ADAPTIVE BEAMFORMING FOR CYCLOSTATIONARY SIGNALS

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ABSTRACT

In this paper, we propose a direction finding beamformer algorithm that exploits cyclostationarity (or periodic correlation). This new adaptive algorithm shows very attractive estimation performance over conventional beamforming methods, as depicted by simulation results.

1. INTRODUCTION

Array of sensors such as radio antennas can be used in the process of detecting the presence of propagating signals, estimating their directions of arrival (DOA) and other parameters. Conventional array processing methods basically exploit the spatial properties of the signals impinging on the array of sensors. Application areas include radar, sonar, biomedical signal processing, communication systems, and others.

More specifically, in this paper we focus on direction estimation methods in the telecommunication systems area (terrestrial or satellite), where almost all signals exhibit a statistical property called “cyclostationarity” [1]. A key point is that this property can be incorporated into signal processors to improve the direction finding performance of existing methods. We can find in the literature several algorithms, that exploit cyclostationarity to outperform conventional methods, such as [2], [3] and [4]. These methods can accommodate partial correlation among signals (e.g., arising from multi-path propagation or smart jamming), but they fail in presence of perfect correlation since they rely on a signal subspace analysis.

In this paper, we propose a new direction finding procedure, that exploits the cyclostationarity of incoming signals. The presented method is based on the conventional adaptive beamforming principle, and it can accommodate perfect correlation among signals, and offers the advantages of simple implementation and computational efficiency. The proposed procedure requires only knowledge of the baud rate, carrier frequency, or other parameters that characterize the underlying periodicity exhibited by the desired signals. Contrary to the above listed existing techniques, in our proposed procedure the *prior* knowledge of cyclostationarity is not used to discriminate a desired signal against undesired signals, but is exploited to outperform conventional methods, that is to improve resolution power and noise robustness without significant growth in complexity.

2. CYCLOSTATIONARY SIGNALS

For a signal $s(t)$, its cyclic autocorrelation function and cyclic conjugate autocorrelation function can be defined [1] as the following infinite-time averages :

$$\begin{aligned} R^{\alpha}_{ss}(\tau) &= \left\langle s(t+\tau/2)s^{*}(t-\tau/2)e^{-j2\pi\alpha t}\right\rangle \\ R^{\alpha}_{ss^{*}}(\tau) &= \left\langle s(t+\tau/2)s(t-\tau/2)e^{-j2\pi\alpha t}\right\rangle \end{aligned} \quad (1)$$

respectively, where the superscript * denotes the complex conjugate.

The temporal signal $s(t)$ is said to be wide-sense cyclostationary if $R^{\alpha}_{ss}(\tau)$ or $R^{\alpha}_{ss^{*}}(\tau)$ are not equal to zero at the cycle frequency α for some lag parameter τ.

Let the data vector received by an array of antennas be designated as $\mathbf{x}(t)$. Then its cyclic autocorrelation matrix and cyclic conjugate autocorrelation matrix are given by :

$$\begin{aligned} \mathbf{R}^{\alpha}_{\mathbf{xx}}(\tau) &= \left\langle \mathbf{x}(t+\tau/2)\mathbf{x}^{H}(t-\tau/2)e^{-j2\pi\alpha t}\right\rangle \\ \mathbf{R}^{\alpha}_{\mathbf{xx}^{*}}(\tau) &= \left\langle \mathbf{x}(t+\tau/2)\mathbf{x}^{T}(t-\tau/2)e^{-j2\pi\alpha t}\right\rangle \end{aligned} \quad (2)$$

respectively, where the superscript ^{H} denotes the conjugate transpose and ^{T} the transpose. Note that in practice, the above correlation matrices are estimated by the finite-time average operator.

In the next sections, we consider that the incoming signals are cyclostationary with cycle frequency α. Depending on the kind of modulation used, the cycle frequency α is often equal to the twice of the carrier frequency or multiples of the baud rate or combinations of these [1]. In this paper we only consider complex base-band signals (their carrier frequency has been removed). This assumption implies that the carrier frequency is not involved in the estimation of the cyclic correlation matrices in the following of this paper.

3. ARRAY DATA MODEL

In this paper we consider an array of M antennas. Suppose K sources impinging on the array from angular directions θ_k, $k = 1, \ldots, K$. The incident waves are assumed to be plane waves, as generated from far-field point sources. Furthermore, the signals are assumed to be narrowband. The signal received by the array from the emitting narrowband sources is described by the following conventional model :

$$\mathbf{z}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k) s_k(t) + \mathbf{n}(t) \quad (3)$$

where the vector $\mathbf{s}(t) = [s_1(t), \ldots, s_K(t)]^T$ contains temporal signals transmitted by the K sources, the vector $\mathbf{n}(t)$ represents spatially white noise. The matrix $\mathbf{A} = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_K)]$ contains the steering vectors of the impinging sources. Noise and signals are assumed to be uncorrelated and the noise is spatially white. The covariance matrix of the array measurements can be estimated in practice by :

$$\mathbf{R}_{\mathbf{zz}} = \left\langle \mathbf{z}(t)\mathbf{z}^H(t) \right\rangle \quad (4)$$

By assuming that the sources emit cyclostationary signals, we can give an extension of the conventional model (3), so that the cyclostationarity can be exploited. We form the following data vector :

$$\mathbf{z}_{\alpha\tau}(t) = \begin{bmatrix} \mathbf{z}(t) \\ \mathbf{z}^*(t-\tau)e^{j2\pi\alpha(t-\frac{\tau}{2})} \end{bmatrix} \quad (5)$$

The extended covariance matrix is then given by :

$$\mathbf{R}_{\alpha\tau} = \left\langle \mathbf{z}_{\alpha\tau}(t)\mathbf{z}_{\alpha\tau}^H(t) \right\rangle \quad (6)$$

According to (3), it can be written :

$$\mathbf{z}_{\alpha\tau}(t) = \sum_{k=1}^{K} \mathbf{B}(\theta_k) \begin{bmatrix} s(t) \\ s^*(t-\tau)e^{j2\pi\alpha(t-\frac{\tau}{2})} \end{bmatrix} + \begin{bmatrix} \mathbf{n}(t) \\ \mathbf{n}^*(t-\tau)e^{j2\pi\alpha(t-\frac{\tau}{2})} \end{bmatrix} \quad (7)$$

where

$$\mathbf{B}(\theta) = \begin{bmatrix} \mathbf{a}_1(\theta) & \mathbf{a}_2(\theta) \end{bmatrix} \quad (8)$$

with

$$\mathbf{a}_1(\theta_k) = \begin{bmatrix} \mathbf{a}(\theta_k) \\ \mathbf{0} \end{bmatrix} \quad \text{and} \quad \mathbf{a}_2(\theta_k) = \begin{bmatrix} \mathbf{0} \\ \mathbf{a}^*(\theta_k) \end{bmatrix} \quad (9)$$

According to the model (7), for any DOA θ_k, there are two signal components : a non conjugate signal component and a conjugate delayed signal component. Moreover, these two signal components are associated with the steering vectors $\mathbf{a}_1(\theta_k)$ and $\mathbf{a}_2(\theta_k)$, respectively. For any angle θ, note that :

$$\begin{aligned} &\mathbf{a}_1^H(\theta)\, \mathbf{a}_2(\theta) = 0 \\ &\mathbf{a}_1^H(\theta)\, \mathbf{a}_1(\theta) = \mathbf{a}_2^H(\theta)\, \mathbf{a}_2(\theta) = \beta \end{aligned} \quad (10)$$

where β is a real positive constant for any angle θ such that $||\mathbf{a}(\theta)||^2 = \beta$. In the following, without losing generality, we consider that β is unity.

4. ADAPTIVE BEAMFORMING FOR CYCLOSTATIONARY SIGNALS

In this section we propose a new direction finding method based on the adaptive beamforming principle. Let the beamformer output be $y(t)$, it can be expressed as the inner product of a $2M$-dimensional weight vector $\mathbf{w}$ and the extended data vector $\mathbf{z}_{\alpha\tau}(t)$:

$$y(t) = \mathbf{w}^H \mathbf{z}_{\alpha\tau}(t) \quad (11)$$

The output power can be estimated by :

$$\left\langle ||y(t)||^2 \right\rangle = \mathbf{w}^H \mathbf{R}_{\alpha\tau} \mathbf{w} \quad (12)$$

The basic idea of this method is to constrain the response of the beamformer so that the contribution of the two signal components (the above mentioned non conjugate signal component and the conjugate delayed one) from the direction of interest are passed with a specified gain. The weight vector $\mathbf{w}$ is chosen to minimize output variance or power subject to the response constraint. This has the effect of preserving the whole contribution (the non conjugate and conjugate delayed parts) of the desired signal while minimizing that due to the noise and the interfering signals coming from directions other than the steered direction.

The response constraint about the contributions of the two signal components can be expressed by the following equation (the whole contribution is normalized) :

$$\mathbf{B}^H(\theta)\mathbf{w} = \frac{\mathbf{c}}{||\mathbf{c}||} \quad \text{with} \quad \mathbf{c} = \begin{bmatrix} c_1 & c_2 \end{bmatrix}^T \quad (13)$$

In this constraint equation, coefficients c_1 and c_2 are complex numbers. Since the contribution of each of the two components of the signal is not known, c_1 and c_2 are unknown. The constraint can also be written as :

$$\mathbf{w}^H \mathbf{B}(\theta) \frac{\mathbf{c}}{||\mathbf{c}||} = 1 \quad (14)$$

Consequently, for any given vector $\mathbf{c}$ and any given angle θ, the linearly constrained minimum variance criterion for choosing the weight vector $\mathbf{w}$ is :

$$\min_{\mathbf{w}} \ \mathbf{w}^H \mathbf{R}_{\alpha\tau} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{B}(\theta) \frac{\mathbf{c}}{||\mathbf{c}||} = 1 \quad (15)$$

The solution to this problem obtained via the method of the Lagrange multipliers is given by :

$$\mathbf{w} = \frac{\mathbf{R}_{\alpha\tau}^{-1}\mathbf{B}(\theta)\mathbf{c}\ \|\mathbf{c}\|}{\mathbf{c}^H\mathbf{B}^H(\theta)\mathbf{R}_{\alpha\tau}^{-1}\mathbf{B}(\theta)\mathbf{c}} \quad (16)$$

Once the minimum variance beamformer weight vector $\mathbf{w}$ is known, it can used to calculate the minimum variance beamformer output power. The weight vector $\mathbf{w}$ is a function of θ and $\mathbf{c}$. According to equations (12) and (10), the beamformer output power is given by :

$$P_{MV\alpha\tau}(\theta, \mathbf{c}) = \frac{\mathbf{c}^H\mathbf{c}}{\mathbf{c}^H\mathbf{B}^H(\theta)\mathbf{R}_{\alpha\tau}^{-1}\mathbf{B}(\theta)\mathbf{c}} \quad (17)$$

DOA estimates are obtained by maximizing the beamformer output power (17), over the angle θ and the vector $\mathbf{c}$. The beamformer output power (17) can first be maximized over $\mathbf{c}$, for any fixed angle θ. The minimum variance beamformer output power spectrum can thus be defined as :

$$P_{MV\alpha\tau}(\theta) = \left[\min_{\mathbf{c}} \frac{\mathbf{c}^H\mathbf{B}^H(\theta)\mathbf{R}_{\alpha\tau}^{-1}\mathbf{B}(\theta)\mathbf{c}}{\mathbf{c}^H\mathbf{c}}\right]^{-1} \quad (18)$$

For a fixed θ, the minimum value in brackets is given by the minimum eigenvalue of the matrix $\mathbf{B}^H(\theta)\mathbf{R}_{\alpha\tau}^{-1}\mathbf{B}(\theta)$, and the minimizing vector $\mathbf{c}$ is the associated eigenvector. $\mathbf{B}^H(\theta)\mathbf{R}_{\alpha\tau}^{-1}\mathbf{B}(\theta)$ being a (2×2)-dimensional hermitian matrix, a low computational cost is required to determine this minimum eigenvalue. The proposed beamformer output power spectrum is then :

$$P_{MV\alpha\tau}(\theta) = \left[\begin{array}{c}\text{Minimum eigenvalue of} \\ \mathbf{B}^H(\theta)\mathbf{R}_{\alpha\tau}^{-1}\mathbf{B}(\theta)\end{array}\right]^{-1} \quad (19)$$

So the search for the minimizing vector $\mathbf{c}$ is replaced by the calculation of the minimum eigenvalue of the matrix $\mathbf{B}^H(\theta)\mathbf{R}_{\alpha\tau}^{-1}\mathbf{B}(\theta)$, which is a substantial reduction in computation. It is only necessary to search the spectrum (19) in angle in order to find the maxima, and the DOA are determined from the location of the spectral peaks.

5. SIMULATION RESULTS

In this section we will compare the simulation results provided by the proposed method with those obtained by the Capon's method [5] (which does not take the cyclostationarity into account). The computational cost for the proposed algorithm is less attractive than that of the Capon's method since it requires an inversion of a $(2M \times 2M)$-dimensional matrix. However it will be shown that the proposed cyclostationarity-exploiting adaptive beamforming method has a better resolution power than that of the Capon's method. These results are obtained by simulating a linear uniformly spaced array with 6 sensors spaced $\lambda/2$ apart. The number of data samples generated at each sensor output is 100. In our simulations, the receiving signals are 16 Mb/s OQPSK modulated signals. In order to choose correctly the parameters α and τ, we have estimated the cyclic conjugate autocorrelation function (figure 1) for a 16 Mb/s OQPSK signal sampled with the frequency 64 MHz during 125 μs. According to this result, the proposed cyclic adaptive beamforming method is simulated with $\alpha = 4\ MHz$ and $\tau = 0$. By choosing these values for the cycle frequency α and the lag parameter τ, we ensure that the cyclic conjugate autocorrelation matrix of incoming OQPSK modulated signals is non zero.

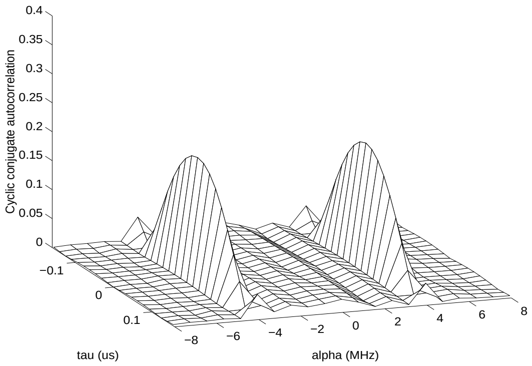

Fig. 1. *OQPSK cyclic conjugate autocorrelation function.*

In the next simulations, the performance of the estimators is evaluated from 200 Monte-Carlo simulations, by calculating the failure rate and the root mean square error (RMSE) of DOA estimates. The failure rate is determined according to the following criteria : for two sources coming from the arrival angles θ_1 and θ_2 $(\theta_1 < \theta_2)$, spaced by $\Delta\theta = \theta_2 - \theta_1$, we consider that the sources are well detected by the algorithms if the two estimated angles are included in the interval $[\theta_1 - \frac{\Delta\theta}{2}, \theta_2 + \frac{\Delta\theta}{2}]$.

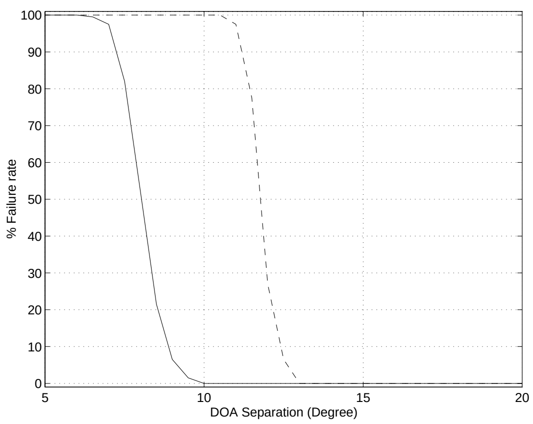

Fig. 2. *DOA estimation failure rate versus DOA separation (SNR=$0dB$), with proposed method (solid line) and with Capon's method (dashed line).*

Suppose two sources emitting OQPSK modulated signals and impinging on the array with a SNR of $0\ dB$. Failure rates versus DOA separation obtained with the proposed method (solid line) and with the Capon's method (dashed line) are provided in figure 2. The proposed method can localize closer sources than the Capon's method does. So this simulation result shows the improvement in terms of resolution power provided by exploiting the cyclostationarity property of incoming signals.

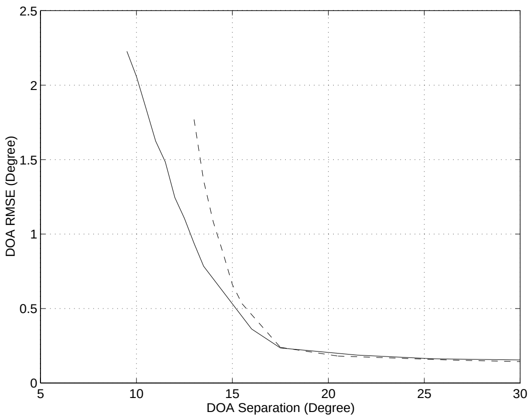

Fig. 3. *RMSE of DOA estimation versus DOA separation (SNR=$0dB$), with proposed method (solid line) and with Capon's method (dashed line).*

Figure 3 shows how DOA separation affects the performance of the proposed method (solid line) compared with the Capon's method (dashed line), in terms of RMSE. As expected, the proposed method provides more accurate DOA estimates, especially for small DOA separation between the two sources. For these last simulations, the failure rate is always keeping lower than $5\ \%$, and so according to the results presented in figure 2, the DOA separation must be at least 9° for the proposed method and 13° for the Capon's method. We can conclude that taking into account the cyclostationarity property of incoming signals improves the estimation accuracy of the beamforming estimators.

Figure 4 shows how SNR affects the performance of the proposed method (solid line), compared with the Capon's method (dashed line), also in terms of RMSE. According to this result, we can say that the proposed method has better noise robustness than the conventional method.

6. CONCLUSION

In this paper we have presented a new direction finding algorithm with an array of antennas. The method relies on the beamforming principle. By exploiting the cyclostationarity property of the incoming signals, the proposed technique shows attractive direction finding performances. Simulation results have shown that the proposed method improves sig-

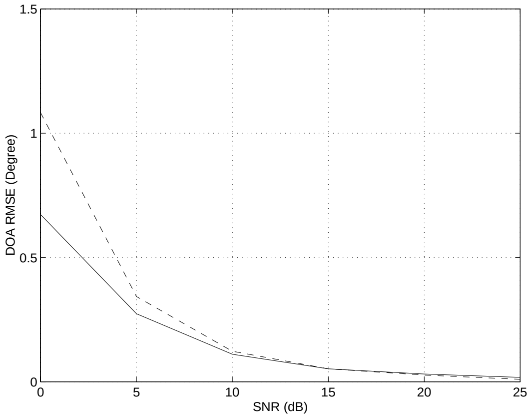

Fig. 4. *RMSE of DOA estimation versus SNR (DOA separation of 14°), with proposed method (solid line) and with Capon's method (dashed line).*

nificantly failure rate, resolution power, accuracy and noise robustness of adaptive array beamforming.

7. REFERENCES

[1] W.A. Gardner *et al.*, "Cyclostationarity in communications and signal processing," *IEEE press*, 1993.

[2] S.V. Schell, R.A. Calabreta, W.A. Gardner and B.G. Agee, "Cyclic MUSIC algorithms for signal-selective DOA estimation," *in Proc. IEEE ICASSP*, pp. 2278-2281, Glasgow, Scotland, May 1989.

[3] L. Izzo, L. Paura and G. Poggi, "An interference-tolerant algorithm for localization of cyclostationary-signal sources," *IEEE Transactions on Signal Processing*. vol. 40, pp. 1682-1686, July 1992.

[4] G. Xu and T. Kailath, "Direction of arrival estimation via exploitation of cyclostationarity – A combination of temporal and spatial processing," *IEEE Transactions on Signal Processing*. vol. 40, pp. 1775-1786, July 1992.

[5] J. Capon, "High resolution frequency-wavenumber spectrum analysis," Proc. IEEE, vol. 57, pp. 1408-1418, August 1969.

[6] P. Chargé, Y. Wang, J. Saillard, "An extended cyclic MUSIC algorithm," *in Proc. IEEE ICASSP*, Orlando, Florida USA, May 2002.