EFFICIENT METHOD TO DETERMINE DIAGONAL LOADING VALUE

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ABSTRACT

It is well known that the performance of adaptive beamforming algorithm is degraded when the sample support is short. The diagonal loading method is a simple and efficient method for improving the robustness of adaptive beamforming for such cases Meanwhile, we are not aware of a formal approach to determine an optimal diagonal value to date. In this paper, we will propose one data dependent method for the determination of the diagonal loading value. The proposed method makes a connection between the diagonal loading value and the estimation error of the estimated covariance matrix. The larger the estimation error, the larger the diagonal loading value is. Thus, the proposed method adjusts the diagonal loading value according to the array data. In addition, this method is efficient in computation.

1. INTRODUCTION

It is well known that the performances of adaptive beamforming algorithms are degraded when the sample support is short. Some studies have shown that while few samples are required for effective interference suppression, the adaptive algorithm generates a beam pattern with distorted main beam and high side lobes. Kelly[1] showed that the expected value of side lobe level is equal to: $E(SLL) = \frac{1}{K+1}$, where K is the number of samples. This indicates that in order to achieve -40dB average side lobes in the adaptive beam pattern, 10,000 samples should be used in estimating the covariance matrix and the conventional beam shape must be below the -40dB level. However, in practice, the requirement for such long data lengths is often limited by stationary conditions, which are often destroyed in cases of fast moving interference or short signal duration.

Carlson [2] proposed to improve the robustness of adaptive beamforming for cases of small sample support by diagonal loading technique, which is a simple and efficient method. However, the diagonal loading value determination has not been reported till now in spite that certain upper and lower bounds have been proposed [3] [4].

In this paper, we will introduce one data dependent method for determining the diagonal loading value. The loading value is related to the error in estimating the covariance matrix. The larger the covariance matrix estimation error is, the larger the diagonal loading value will be. The data dependent method has the advantage over the fixed diagonal loading value in that it adjusts the diagonal loading value according to available data. Therefore, the beamformer performance can be maintained at a certain level even when the data quality is changing.

In order to implement this method, we also proposed a simple approach for estimating the error in the covariance matrix estimation. Thus the proposed diagonal loading value determination method is data dependent and efficient in computation.

2. UPPER AND LOWER BOUNDS

Assume the received signal by an array of M sensors is:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \qquad (1)$$

where $\mathbf{A}$ is the array manifold, $\mathbf{n}$(t) is white noise with variance $\sigma_n{}^2$ and $\mathbf{s}$(t) is a vector of the signals emitted by P independent sources with P<M. The covariance matrix of the array output is:

$$\mathbf{R_{xx}} = \sum_{i=1}^{P} \sigma_i{}^2 \mathbf{a}_i \mathbf{a}_i{}^H + \sigma_n{}^2 \mathbf{I} = \mathbf{A}\mathbf{\Lambda_s}\mathbf{A}^H + \sigma_n{}^2 \mathbf{I} \qquad (2)$$

where $\sigma_n{}^2$ is the noise power, $\sigma_i{}^2$ indicates the i[th] signal power, $\mathbf{A}$=[$\mathbf{a}_1$,…$\mathbf{a}_P$] and $\mathbf{\Lambda_s}$ is the signals' covariance matrix which is a diagonal matrix in which each diagonal element corresponds to one signal power.

From this equation, the diagonal elements of the covariance matrix have the same value: $\sum_{i=1}^{P}\sigma_i^2+\sigma_n^2$ which is the sum of the signal and noise power, when the array response is normalized: $\mathbf{a}_i'\mathbf{a}_i=1$, i=1…P.

In practice, the adaptive beamformming weights are estimated using sensor outputs as:

$$\mathbf{w}'=\beta\mathbf{a}'\hat{\mathbf{R}}_{\mathbf{xx}}^{-1} \quad (3)$$

where **a** is the steering vector and $\hat{\mathbf{R}}_{\mathbf{xx}}$ is the estimated covariance matrix of sensor output data which is calculated by:

$$\hat{\mathbf{R}}_{\mathbf{xx}}=\frac{1}{K}\sum_{k=1}^{K}\mathbf{X}(k)\mathbf{X}(k)' \quad (4)$$

and the scalar factor β is:

$$\beta=\frac{1}{\mathbf{a}'\hat{\mathbf{R}}_{\mathbf{xx}}^{-1}\mathbf{a}} \quad (5)$$

As the data length is limited, the estimated covariance matrix has errors and it can be rewritten as [7]:

$$\hat{\mathbf{R}}_{\mathbf{xx}}=\mathbf{R}_{\mathbf{xx}}+\varepsilon\mathbf{B} \quad (6)$$

where $\mathbf{R}_{\mathbf{xx}}$ is the true covariance matrix as given in (2), **B** is a zero mean random matrix with unit variance and ε is a positive constant which indicates the estimation error of the estimated covariance matrix. Evidently, the larger the estimation error is, the worse the beamformer performance is.

The diagonal loaded data covariance matrix is:

$$\mathbf{R}_{DL}=\mathbf{R}_{\mathbf{xx}}+\lambda_{DL}\mathbf{I}+\varepsilon\mathbf{B} \quad (7)$$

where λ_{DL} is the diagonal loading value to be determined. Assume $\varepsilon\|\mathbf{B}\| << \|\mathbf{R}_{\mathbf{xx}}+\lambda_{DL}\mathbf{I}\|$, the inverse of the diagonal loaded covariance matrix can be approximately expressed as:

$$\begin{aligned}\mathbf{R}_{DL}^{-1}&=(\mathbf{R}_{\mathbf{xx}}+\lambda_{DL}\mathbf{I})^{-1}\left[\mathbf{I}+\varepsilon\mathbf{B}(\mathbf{R}_{\mathbf{xx}}+\lambda_{DL}\mathbf{I})^{-1}\right]^{-1}\\&\approx(\mathbf{R}_{\mathbf{xx}}+\lambda_{DL}\mathbf{I})^{-1}\left[\mathbf{I}-\varepsilon\mathbf{B}(\mathbf{R}_{\mathbf{xx}}+\lambda_{DL}\mathbf{I})^{-1}\right]\\&=(\mathbf{R}_{\mathbf{xx}}+\lambda_{DL}\mathbf{I})^{-1}\left\{\mathbf{I}-\frac{\varepsilon}{\lambda_{DL}+\sigma_n^2}\mathbf{B}\right.\\&\left.\left[\mathbf{I}-\mathbf{A}\left(\mathbf{A}^H\mathbf{A}+(\sigma_n^2+\lambda_{DL})\mathbf{\Lambda}_s^{-1}\right)^{-1}\mathbf{A}^H\right]\right\}\end{aligned} \quad (8)$$

From this equation, the term inside the first brackets should be close to $\mathbf{R}_{\mathbf{xx}}$, thus the diagonal loading value should be much smaller than the diagonal element value of the covariance matrix $\mathbf{R}_{\mathbf{xx}}$:

$$\lambda_{DL} << \mathbf{R}_{\mathbf{xx}}(i,i) \quad (9)$$

where i can be any value from 1 to M because all the diagonal values of $\mathbf{R}_{\mathbf{xx}}$ are equal, where M is the number of sensors.

From equation (8), it is observed that the beamformer performance degradation is caused by the second term in the curly brackets. If the second term is zero, the optimal beamforming will be obtained. Thus it is desirable to have:

$$\frac{\varepsilon}{\lambda_{DL}+\sigma_n^2}<1 \quad (10)$$

The diagonal value should satisfy:

$$\lambda_{DL}+\sigma_n^2>\varepsilon \quad (11)$$

As $\lambda_{DL}>0$, $\sigma_n^2>0$, it has:

$$\lambda_{DL}\geq\varepsilon \quad (12)$$

Combining condition (9) and (12), we get a set of upper and lower bounds for the diagonal loading value:

$$\varepsilon\leq\lambda_{DL}<\mathbf{R}_{\mathbf{xx}}(i,i) \quad (13)$$

From (10), if

$$\lambda_{DL}=n\varepsilon \quad (14)$$

it has:

$$\frac{\varepsilon}{\lambda_{DL}+\sigma_n^2}=\frac{1}{n+\sigma_n^2/\varepsilon}<\frac{1}{n} \quad (15)$$

In order to maintain deep nulls in the beam pattern, the diagonal loading value should not be too large while the side lobe reaches certain level. From (15), we suggest to select n=1 or 2.

In this method, it needs to know the estimation error of the estimated covariance matrix. In next part, we will propose a simple method to estimate it.

3. DIAGONAL LOADING VALUE AND ESTIMATED COVARIANCE MATRIX

Since it is impossible to obtain the true covariance matrix in real application, we will estimate the covariance matrix diagonal element value and the covariance matrix estimation error from the estimated covariance matrix.

From expression of the true covariance matrix, it is known that the diagonal elements have the same value. But the estimated covariance matrix has errors in each element. From equation (6), the error matrix B is a random matrix with zeros mean, thus, the diagonal element value of the true covariance matrix can be estimated by the average of the estimated covariance matrix diagonal element values:

$$\tilde{\mathbf{R}}_{\mathbf{xx}}(i,i)=trace(\hat{\mathbf{R}}_{\mathbf{xx}})/M \quad (16)$$

where M is the number of sensors.

Using the same observation, the standard deviation of the diagonal elements can be used as an indication of the covariance matrix estimation error [6]:

$$\varepsilon = Std(diag(\hat{\mathbf{R}}_{\mathbf{xx}})) \quad (17)$$

where *diag* means the diagonal elements of the matrix, *Std* means the standard deviation. Therefore, the diagonal loading value should satisfy:

$$Std(diag(\hat{\mathbf{R}}_{\mathbf{xx}})) \le \lambda_{DL} < trace(\hat{\mathbf{R}}_{\mathbf{xx}})/M \quad (18)$$

The diagonal loading value can be selected as:

$$\lambda_{DL} = Std(diag(\hat{\mathbf{R}}_{\mathbf{xx}})) \quad (19)$$

As the standard deviation of the diagonal elements of the estimated covariance matrix is very easy to be calculated, this method is efficient in computation. We selected n=1 in (19) compared with (15). This selection will be discussed in next part.

4. SIMULATION AND COMPARISON

Assume an interference of 30dB located at -25° and a source of interest located at 0° with a SNR of -6dB. A Chebyshev window of -35dB is used for estimating the beamformer weights. The inter-element distance is λ/2 in all the simulation. Fig.1 and 2 present the beam patterns for the ideal Minimum Variance Distortionless Response (MVDR), Sample Matrix Inverse (SMI) and loaded SMI (LSMI) beamformers with diagonal loading value calculated by (19) for two different array and data lengths. Fig.1 is computed for a linear array of 21 sensors with a data length of 42. The results in Fig.2 are computed for a linear array of 15 sensors and a data length of 30.

These results show that the diagonal loading value calculated by our proposed method can provide low side lobe level and maintain the null as deep as about 25dB below the side lobe level.

In order to investigate the performance of the proposed methods, statistical analysis results based on 100 independent realizations are presented in Fig.3-Fig.5. In these simulations, there are 3 interference sources of equal power with INR 30dB and a signal of interest is located at -7° and has an SNR of -1.5dB.

Fig.3 presents the output SINRs versus data length for SMI, LSMIcst with a constant diagonal loading value of 10dB above the noise and LSMIerror with the diagonal loading value calculated by the proposed methods. The number of sensors is 21. This result shows that the proposed method performs better than the constant diagonal value.

Fig.4 is the output SINR versus the number of sensors while the data length is twice the number of sensors. This simulation shows that the SINR of the optimal MVDR increases with the number of sensors, while the SINRs of the SMI and the LSMI with constant diagonal loading value decrease with the number of sensors. The proposed diagonal loading value based LSMI has the SINR increasing with the number of sensors. This result shows the advantage of the data dependent approach to determining the diagonal loading value over the constant diagonal loading value.

In order to verify the selection of the diagonal loading value, the output SINR versus n for $\lambda_{DL} = nStd(diag(\hat{\mathbf{R}}_{\mathbf{xx}}))$ is presented in Fig.5. These results show that n=1 is a suitable choice for arrays with different number of sensors.

5. CONCLUSION

In this paper, we have proposed an automatic data dependent method to determine the diagonal loading value for improving the performance of adaptive beamforming with short sample supports. The method requires an estimation of the covariance matrix estimation error and we have proposed a simple approach to achieve it, resulting in an efficient algorithm.

Since the proposed method relates the diagonal loading value with available data, the LSMI with the proposed diagonal loading value performs better than that using a fixed diagonal loading value in general.

6. REFERENCE

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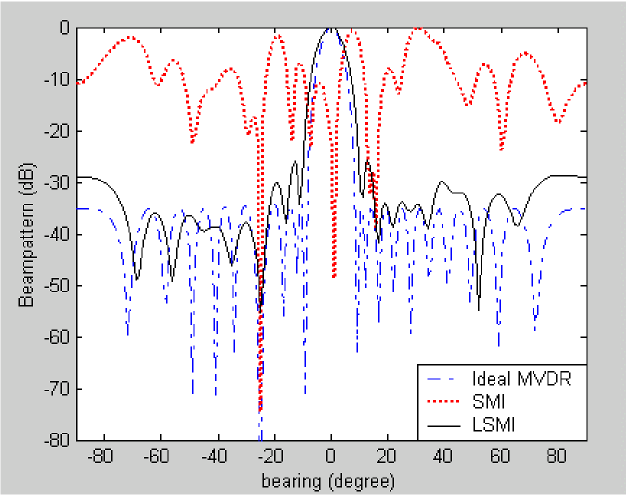


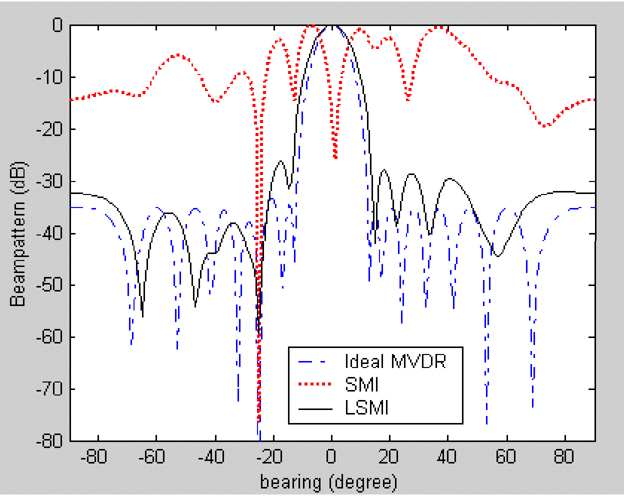
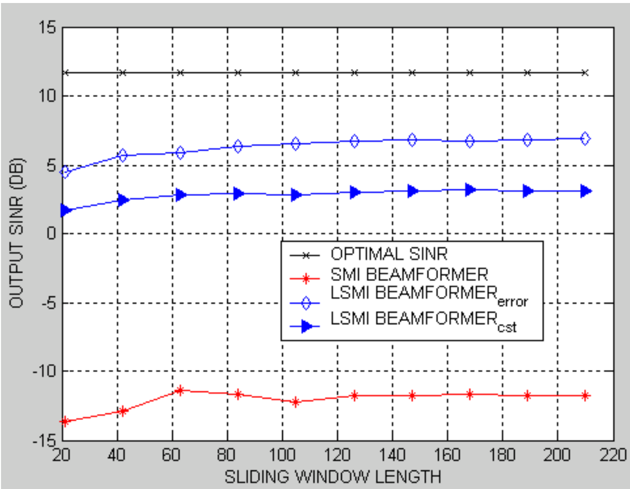


Fig.1 Beam pattern of MVDR, SMI and LSMI with 21 sensors

Fig.2 Beam pattern of MVDR, SMI and LSMI with 15 sensors



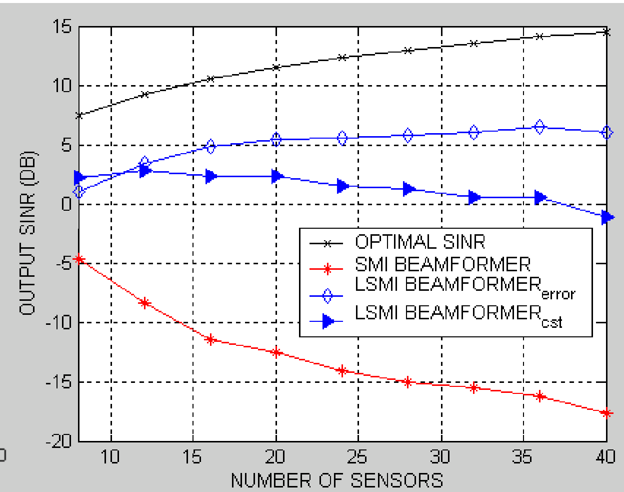


Fig.3 Output SINR versus data length (M=21, INR=30dB, SNR=-1.5dB)

Fig.4 Output SINR versus the number of sensors (Data length K=2M, INR=30dB, SNR=-1.5dB)

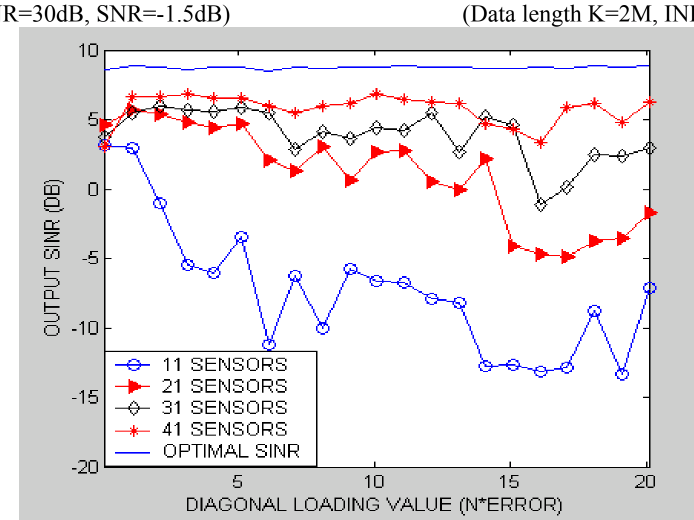


Fig.5 Output SINR versus the diagonal loading values (INR=30dB, SNR=-1.5dB, Data length K=2M)