# CONDITIONS ON SOURCES AND MIXING MATRIX FOR SOLVING THE PERMUTATION INDETERMINACY IN $2 \times 2$ INSTANTANEOUS BLIND SIGNAL SEPARATION

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**ABSTRACT**

This paper describes a closed-form solution for $2 \times 2$ Instantaneous Blind Signal Separation (IBSS) that is based on second order statistics and explicitly exploits the non-stationarity and the colored-ness (difference between spectra) of the source signals. A criterion is derived which specifies the requirements on the statistics of the source signals for which the proposed method will work. This criterion unifies the non-stationarity and colored-ness criteria encountered separately in other work. Moreover, it is shown that if this criterion is satisfied a condition on the scalar mixing coefficients can be derived under which the permutation indeterminacy is solved. This is very important, since it makes it possible to predict for which of the two resulting solutions the sources are permuted. The algorithm is based on the division of available data into two different time blocks each time the de-mixing system is estimated. For each block, certain correlation values are computed that are subsequently combined to provide two separating solutions. One of these solutions provides the sources in the original order and the other in the reversed order.

## 1. INTRODUCTION

This paper is concerned with the Blind Signal Separation (BSS) problem in its simplest form, namely instantaneous BSS. Instantaneous Blind Signal Separation (IBSS) deals with the problem of separating independent sources from their observed instantaneous mixtures only, while both the mixing process and original sources are unknown. It is widely recognized that many possible applications exist for BSS, such as separating speech signals from competing talkers in the so-called ”cocktail party” problem, removing additive noise from signals (including images), revealing independent sources in different kinds of biological signals like EEG's and MEG's, etc.

Until recently, most BSS algorithms employing second order statics only were based on the assumption of stationary mutually uncorrelated sources. Most methods try to decorrelate the output signals for different lags, thereby exploiting the spectral difference between the sources (colored-ness). However, nowadays it is recognized by several authors that also the non-stationarity of the source signals is something that can and must be exploited (see for example [1] and [2]).

In this work, a closed-form solution for $2 \times 2$ Instantaneous Blind Signal Separation (IBSS) based on second order statistics is developed, which explicitly exploits both the non-stationarity and the colored-ness of the source signals. A criterion is derived which specifies the requirements on the statistics of the source signals for which the proposed method will work. This criterion unifies the non-stationarity and colored-ness criteria encountered very often separately in other work on BSS. Moreover, it is shown that if the mentioned criterion is satisfied a condition on the scalar mixing coefficients can be derived under which the permutation indeterminacy is solved.

In Section 2 the considered mixing model is explained together with the adopted de-mixing structure. Section 3 presents the estimation of the de-mixing system by means of closed-form formulas. In Section 4 the requirement on the source signals for which the proposed method will work, as well as a condition for solving the permutation indeterminacy in $2 \times 2$ IBSS is derived. The practical implementation and simulation results are discussed in Section 5. Finally, Section 6 presents the conclusions.

## 2. INSTANTANEOUS 2×2 MIXING MODEL

The following instantaneous mixing model is considered:

$$\begin{pmatrix} x_1[n] \\ x_2[n] \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} s_1[n] \\ s_2[n] \end{pmatrix} \tag{1}$$

which can shortly be written as $\underline{x}[n] = \mathbf{A}\underline{s}[n]$.

Because scalings of the columns can be absorbed by the sources, without loss of generality the following simplified mixing model is discussed in the remainder of the paper:

$$\begin{pmatrix} x_1[n] \\ x_2[n] \end{pmatrix} = \begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \begin{pmatrix} s_1[n] \\ s_2[n] \end{pmatrix} \tag{2}$$

where $a$ and $b$ are unknown mixing constants. It is assumed that $a \neq 0$ and $b \neq 0$ because either of these cases corresponds to the situation that one of the sensor signals contains only the contribution from one source, which makes the problem trivial.

If the mixing matrix would be known, the original source signals could be obtained by computing $\underline{s}[n] = \mathbf{A}^{-1}\underline{x}[n]$:

$$\begin{pmatrix} s_1[n] \\ s_2[n] \end{pmatrix} = \frac{1}{1-ab} \begin{pmatrix} 1 & -a \\ -b & 1 \end{pmatrix} \begin{pmatrix} x_1[n] \\ x_2[n] \end{pmatrix} \tag{3}$$

Subsequently, it is be assumed that $1 - ab \neq 0$. If this assumption is not satisfied, $\mathbf{A}$ is not invertible and the source signals cannot be recovered, even in case $\mathbf{A}$ is known. Motivated by this form of the inverse system, the estimates $y_1[n]$ and $y_2[n]$ of the source signals are computed by:

$$\begin{pmatrix} y_1[n] \\ y_2[n] \end{pmatrix} = \frac{1}{1-\alpha\beta} \begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix} \begin{pmatrix} x_1[n] \\ x_2[n] \end{pmatrix} \tag{4}$$

where $\alpha$ and $\beta$ are de-mixing constants that have to be estimated. It can easily be shown (consider equation (19)) that two separating solutions exist, given by:

$$(\alpha_o, \beta_o) = (-a, -b) \tag{5a}$$

$$(\alpha_p, \beta_p) = \left(-\tfrac{1}{b}, -\tfrac{1}{a}\right) \tag{5b}$$

The solution $(\alpha_o, \beta_o)$ corresponds to the situation that the sources are recovered in the right order, i.e. without permutation of the sources at the outputs ($y_1$ is an estimate of $s_1$ and $y_2$ is an estimate of $s_2$). The solution $(\alpha_p, \beta_p)$ corresponds to the situation that the sources are permutated ($y_1$ is an estimate of $s_2$ and $y_2$ is an estimate of $s_1$).

## 3. ESTIMATION OF DE-MIXING SYSTEM

In order to estimate the de-mixing constants $\alpha$ and $\beta$, it is assumed that $s_1$ and $s_2$ are zero-mean statistically uncorrelated non-stationary real-valued processes, such that:

$$r_{s_i s_j}[m,n] = 0 \quad \forall \quad m,n \tag{6}$$

where $r_{s_i s_j}[m,n] \triangleq E\{s_i[m]s_j[n]\}$, $i,j = 1,2$. The unknown de-mixing constants are estimated by applying the same criterion to the estimated source signals $y_1$ and $y_2$:

$$r_{y_1 y_2}[m,n] \triangleq E\{y_1[m]y_2[n]\} = 0 \quad \forall \quad m,n \tag{7}$$

Substituting equation (4) into equation (7) yields:

$$\begin{aligned}(1-\alpha\beta)^2 r_{y_1 y_2}[m,n] &= \alpha\beta\, r_{x_2 x_1}[m,n] + \alpha\, r_{x_2 x_2}[m,n] \\ &+ \beta\, r_{x_1 x_1}[m,n] + r_{x_1 x_2}[m,n] = 0 \quad \forall \quad m,n \end{aligned} \tag{8}$$

where $r_{x_i x_j}[m,n] = E\{x_i[m]x_j[n]\}$ for $i,j = 1,2$. Each equation represents a hyperbolic set of solutions.

In the next sections, it will be shown that solving a system consisting of two of these nonlinear equations for two different sets of time indices yields the two separating solutions given in equations (5a) and (5b). Experiments have revealed that for many real-world signals, such as speech, only two equations provide already enough information to solve the problem with good quality (see Section 5), i.e. the equations are independent enough.

Combining two equations, one with time indices $[m,n]$ and the other with $[k,l]$, and eliminating $\beta$ results in a second order polynomial in the variable $\alpha$ (it is equally well possible to obtain a polynomial in $\beta$ by eliminating $\alpha$):

$$p_2\alpha^2 + p_1\alpha + p_0 = 0 \tag{9}$$

where the coefficients $p_i$ with $i = 0,1,2$ are given by:

$$p_0 = r_{x_1 x_1}[m,n]r_{x_1 x_2}[k,l] - r_{x_1 x_1}[k,l]r_{x_1 x_2}[m,n] \tag{10a}$$

$$\begin{aligned} p_1 &= (r_{x_2 x_1}[m,n]r_{x_1 x_2}[k,l] - r_{x_2 x_1}[k,l]r_{x_1 x_2}[m,n]) \\ &+ (r_{x_1 x_1}[m,n]r_{x_2 x_2}[k,l] - r_{x_1 x_1}[k,l]r_{x_2 x_2}[m,n]) \end{aligned} \tag{10b}$$

$$p_2 = r_{x_2 x_1}[m,n]r_{x_2 x_2}[k,l] - r_{x_2 x_1}[k,l]r_{x_2 x_2}[m,n] \tag{10c}$$

The discriminant of equation (9) is defined as: $D \triangleq p_1^2 - 4p_0p_2$. If $p_2 \neq 0$ due to the non-stationary character of the sources and if $D > 0$, two real solutions exist for $\alpha$. These solutions can be computed by completing the square for equation (9), resulting in:

$$\alpha_{1,2} = -\frac{p_1}{2p_2} \pm \frac{1}{2}\sqrt{\frac{D}{p_2^2}} \tag{11}$$

Once the solutions for $\alpha$ have been computed, the corresponding solutions for $\beta$ can be computed by summing the two equations used to derive equation (9) and solving for $\beta$:

$$\beta(\alpha) = -\frac{(r_{x_2 x_2}[m,n] + r_{x_2 x_2}[k,l])\alpha + (r_{x_1 x_2}[m,n] + r_{x_1 x_2}[k,l])}{(r_{x_2 x_1}[m,n] + r_{x_2 x_1}[k,l])\alpha + (r_{x_1 x_1}[m,n] + r_{x_1 x_1}[k,l])}$$

It can easily be verified that $\beta(-a) = -b$ and $\beta(-\frac{1}{b}) = -\frac{1}{a}$. Hence, another way to compute $\beta_1 \triangleq \beta(\alpha_1)$ and $\beta_2 \triangleq \beta(\alpha_2)$ is:

$$\beta_1 = \beta(\alpha_1) = \tfrac{1}{\alpha_2} \quad \text{and} \quad \beta_2 = \beta(\alpha_2) = \tfrac{1}{\alpha_1} \tag{12}$$

## 4. NON-STATIONARITY REQUIREMENT AND PERMUTATION

This section has multiple objectives. First, a criterion is derived which specifies the requirements on the statistics of the source signals for which the proposed method will work. This criterion unifies the non-stationarity and colored-ness criteria encountered separately in other work on blind signal separation. Next, it will be proven that the solutions given in equation (11) are the separating solutions. Finally, the required amount of a priori knowledge about the two mixing coefficients, that is necessary for solving the permutation indeterminacy, is derived.

### 4.1. Non-stationarity requirement

To start the derivations, the relevant quantities for examining the solutions in equation (11), namely $p_0, p_1, p_2$ and $D$, are expressed in the source correlation functions:

$$p_0 = a(1-ab)\,\psi_s[k,l,m,n] \tag{13a}$$

$$p_1 = (1-ab)(1+ab)\,\psi_s[k,l,m,n] \tag{13b}$$

$$p_2 = b(1-ab)\,\psi_s[k,l,m,n] \tag{13c}$$

$$D = (1-ab)^4\psi_s^2[k,l,m,n] \tag{13d}$$

where

$$\psi_s[k,l,m,n] \triangleq r_{s_1 s_1}[m,n]r_{s_2 s_2}[k,l] - r_{s_1 s_1}[k,l]r_{s_2 s_2}[m,n]$$

From these equations it is clear that if the assumptions in Section 1 are satisfied (i.e. $a \neq 0, b \neq 0, (1-ab) \neq 0$), the condition for having a non-degenerate polynomial in equation (9) is that $\psi_s[k,l,m,n] \neq 0$, or:

$$\frac{r_{s_1 s_1}[m,n]}{r_{s_2 s_2}[m,n]} \neq \frac{r_{s_1 s_1}[k,l]}{r_{s_2 s_2}[k,l]}, \quad [m,n] \neq [k,l] \tag{14}$$

This equation means that the source signals must be either non-stationary, or stationary but differently colored (or both) in order to be able to achieve source separation. In other words, the source correlation functions must not be scaled versions of each other.

This requirement on the source correlation functions is a generalization of the one mentioned in [2] and [3]. There, only time-varying (non-stationary) autocorrelations are considered resulting in the following non-stationarity criterion:

$$\frac{E\{s_1^2[m]\}}{E\{s_2^2[m]\}} \neq \frac{E\{s_1^2[n]\}}{E\{s_2^2[n]\}}, \quad m \neq n \tag{15}$$

This equation says that if the ratio of source powers varies in time, it is possible to achieve separation using decorrelation.

When the source signals are stationary, the criterion reduces to:

$$\frac{r_{s_1 s_1}[\tau_1]}{r_{s_2 s_2}[\tau_1]} \neq \frac{r_{s_1 s_1}[\tau_2]}{r_{s_2 s_2}[\tau_2]}, \quad \tau_1 \neq \tau_2 \tag{16}$$

where $r_{s_i s_i}[\tau] = E\{s_i[k]s_i[k-\tau]\}$. Hence, in this case it is still possible to achieve separation if the source correlation functions are not scaled versions of each other (i.e. the spectra are different).

From the above, it can be concluded that the general criterion in equation (14) *unifies the non-stationarity and colored-ness criteria* encountered separately in other work on blind source separation (only a few papers exploit both criteria simultaneously, see for example [4]).

### 4.2. Separating solutions and permutation indeterminacy

In order to prove that the two possible solutions for the de-mixing constant $\alpha$ given in equation (11) are the separating solutions, they are expressed in the source correlation functions using equations (13a) through (13d). The first solution becomes:

$$\alpha_1 = -\frac{p_1}{2p_2} + \frac{1}{2}\sqrt{\frac{D}{p_2^2}} = \frac{1}{2}\left\{\frac{|1-ab|}{|b|} - \frac{(1+ab)}{b}\right\} \qquad (17)$$

For this expression two different situations can be discerned. Firstly, if $(1-ab)$ and $b$ have the same sign, expression (17) yields $\alpha_1 = -a$, which means that the estimated sources computed with solution $(\alpha_1, \beta_1)$ are in the right order: $(\alpha_1, \beta_1) = (\alpha_o, \beta_o)$. Secondly, if $(1-ab)$ and $b$ have different signs, equation (17) results in $\alpha_1 = -\frac{1}{b}$, which means that the estimated sources corresponding to $(\alpha_1, \beta_1)$ are permuted: $(\alpha_1, \beta_1) = (\alpha_p, \beta_p)$.

Likewise, the second solution given in equation (11) becomes:

$$\alpha_2 = -\frac{p_1}{2p_2} - \frac{1}{2}\sqrt{\frac{D}{p_2^2}} = -\frac{1}{2}\left\{\frac{|1-ab|}{|b|} + \frac{(1+ab)}{b}\right\} \qquad (18)$$

Again, two different cases can be distinguished. Firstly, if $(1-ab)$ and $b$ have the same sign, expression (18) yields $\alpha_2 = -\frac{1}{b}$, which means that the estimated sources computed with solution $(\alpha_2, \beta_2)$ are permuted: $(\alpha_2, \beta_2) = (\alpha_p, \beta_p)$. Secondly, if $(1-ab)$ and $b$ have different signs the expression results in $\alpha_2 = -a$, which means that the estimated sources corresponding to $(\alpha_2, \beta_2)$ are not permuted: $(\alpha_2, \beta_2) = (\alpha_o, \beta_o)$.

In summary, the results derived above can be formulated as:

1. If $(1-ab)$ and $b$ have the same sign: $(\alpha_1, \beta_1) = (\alpha_o, \beta_o)$ and $(\alpha_2, \beta_2) = (\alpha_p, \beta_p)$
2. If $(1-ab)$ and $b$ have different signs: $(\alpha_1, \beta_1) = (\alpha_p, \beta_p)$ and $(\alpha_2, \beta_2) = (\alpha_o, \beta_o)$

For situation 1 two possibilities exist: $(1-ab>0 \wedge b>0) \equiv (a<\frac{1}{b} \wedge b>0)$ and $(1-ab<0 \wedge b<0) \equiv (a<\frac{1}{b} \wedge b<0)$. Hence, the statement that $(1-ab)$ and $b$ have the same sign can also be formulated as: $(a<\frac{1}{b} \wedge b \neq 0)$. In a similar way, for situation 2 the statement that $(1-ab)$ and $b$ have different signs can be formulated as: $(a>\frac{1}{b} \wedge b \neq 0)$.

Hence, we can conclude the following (omitting the condition $b \neq 0$ because this is an initial assumption):

- If $a<\frac{1}{b}$: $(\alpha_1, \beta_1) = (\alpha_o, \beta_o)$ and $(\alpha_2, \beta_2) = (\alpha_p, \beta_p)$
- If $a>\frac{1}{b}$ : $(\alpha_1, \beta_1) = (\alpha_p, \beta_p)$ and $(\alpha_2, \beta_2) = (\alpha_o, \beta_o)$

In words this means the following (assuming that $a \neq \frac{1}{b}$ which is equivalent to $(1-ab) \neq 0$):

***If*** $a<\frac{1}{b}$***, then the output signals computed with solution*** $(\alpha_1, \beta_1)$ ***are in order and the output signals computed with solution*** $(\alpha_2, \beta_2)$ ***are permuted and vice versa.***

This statement is very important since it solves the permutation indeterminacy under the given required knowledge about the relationship between the mixing coefficients $a$ and $b$.

The shaded area in Figure 1 indicates the points $(a, b)$ where $a<\frac{1}{b}$ ($a$ vertical and $b$ horizontal). For example, if it is known that $0<a<1$ and $0<b<1$ (see box in figure), this implies that $a<\frac{1}{b}$ and thus the output signals computed with solution $(\alpha_1, \beta_1)$ are always in the right order. Intuitively, this can be understood in the following way: $0<a<1$ means that at sensor 1 source 2 is weaker than source 1, and vice versa for $0<b<1$, hence it must be possible to exploit this knowledge to reconstruct the sources in the desired order.

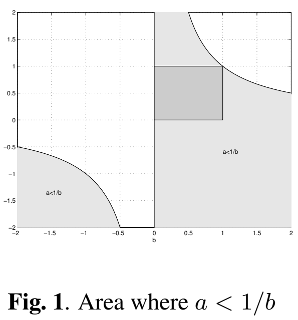


**Fig. 1**. Area where $a<1/b$

With the described method, using different sets of (two blocks of) data from the same set of mixtures always results in the same permutation. For example, if two data blocks of the available data set are used to compute the solutions and solution $(\alpha_1, \beta_1)$ yields output signals that are in the right order, the same holds for the solutions given by two other data blocks of the same data set.

## 5. IMPLEMENTATION AND SIMULATIONS

The implementation of the algorithm in batch mode is very straightforward. First, two different pairs of time indices $[m, n]$ and $[k, l]$ are chosen and then the sensor correlations required for computing the $p_i$'s are estimated by means of time averaging over block length $P$:

$$r_{x_i x_j [s,t]} \approx \sum_{p=0}^{P-1} x_i[p+s]x_j[p+t], \begin{cases} i, j = 1, 2 \\ [s,t] \in \{[m,n],[k,l]\} \end{cases}$$

The block size $P$ must be chosen large enough such that temporal correlations between $s_1[m]$ and $s_2[m]$ are approximately zero. The pairs $[m, n]$ and $[k, l]$ must be chosen in such a way that equation (14) is satisfied. For example, if it is desired to exploit both the non-stationarity and the colored-ness of the source signals, the pairs $[m, n]$ and $[k, l]$ must be far enough apart (to exploit the non-stationarity), and $m \neq n$ or $k \neq l$ in order to exploit the coloredness. Once the sensor correlations have been computed, $p_0, p_1$ and $p_2$ are computed by means of equations (10a), (10b) and (10c). Subsequently, the two possible solutions for the de-mixing parameter $\alpha$ are computed with equation (11). Finally, equation (12) is used to compute the corresponding values of $\beta$.

For the simulations, two speech signals sampled at 8 kHz (see Figures 2a and 2b) are mixed according to equation (2). The used parameter values are: $P = 2000, m = 1, n = m, k = m + P$ and $l = k + 5$. Since the source signals are artificially mixed, the known values of $a$ and $b$ can be used for the evaluation. For each of the two possible solutions, the total transfer $T$ from sources to outputs is computed according to equations (2) and (4), where the scaling factor $\frac{1}{1-\alpha\beta}$ is excluded for convenience:

$$T \triangleq \begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} = \begin{pmatrix} 1+\alpha b & a+\alpha \\ \beta+b & 1+a\beta \end{pmatrix} \qquad (19)$$

The separation quality for each solution is defined as:

$$Q(T) = \frac{1}{2}\sum_{i=1}^{2} \frac{\max_j |t_{ij}|}{\min_j |t_{ij}|} \qquad (20)$$

where $t_{ij}$ with $i, j = 1, 2$ denotes the $ij$th element of matrix $T$.

Several values for $a$ and $b$ were tested and in all cases the algorithm performed very well. The simulation results for $(a, b) = (-0.52, 0.75)$ will now be presented in more detail. Since $a<\frac{1}{b}$, according to the statement made in the previous section, it is expected that the output signals computed with solution $(\alpha_1, \beta_1)$ are in the right order and the output signals computed with solution $(\alpha_2, \beta_2)$ are permuted, i.e. $(\alpha_1, \beta_1) = (\alpha_o, \beta_o)$ and $(\alpha_2, \beta_2) = (\alpha_p, \beta_p)$.

The algorithm gives the following results. The first solution $(\alpha_1, \beta_1) = (0.508, -0.749)$, which indeed corresponds to the solution in equation (5a): $(\alpha_1, \beta_1) \approx (\alpha_o, \beta_o)$. The second solution $(\alpha_2, \beta_2) = (-1.319, 1.943)$ corresponds to equation (5b): $(\alpha_2, \beta_2) \approx (\alpha_p, \beta_p)$. Substituting these solutions into equation (4), the corresponding set of output signals is computed for 10000 samples (only $2P$ samples are used in the estimation process). Figure 2 plots all relevant signals. Figures (a) and (b) show the source signals $s_1[n]$ and $s_2[n]$, whereas (c) and (d) show the mixtures $x_1[n]$ and $x_2[n]$. The vertical dotted lines in Figures (c) and (d)

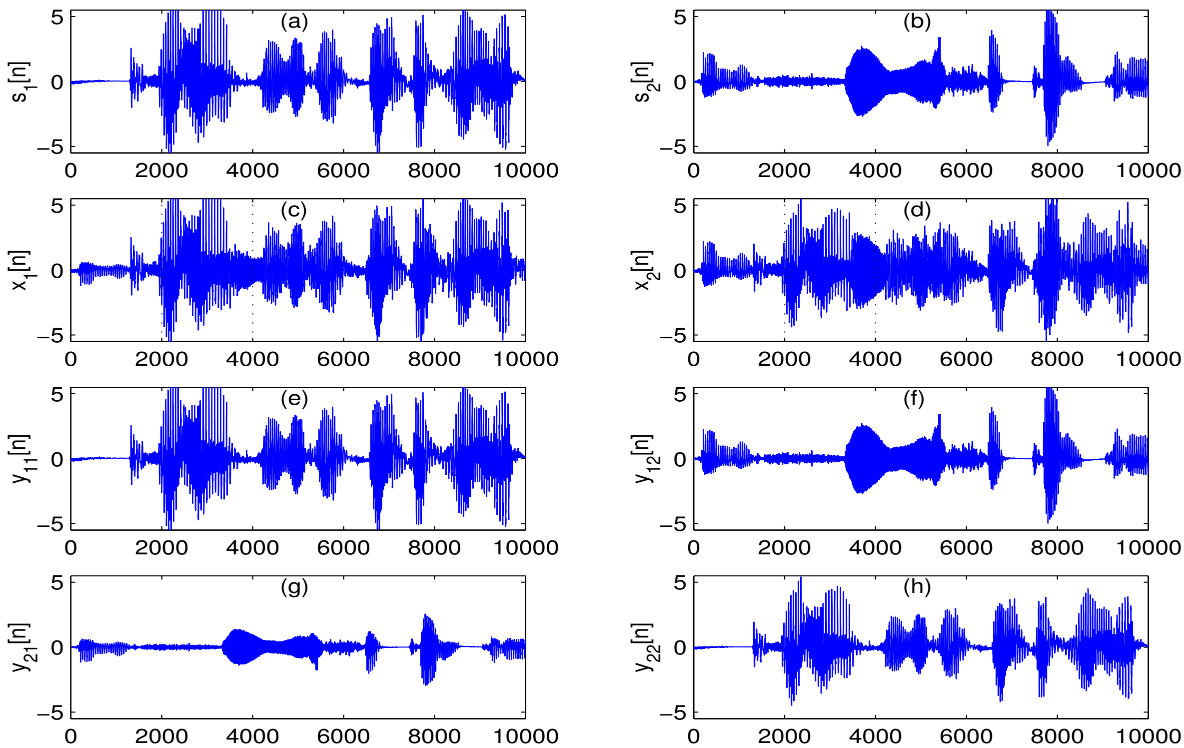


**Fig. 2**. $s_1[n], s_2[n]$: source signals, $x_1[n], x_2[n]$: mixtures, $y_{11}[n], y_{12}[n]$: estimated sources obtained with $(\alpha_1, \beta_1)$, $y_{21}[n], y_{22}[n]$: estimated sources obtained with $(\alpha_2, \beta_2)$

indicate the data blocks used for processing. Figures (e) and (f) show the output signals $y_{11}[n]$ and $y_{12}[n]$ corresponding to solution $(\alpha_1, \beta_1)$, while (g) and (h) show the output signals $y_{21}[n]$ and $y_{22}[n]$ corresponding to solution $(\alpha_2, \beta_2)$. The separation quality for solution $(\alpha_1, \beta_1)$ is $Q(T_1) = 528$ and for $(\alpha_2, \beta_2)$ is $Q(T_2) = 218$. It must be remarked that the quality measuring values are very specific to the chosen block positions. However, experiments reveal that in almost all cases these values are larger than 50. The figures also reveal that the scaling of the outputs corresponding to the solution that is not permuted ($y_{11}$ and $y_{12}$) is correct, as expected from equations (3) and (4).

## 6. CONCLUSIONS

A closed-form solution has been presented for the $2 \times 2$ Instantaneous Blind Signal Separation (IBSS) problem. The algorithm is based on second order statistics and explicitly exploits the non-stationarity and the colored-ness (difference between spectra) of the source signals. A criterion has been derived which specifies the requirements on the statistics of the source signals for which the proposed method will work. This criterion unifies the non-stationarity and colored-ness criteria encountered separately in other work on BSS. Moreover, it has been shown that if the criterion is satisfied a condition on the scalar mixing coefficients can be derived under which the permutation indeterminacy is solved using the proposed method. This is very important, since it makes it possible to predict for which of the two resulting solutions the sources are permuted. The algorithm is based on the division of (a part of the) available data into two different time blocks for each time the de-mixing system is estimated. For each block, certain correlation values are computed that are subsequently combined to provide two separating solutions. One of these solutions provides the sources in the original order and the other in the reversed order, i.e. the sources are permuted at the outputs. The algorithm has been evaluated for different values of the mixing constants and different speech input signals, from which it can be concluded that the separation quality is very good in all cases and that the permutation of the sources is predicted correctly when it is known that the derived condition is satisfied. From the experiments, it can also be concluded that for many real-world signals, such as speech, only two equations provide already enough information to solve the problem with good quality, i.e. the equations are independent enough. Since the solution to the IBSS problem becomes very simple in this case, it follows that it may not always be necessary to resort to more advanced methods solving many nonlinear equations simultaneously.

## 7. REFERENCES


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