ \hat{\mathbf{z}}_i \end{bmatrix}$ from

$$\begin{aligned} \hat{\mathbf{z}}_i &= \arg\min_{\mathbf{z}_i} \left\| [\mathbf{a}_1 \mid \mathbf{A}_2] \begin{bmatrix} y_i \\ \hline \mathbf{z}_i \end{bmatrix} - \mathbf{b}_i \right\|_2 \\ &= \mathbf{A}_2{}^\dagger [\mathbf{b}_i - \mathbf{a}_1 y_i], \end{aligned} \tag{11}$$

where $\mathbf{V}_d{}^H \mathbf{Q} = [\mathbf{a}_1 \mid \mathbf{A}_2]$, $y_i = \tilde{v}_t^i / r_1$ and $\mathbf{A}_2{}^\dagger$ is the pseudo-inverse of $\mathbf{A}_2$. The solution has excellent interpolation properties because the $\rho_{DIS} \ll MN$. The rank p of the interpolation matrix $\mathbf{W} = \mathbf{T}^H$ can be shown to be one more than 1D Brennan's rule for the virtual ULA. It follows that the interpolated clutter rank is $\le p$. From the SVD of $\mathbf{W} = \mathbf{U\Lambda V}^H$, the distortion compensated STAP weight vector can be computed as

$$\mathbf{w} = \mathbf{V}(\mathbf{V}^H \mathbf{R}_x \mathbf{V})^{-1} \mathbf{V}^H \mathbf{v}_t \tag{12}$$

where $\mathbf{V}$ is the $MN \times p$ matrix of right singular vectors of $\mathbf{W}$. For finite sample support, when the estimated covariance matrix $\hat{\mathbf{R}}_x$ is used instead of $\mathbf{R}_x$, the reduced column dimension of $\mathbf{V}$ provides improved statistical stability over conventional STAP. Note that the expression for the STAP weight in (12) is similar to that obtained using partially adaptive STAP schemes [1] when the transformation therein is replaced by $\mathbf{V}$. Also note that the STINT technique can be easily generalized to include the presence of jamming by incorporating constraints which preserve the jammer response, into (9).

5. SIMULATION EXPERIMENTS

Simulations were performed using a 30 element distorted array with $M = 5$, $\lambda = .5m$, $\beta = 1$, clutter to noise ratio $CNR = 50dB$ and $x_n = (n-1)\lambda/2$. For all finite time simulations, 80 training data snapshots were used to compute the STAP weights. From Fig. 2, we see that the clutter rank is 34 for the uniform linear array and approximately 72 (corresponding to 2-D Brennan's rule) for the distorted array. Figures 3 and 4 show the finite sample azimuth vs normalized Doppler adapted patterns for the distorted array and the compensated array respectively, steered towards a target with $\theta_t = 45^\circ$ and $\varpi_t = -.3$. Note the substantially reduced sidelobe levels for the compensated array. The second mainlobe at $\theta_t = 135^\circ, \varpi_t = -.3$ is due to the left-right ambiguity of the linear array. Fig. 5 indicates the SINR loss relative to diffuse-noise limited optimum processor at the target azimuth for all Doppler frequencies for the various array configurations. Clearly, we see $\approx 5dB$ SINR improvement is obtained using the interpolated array. Also note that the dip in SINR is narrower for the interpolated array compared to the distorted array indicating a lower MDV using STINT. Note the linear array SINR performance serves as an upper bound for the asymptotic as well as finite sample performance of the interpolation technique. Fig. 6 shows the loss in performance, relative to that with perfect covariance knowledge, due to covariance matrix estimation. Averaged over Doppler and azimuth, STAP with interpolation filtering offers as much as 10 dB gain at low number of snapshots, with performance comparable to the linear array. Note that both the linear and interpolated arrays require fewer snapshots compared to the distorted array, to achieve SINR close to their asymptotic values. This stems from the fact that the clutter rank for the interpolated array is approximately the same as for the linear array and therefore fewer snapshots are required to estimate the clutter covariance matrix [1] as compared to the distorted array which requires approximately twice the number of snapshots to achieve similar performance.

6. CONCLUSION

Distortion of nominally linear arrays with limited backlobe rejection leads to substantial increase in the clutter covariance matrix rank. To reduce the clutter rank we design constrained MMSE interpolating filters that “interpolate” the clutter space-time steering vectors from the distorted array to fit those from a virtual ULA. With relatively low sample support, the proposed interpolation technique is found to yield an improvement in low sample support SINR on the order of 10 dB.

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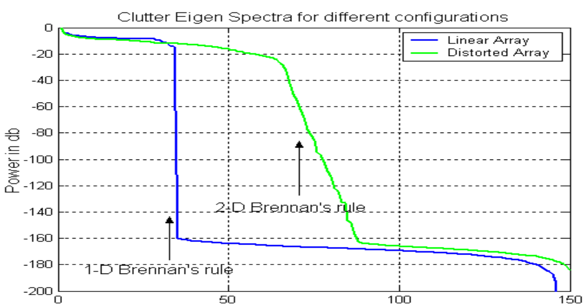

Fig. 2. Clutter Eigen Spectra

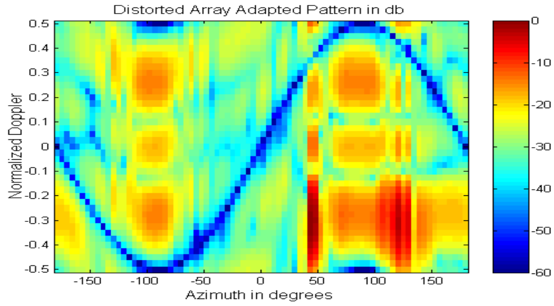

Fig. 4. Distorted Array Adapted Pattern

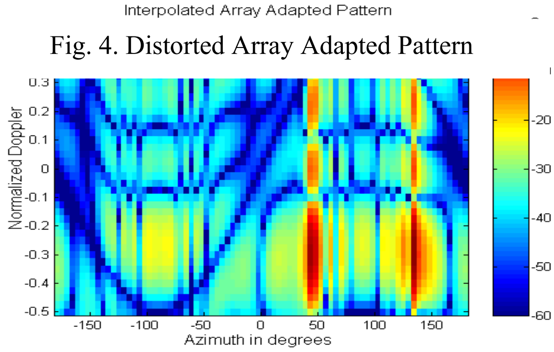

Fig. 3. Interpolated Array Adapted Pattern

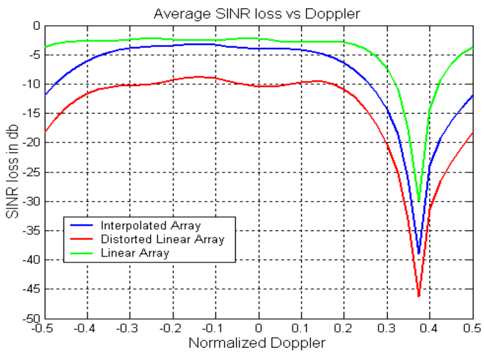

Fig. 5. SINR loss vs Doppler

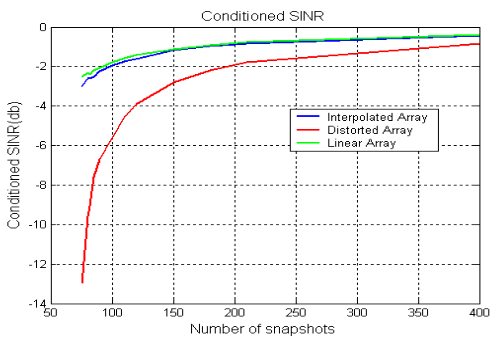

Fig. 6. Conditioned SINR