ROBUST ADAPTIVE BEAMFORMING USING WORST-CASE SINR OPTIMIZATION: A NEW DIAGONAL LOADING-TYPE SOLUTION FOR GENERAL-RANK SIGNAL MODELS

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ABSTRACT

The performance of adaptive beamforming methods may degrade in the presence of even slight mismatches between the actual and presumed array responses to the desired signal. This paper addresses the problem of robust adaptive beamforming in the presence of unknown arbitrary (yet norm-bounded) mismatches of such type as well as interference-plus-noise covariance matrix mismatch. Our approach is developed for the case of an arbitrary dimension of the signal subspace and, therefore, it can be applied to both rank-one and higher-rank signal models. The proposed beamformer is based on the optimization of the worst-case signal-to-interference-plus-noise ratio (SINR). The obtained closed-form solution combines two different types of diagonal loading (DL) applied to the signal and data covariance matrices. An efficient on-line implementation of our beamformer is developed. Simulations validate substantial performance improvements relative to other popular adaptive beamforming techniques.

1. INTRODUCTION

The robustness of adaptive beamformers is known to depend essentially on the availability of signal-free data snapshots [1]-[3]. For the signal-free data case, many powerful and computationally efficient algorithms have been proposed [4]. However, in several applications such as wireless communications, microphone array speech processing, medical imaging, radio astronomy, etc., the signal-free snapshots are unavailable. In such applications, adaptive beamforming methods become very sensitive to mismatches of the array response to the desired signal. This phenomenon is sometimes referred to as the *signal self-nulling*. Similar types of degradation can take place when the signal array response is known exactly but the sample size is small [1], [3].

There are several efficient approaches to robust adaptive beamforming (see [1]-[3] and references therein) but most of them use *ad hoc* ways to improve the robustness. Recently, a more theoretically motivated approach has been developed in [5]-[6] that explicitly models an arbitrary (but bounded in norm) steering vector mismatch and uses *worst-case performance optimization* to improve the robustness of the minimum variance distortionless response (MVDR) beamformer. This method makes use of the second-order cone (SOC) programming based convex optimization approach to compute beamformer weights although it can be interpreted as a DL technique whose DL factor is optimally matched to the level of uncertainty of the signal steering vector. A related approach is reported in [7] where another DL-based iterative algorithm is considered as an alternative to the SOC programming approach.

A serious shortcoming of the above-mentioned robust techniques is that they are only applicable to the point (rank-one) signal model. In this paper, we propose a new robust approach to adaptive beamforming applicable to the general-rank signal model case (where the dimension of the signal subspace is not necessarily equal to one but can be arbitrary). The proposed approach is based on the optimization of the worst-case SINR and yields a closed-form solution which combines two types of DL applied to the signal and the array data covariance matrices.

An efficient on-line implementation of our beamformer is developed.

Supported in parts by the Natural Sciences and Engineering Research Council (NSERC) of Canada, Communications and Information Technology Ontario (CITO), Premier Research Excellence Award Program of the Ministry of Energy, Science, and Technology (MEST) of Ontario, Canada Research Chairs Program, and Wolfgang Paul Award Program of the Alexander von Humboldt Foundation.

2. BACKGROUND

The output of a narrowband beamformer is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k) \tag{1}$$

where k is the time index, $\mathbf{x}(k)$ is the $M \times 1$ complex vector of array observations, $\mathbf{w}$ is the $M \times 1$ complex vector of beamformer weights, M is the number of array sensors, and $(\cdot)^T$ and $(\cdot)^H$ are the transpose and Hermitian transpose, respectively. The data snapshot vector is given by

$$\mathbf{x}(k) = \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k) \tag{2}$$

where $\mathbf{s}(k)$, $\mathbf{i}(k)$, and $\mathbf{n}(k)$ are the statistically independent components of the desired signal, interference, and sensor noise, respectively. The optimal weight vector can be obtained through maximizing the signal-to-interference-plus-noise ratio (SINR) [4]

$$\mathrm{SINR} = \frac{\mathbf{w}^H \mathbf{R}_{\mathrm{s}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{i+n}} \mathbf{w}} \tag{3}$$

where $\mathbf{R}_{\mathrm{s}} = \mathrm{E}\left\{\mathbf{s}(k)\mathbf{s}^H(k)\right\}$ and $\mathbf{R}_{\mathrm{i+n}} = \mathrm{E}\{(\mathbf{i}(k) + \mathbf{n}(k))(\mathbf{i}(k) + \mathbf{n}(k))^H\}$ are the $M \times M$ signal and interference-plus-noise covariance matrices, respectively, and $\mathrm{E}\{\cdot\}$ denotes the statistical expectation. In the general case, the rank of $\mathbf{R}_{\mathrm{s}}$ can be arbitrary, i.e., $1 \leq \mathrm{rank}\{\mathbf{R}_{\mathrm{s}}\} \leq M$. In the specific *point signal source* case,

$\mathbf{s}(k) = s(k)\mathbf{a}_{\rm s}$ and $\mathbf{R}_{\rm s} = \sigma_{\rm s}^2\mathbf{a}_{\rm s}\mathbf{a}_{\rm s}^H$, where $s(k)$ is the zero-mean signal waveform, $\sigma_{\rm s}^2 = {\rm E}\{|s(k)|^2\}$ is the variance of $s(k)$, and $\mathbf{a}_{\rm s}$ is the source steering vector/spatial signature.

In many practical situations $\mathrm{rank}\{\mathbf{R}_{\rm s}\} > 1$ with typical examples being scenarios with incoherently scattered (spatially spread) signal sources or signals with random fluctuations of wavefronts (multiplicative noise) [8], [9]. In the general-rank case, the optimal solution for the weight vector maximizing the SINR in (3) is given by [3], [10]

$$\mathbf{w}_{\rm opt} = \mathcal{P}\{\mathbf{R}_{\rm i+n}^{-1}\mathbf{R}_{\rm s}\} \tag{4}$$

where $\mathcal{P}\{\cdot\}$ is the operator yielding the *principal eigenvector* of a matrix.

In the point signal source case, (4) is reduced to $\mathbf{w}_{\rm opt} = \mathcal{P}\{\mathbf{R}_{\rm i+n}^{-1}\mathbf{a}_{\rm s}\mathbf{a}_{\rm s}^H\}$ where we make use of the fact that in this particular case $\mathbf{R}_{\rm s} = \sigma_{\rm s}^2\mathbf{a}_{\rm s}\mathbf{a}_{\rm s}^H$. Obviously, the principal eigenvector of the matrix $\mathbf{R}_{\rm i+n}^{-1}\mathbf{a}_{\rm s}\mathbf{a}_{\rm s}^H$ is $\alpha\mathbf{R}_{\rm i+n}^{-1}\mathbf{a}_{\rm s}$ where α is an arbitrary constant which does not affect the output SINR. Hence, (4) is reduced to $\mathbf{w}_{\rm opt} = \mathbf{R}_{\rm i+n}^{-1}\mathbf{a}_{\rm s}$ which is the classic solution for the weight vector of the optimal beamformer in the rank-one signal case [4].

In practical situations, the $\mathbf{R}_{\rm i+n}$ is unavailable but can be estimated from the data snapshots. Therefore, the sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n) \tag{5}$$

is used instead of $\mathbf{R}_{\rm i+n}$ [4]. This yields the generalized version of the well-known sample matrix inverse (SMI) beamformer [4]

$$\mathbf{w}_{\rm SMI} = \mathcal{P}\{\hat{\mathbf{R}}^{-1}\mathbf{R}_{\rm s}\} \tag{6}$$

3. A NEW APPROACH TO ROBUST BEAMFORMING

In practical situations, both the signal and interference-plus-noise covariance matrices are known with some errors. Indeed, there is always a certain mismatch between the *presumed* signal covariance matrix $\mathbf{R}_{\rm s}$ and the *actual* signal covariance matrix $\tilde{\mathbf{R}}_{\rm s}$. For example, in cellular communications, the signal covariance matrix is usually estimated at the base station antenna array in the uplink mode for each mobile user during the interval when this particular user transmits its training sequence. However, such an estimate is always subject to some errors because of multiuser interference, user mobility, time variability of the communication channel, etc.

Furthermore, there is always a certain mismatch between the *presumed* and *actual* interference-plus-noise covariance matrices $\mathbf{R}_{\rm i+n}$ and $\tilde{\mathbf{R}}_{\rm i+n}$, respectively. These mismatches are caused by the presence of the desired signal in data snapshots, interferer mobility, channel variability, and small sample size effects. The errors in the signal and the interference-plus-noise covariance matrices can be modeled as

$$\tilde{\mathbf{R}}_{\rm s} = \mathbf{R}_{\rm s} + \mathbf{\Delta}_1 \tag{7}$$
$$\tilde{\mathbf{R}}_{\rm i+n} = \mathbf{R}_{\rm i+n} + \mathbf{\Delta}_2 \tag{8}$$

respectively, where $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ are the corresponding unknown matrix mismatches. Then, the equation (3) for the output SINR of an adaptive array has to be rewritten as

$$\mathrm{SINR} = \frac{\mathbf{w}^H\tilde{\mathbf{R}}_{\rm s}\mathbf{w}}{\mathbf{w}^H\tilde{\mathbf{R}}_{\rm i+n}\mathbf{w}} = \frac{\mathbf{w}^H(\mathbf{R}_{\rm s}+\mathbf{\Delta}_1)\mathbf{w}}{\mathbf{w}^H(\mathbf{R}_{\rm i+n}+\mathbf{\Delta}_2)\mathbf{w}} \tag{9}$$

We assume that the norms of the mismatch matrices $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ can be bounded by some known constants [5]

$$\|\mathbf{\Delta}_1\| \le \varepsilon\,, \qquad \|\mathbf{\Delta}_2\| \le \gamma \tag{10}$$

To provide robustness against possible norm-bounded mismatches (10), we propose to obtain the beamformer weight vector by means of maximizing the worst-case output SINR, i.e., by means of solving the following optimization problem:

$$\max_{\mathbf{w}}\min_{\mathbf{\Delta}_1,\,\mathbf{\Delta}_2}\frac{\mathbf{w}^H(\mathbf{R}_{\rm s}+\mathbf{\Delta}_1)\mathbf{w}}{\mathbf{w}^H(\mathbf{R}_{\rm i+n}+\mathbf{\Delta}_2)\mathbf{w}} \quad \forall \quad \|\mathbf{\Delta}_1\|\le\varepsilon\,,\ \|\mathbf{\Delta}_2\|\le\gamma \tag{11}$$

The problem (11) can be rewritten as

$$\max_{\mathbf{w}}\frac{\min\limits_{\|\mathbf{\Delta}_1\|\le\varepsilon}\mathbf{w}^H(\mathbf{R}_{\rm s}+\mathbf{\Delta}_1)\mathbf{w}}{\max\limits_{\|\mathbf{\Delta}_2\|\le\gamma}\mathbf{w}^H(\mathbf{R}_{\rm i+n}+\mathbf{\Delta}_2)\mathbf{w}} \tag{12}$$

We will make use of the following lemma.

Lemma 1: For any $M\times 1$ vector $\mathbf{w}$, $M\times M$ Hermitian matrix $\mathbf{C}$, and scalar $\delta > 0$,

$$\min_{\|\mathbf{\Delta}\|\le\delta}\mathbf{w}^H(\mathbf{C}+\mathbf{\Delta})\mathbf{w} = \mathbf{w}^H(\mathbf{C}-\delta\mathbf{I})\mathbf{w} \tag{13}$$
$$\max_{\|\mathbf{\Delta}\|\le\delta}\mathbf{w}^H(\mathbf{C}+\mathbf{\Delta})\mathbf{w} = \mathbf{w}^H(\mathbf{C}+\delta\mathbf{I})\mathbf{w} \tag{14}$$

where $\mathbf{I}$ is the identity matrix.

Proof: Let us consider the following problems

$$\min_{\mathbf{\Delta}}\mathbf{w}^H(\mathbf{C}+\mathbf{\Delta})\mathbf{w} \quad \text{subject to} \quad \|\mathbf{\Delta}\|\le\delta \tag{15}$$
$$\max_{\mathbf{\Delta}}\mathbf{w}^H(\mathbf{C}+\mathbf{\Delta})\mathbf{w} \quad \text{subject to} \quad \|\mathbf{\Delta}\|\le\delta \tag{16}$$

From the linearity of the objective function $\mathbf{w}^H(\mathbf{C}+\mathbf{\Delta})\mathbf{w}$ with respect to $\mathbf{\Delta}$, it follows that the inequality constraint $\|\mathbf{\Delta}\|\le\delta$ in (15) and (16) can be replaced by the equality constraint $\|\mathbf{\Delta}\| = \delta$. Therefore, the solutions to (15) and (16) can be obtained using the Lagrange multiplier method, by means of minimizing/maximizing the function

$$L(\mathbf{\Delta},\lambda) = \mathbf{w}^H(\mathbf{C}+\mathbf{\Delta})\mathbf{w} - \lambda(\|\mathbf{\Delta}\|^2 - \delta^2) \tag{17}$$

Equating the gradient $\partial L(\mathbf{\Delta},\lambda)/\partial\mathbf{\Delta}$ to zero and taking into account the constraint $\|\mathbf{\Delta}\| = \delta$, we obtain $\mathbf{\Delta} = \mp\delta\,\mathbf{w}\mathbf{w}^H/\|\mathbf{w}\|^2$. Inserting the latter equation into the objective function $\mathbf{w}^H(\mathbf{C}+\mathbf{\Delta})\mathbf{w}$ yields $\mathbf{w}^H(\mathbf{C}\mp\delta\mathbf{I})\mathbf{w}$. Since δ is positive, $\mathbf{w}^H(\mathbf{C}-\delta\mathbf{I})\mathbf{w} < \mathbf{w}^H(\mathbf{C}+\delta\mathbf{I})\mathbf{w}$ and this proves equations (13) and (14). □

Using this lemma, the optimization problem (12) becomes

$$\max_{\mathbf{w}}\frac{\mathbf{w}^H(\mathbf{R}_{\rm s}-\varepsilon\mathbf{I})\mathbf{w}}{\mathbf{w}^H(\mathbf{R}_{\rm i+n}+\gamma\mathbf{I})\mathbf{w}} \tag{18}$$

The solution to (18) can be expressed in a closed-form

$$\mathbf{w}_{\rm rob} = \mathcal{P}\{(\mathbf{R}_{\rm i+n}+\gamma\mathbf{I})^{-1}(\mathbf{R}_{\rm s}-\varepsilon\mathbf{I})\} \tag{19}$$

Note that if ε is larger than the maximal eigenvalue of $\mathbf{R}_{\rm s}$, the matrix $\mathbf{R}_{\rm s}-\varepsilon\mathbf{I}$ becomes negative definite and the non-negativeness constraint on the worst-case signal power is violated. Therefore, the parameter ε smaller than the maximal eigenvalue of $\mathbf{R}_{\rm s}$ must be chosen.

In practice, the matrix $\mathbf{R}_{\rm i+n}$ is unavailable and the sample covariance matrix $\hat{\mathbf{R}}$ should be used instead of $\mathbf{R}_{\rm i+n}$ in (18). The solution to such modified problem yields a more practical (sample) version of the robust beamformer (19)

$$\mathbf{w}_{\rm rob} = \mathcal{P}\{(\hat{\mathbf{R}} + \gamma\mathbf{I})^{-1}(\mathbf{R}_{\rm s} - \varepsilon\mathbf{I})\} \tag{20}$$

From (20) it is clear that the worst-case performance optimization approach leads to a new DL-based beamformer. Indeed, both the *negative* and *positive* types of DL are combined in (20), where the negative loading is applied to the presumed covariance matrix of the desired signal, while the positive loading is applied to the sample data covariance matrix. The optimal values of the DL factors γ and ε are obtained based on known levels of uncertainty of the signal and interference-plus-noise covariance matrices.

In the point source case, assuming without loss of generality that $\sigma_{\rm s}^2 = 1$ (i.e., absorbing $\sigma_{\rm s}^2$ in ε), we have that (20) can be rewritten as

$$\mathbf{w}_{\rm rob} = \mathcal{P}\{(\hat{\mathbf{R}} + \gamma\mathbf{I})^{-1}(\mathbf{a}_{\rm s}\mathbf{a}_{\rm s}^H - \varepsilon\mathbf{I})\} \tag{21}$$

4. ON-LINE IMPLEMENTATION

In the previous section, the robust algorithm (20) is formulated in a form suitable for the batch processing mode. In practical systems, on-line implementations are often required where the weight vector must be updated with each new data snapshot. In this section, a computationally efficient on-line implementation of the algorithm (20) is developed.

We will make use of the following lemma.

Lemma 2: For arbitrary $M \times M$ Hermitian matrix $\mathbf{X}$ and arbitrary $M \times M$ full-rank Hermitian matrix $\mathbf{Y}$ the following relationship holds

$$\mathcal{P}\{\mathbf{XY}\} = \mathbf{Y}^{-1/2}\mathcal{P}\{\mathbf{Y}^{1/2}\mathbf{X}\mathbf{Y}^{1/2}\}\,. \tag{22}$$

Proof: Write the characteristic equation for $\mathbf{XY}$ as

$$\mathbf{XY}\mathbf{u}_i = \lambda_i\mathbf{u}_i \tag{23}$$

where $\{\lambda_i\}_{i=1}^M$ and $\{\mathbf{u}_i\}_{i=1}^M$ are the eigenvalues and corresponding eigenvectors of $\mathbf{XY}$. Multiplying this equation by $\mathbf{Y}^{1/2}$ gives

$$\mathbf{Y}^{1/2}\mathbf{X}\underbrace{\mathbf{Y}^{1/2}\mathbf{Y}^{1/2}}_{\mathbf{Y}}\mathbf{u}_i = \lambda_i\mathbf{Y}^{1/2}\mathbf{u}_i \tag{24}$$

which is the characteristic equation for $\mathbf{Y}^{1/2}\mathbf{X}\mathbf{Y}^{1/2}$:

$$\mathbf{Y}^{1/2}\mathbf{X}\mathbf{Y}^{1/2}\mathbf{v}_i = \lambda_i\mathbf{v}_i \tag{25}$$

where the eigenvectors of the matrices $\mathbf{XY}$ and $\mathbf{Y}^{1/2}\mathbf{X}\mathbf{Y}^{1/2}$ are related as $\mathbf{v}_i = \mathbf{Y}^{1/2}\mathbf{u}_i$ for all $i = 1, 2, \ldots, M$. Applying this result to the principal eigenvectors of $\mathbf{XY}$ and $\mathbf{Y}^{1/2}\mathbf{X}\mathbf{Y}^{1/2}$, we obtain (22). □

Applying this lemma to the beamformer (20) can we rewrite it as

$$\begin{aligned}\mathbf{w}_{\rm rob} &= (\mathbf{R}_{\rm s} - \varepsilon\mathbf{I})^{-1/2}\,\mathcal{P}\{\mathbf{G}^{-1}\} \\ &= (\mathbf{R}_{\rm s} - \varepsilon\mathbf{I})^{-1/2}\,\mathcal{M}\{\mathbf{G}\}\end{aligned} \tag{26}$$

where $\mathcal{M}\{\cdot\}$ is the operator yielding the *minor eigenvector* of a matrix [10] and

$$\mathbf{G} \triangleq (\mathbf{R}_{\rm s} - \varepsilon\mathbf{I})^{-1/2}(\hat{\mathbf{R}} + \gamma\mathbf{I})(\mathbf{R}_{\rm s} - \varepsilon\mathbf{I})^{-1/2} \tag{27}$$

It is worth noting that even if $\mathbf{R}_{\rm s}$ is singular or ill-conditioned, the matrix $\mathbf{R}_{\rm s} - \varepsilon\mathbf{I}$ can be made full-rank (well-conditioned) by a proper choice of the parameter ε. Furthermore, for any nonzero ε, $\text{rank}\{\mathbf{R}_{\rm s} - \varepsilon\mathbf{I}\} = M$ *almost surely*.

Let us consider the case of rectangular sliding window of the length N where the update of the matrix $\breve{\mathbf{R}} \triangleq \hat{\mathbf{R}} + \gamma\mathbf{I}$ in the nth step can be computed as

$$\begin{aligned}\breve{\mathbf{R}}(n) &= \breve{\mathbf{R}}(n-1) + \frac{1}{N}\mathbf{x}(n)\mathbf{x}^H(n) \\ &- \frac{1}{N}\mathbf{x}(n-N)\mathbf{x}^H(n-N)\end{aligned} \tag{28}$$

Note that (28) represents the so-called rank-2 update. The diagonal load should be added in the initialization step of (28), that is, we initialize the matrix $\breve{\mathbf{R}}$ with $\gamma\mathbf{I}$. Using (28), we can rewrite the update of the matrix (27) as

$$\begin{aligned}\mathbf{G}(n) &= \mathbf{G}(n-1) + \tilde{\mathbf{x}}(n)\tilde{\mathbf{x}}^H(n) \\ &- \tilde{\mathbf{x}}(n-N)\tilde{\mathbf{x}}^H(n-N)\end{aligned} \tag{29}$$

where the *transformed* data snapshots are defined as $\tilde{\mathbf{x}}(n) = \frac{1}{\sqrt{N}}(\mathbf{R}_{\rm s} - \varepsilon\mathbf{I})^{-1/2}\mathbf{x}(n)$ and, according to (27), $\gamma(\mathbf{R}_{\rm s} - \varepsilon\mathbf{I})^{-1}$ should be chosen to initialize the matrix $\mathbf{G}$.

According to equations (26) and (29), the on-line algorithm for updating the weight vector $\mathbf{w}_{\rm rob}$ involves updating the matrix $\mathbf{G}$ and tracking the minor eigenvector of this matrix. An alternative way is to make use of the matrix inversion lemma to find the update of the matrix $\mathbf{G}^{-1}$ and then track the principal eigenvector of this matrix. Any of subspace tracking algorithms available in the literature can be used for this purpose. Note that the complexities of the existing subspace tracking techniques lie between $O(M)$ and $O(M^2)$ per step. Therefore, the total complexities of the on-line implementations developed are given by $O(M^2)$ per step because regardless of the complexity of the subspace tracking algorithm used, $O(M^2)$ additional operations per step are required to update the weight vector (26).

5. SIMULATIONS

We assume a uniform linear array (ULA) of $p = 20$ omnidirectional sensors spaced half-wavelength apart. There is one desired source and one interferer. The desired signal is always present in the data samples and the interference-to-noise ratio (INR) is equal to 20 dB. We compare the averaged performances of the *benchmark* SMI algorithm (which corresponds to the ideal case when $\mathbf{R}_{\rm s}$ in (6) is known exactly and is included in our simulations for comparison reasons only), the SMI algorithm (6), the diagonally loaded SMI (LSMI) algorithm (see [2]-[3] and references therein) with the DL factor $\gamma = 30\sigma^2$ (where σ^2 is the noise variance) and the proposed robust algorithm (20) with the DL parameters $\gamma = 30\sigma^2$ and $\varepsilon = 9$. Note that the presumed matrix $\mathbf{R}_{\rm s}$ in (20) is normed so that the signal power in a single sensor is equal to one. Furthermore, note that these values of γ and ε are nearly optimal for each algorithm. Additionally, the optimal SINR curve is displayed.

We assume that the desired signal and interferer are locally incoherently scattered sources but the shape of the signal angular power density is known with a substantial error. Following the results of the experimental work [8], the actual signal angular power density is assumed to be a Laplacian function which is distorted

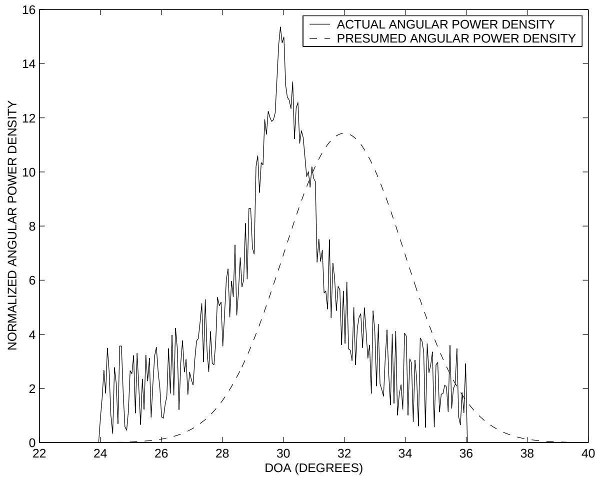

Fig. 1. Actual and presumed signal angular power densities.

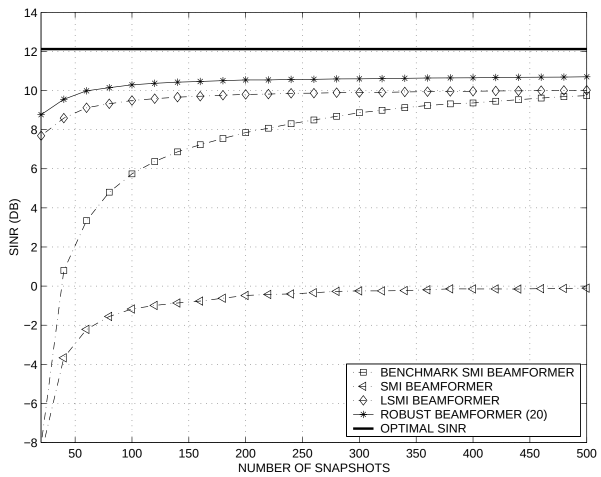

Fig. 2. Output SINR versus N.

by severe fluctuations and has the central angle and the angular spread equal to $30°$ and $4°$, respectively. The presumed signal angular power density is a Gaussian function where the central angle and the angular spread are assumed to be $32°$ and $6°$, respectively. The presumed and actual signal angular power densities are plotted in Fig. 1. The interferer is assumed to have a uniform angular power density characterized by the central angle $-30°$ and angular spread $4°$. Figure 2 displays the performances of the techniques tested versus N for the fixed $\mathrm{SNR} = 0$ dB. Their performances for the fixed data length $N = 100$ are displayed versus the SNR in Fig. 3. From Figs. 2 and 3, it can be seen that our beamformer outperforms the conventional and benchmark SMI techniques as well as the LSMI algorithm with the fixed DL factor.

6. CONCLUSIONS

A new closed-form robust approach to adaptive beamforming using the optimization of the worst-case SINR has been proposed. Our technique has been developed for the general case of an arbitrary rank of the desired signal model and combines two types of DL applied to the signal and the data covariance matrices. Its computationally efficient on-line implementation has been derived.

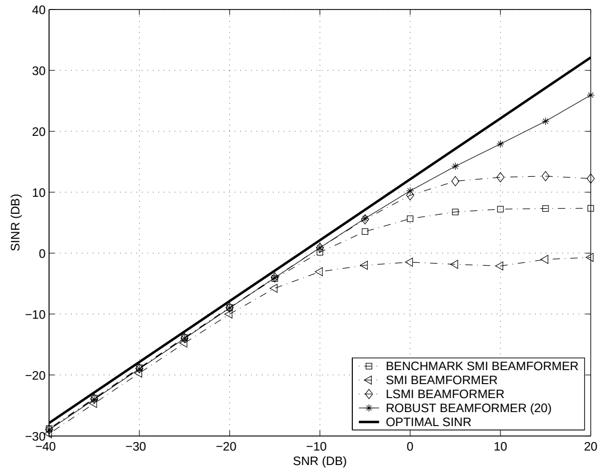

Fig. 3. Output SINR versus SNR.

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