BLIND SOURCES SEPARATION BASED ON QUADRATIC TIME-FREQUENCY REPRESENTATIONS: A METHOD WITHOUT PRE-WHITENING

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ABSTRACT

In this communication, we are interested in blind sources separation methods based on joint-diagonalization of combined sets of "spatial *t-f* distributions" matrices (STFD). Our aim is to perform sources separation with no pre-whitening, thanks to the non-orthogonal joint-diagonalization procedure recently proposed in [7]. We also show how such an approach makes it possible to give up the classical hypothesis of independence of sources. The problem is, in fact, reformulated in order to treat the case of even correlated sources. Computer simulations are provided in order to illustrate the effectiveness of the proposed approach and to evaluate its advantages with regard to other techniques.

1. INTRODUCTION

During the past decade, many "blindly" operating approaches have treated of a model called *sources separation.* In such a problem, the coupling channels are assumed to have unknown constant gains, the aim being to recover the inputs from the only outputs, without the explicit use of the unobservable sources (supposed independent) and without any model of the mixture matrix. Many solutions have been proposed to solve this problem (contrasts functions [5], maximum likelihood functions...), among which the one we are interested in and that was recently introduced in [1] [2]. This method plays upon the joint-diagonalization (JD) of a combined set of "spatial quadratic *t-f* distributions" (SQTFD) matrices. In [3], we have introduced new criteria of automatic selection of the *t-f* points to use in the building of matrices sets to JD and/or to joint anti-diagonalize (JAD). In this communication, our aim is to show that blind sources separation based on SQTFD can be performed without the classical preliminary operation of whitening of the observations. To that aim, we generalize one of the selection criteria we have proposed in [3] to make it suitable for the case of non pre-whitened data (the trace of SQTFD cannot be used any more, that is why a 2D-filtering operation is now introduced). The classical joint-diagonalization procedure is then replaced by the non-orthogonal one proposed by A. Yeredor in [7]. The main advantage of this approach is to allow to separate even correlated sources. Finally, computer simulations are provided in order to evaluate the effectiveness of the proposed method.

2. PROBLEM STATEMENT

2.1. Model & Assumptions

We consider the blind sources separation problem, assuming that N sources signals are received on an antenna of the same number N of sensors. In matrix and vector notations, the input/output relationship of the mixing system is then given, in the noiseless case, by: $\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t)$, with $\boldsymbol{A}$ the $N \times N$ instantaneous mixing matrix (assumed invertible), $\boldsymbol{x}(t) = [x_1(t), ..., x_N(t)]^T$ the $N \times 1$ observations vector (superscript ^{T} denoting transposition), $\boldsymbol{s}(t) = [s_1(t), ..., s_N(t)]^T$ the $N \times 1$ sources vector. In our case, the sources $s_i(t)$, $i = 1, \ldots, N$ are supposed to be deterministic signals. Their correlation matrix $\boldsymbol{C_s}(\tau) = (C_{s_i s_j}(\tau))$ is defined component-wise as: $C_{s_i s_j}(\tau) = \langle s_i(t) s_j^*(t-\tau)\rangle \; \forall i, j$, where $\langle . \rangle$ stands for a temporal mean over t. Its expression depends on the class of considered deterministic signals. In the case of finite (mean) power signals the temporal mean is defined according to $\langle z(t)\rangle = \lim_{T\to+\infty} \frac{1}{T}\int_{-T/2}^{T/2} z(t)dt$. To deal with the important practical case of finite energy time limited signals, we consider that they correspond to one period of a periodic (finite power) signal. In this latter case the mean can be written as $\langle z(t)\rangle = \frac{1}{T}\int_{-T/2}^{T/2} z(t)dt$, where T is now the period of $z(t)$ corresponding to the time duration of the considered time limited signal. In the following, two assumptions are made: the structure of the sources is such that the localization of their signatures in the *t-f* plane is different and both the mixing matrix and the sources are *real*. The problem of blind sources separation is then to identify the mixing matrix in order to restore the unknown sources.

2.2. Problem indeterminacies

It is well known that the sources separation problem can be solved only up to a diagonal matrix $\boldsymbol{D}$ (corresponding to arbitrary attenuations for the restored sources), and a permutation matrix $\boldsymbol{P}$ (corresponding to an arbitrary order of restitution), that is why the unit power assumption on sources can be done without loss of generality (consequently, for decorrelated sources it is assumed that $\boldsymbol{C_s}(0) = \boldsymbol{I}_N$).

2.3. Spatial Time-Frequency Representations

A Bilinear Transform (BTr), applied to a couple (x_i, x_j) and denoted $D_{x_i x_j}$, is defined as follows [6]:

$$(x_i, x_j) \overset{\text{BTr}}{\longleftrightarrow} D_{x_i x_j}$$

$$D_{x_i x_j}(t,\nu;R) = \int_{\mathsf{R}^2} x_i(\theta) x_j^*(\theta') R(\theta,\theta';t,\nu) d\theta d\theta'$$

A Quadratic Transform (QTr) associated to a signal $x_i(t)$, is the restriction to $x_j(t) = x_i(t)$ of a BTr applied to a couple (x_i, x_j) [6]. In the case of a vectorial signal $\boldsymbol{x}(t) = [x_1(t), \ldots, x_M(t)]^T$, the following quadratic transform is spatial:
$\boldsymbol{D_{xx}}(t,\nu) = \int_{\mathsf{R}^2} \boldsymbol{x}(\theta)\boldsymbol{x}^H(\theta') R(\theta,\theta';t,\nu) d\theta d\theta'$ with R, the kernel of the transform. Terms on the diagonal of $\boldsymbol{D_{xx}}(t,\nu)$ are auto-terms (or quadratic terms $D_{x_p x_p}$) whereas other terms are inter-terms (or bilinear terms $D_{x_p x_q}$). In the case of sources separation, it is interesting to take advantage of the *Hermitian symmetry property* of bilinear transforms. A BTr exhibits hermitian symmetry if it satisfies $D_{x_i x_j}(t,\nu) = D^*_{x_j x_i}(t,\nu)$. It also involves the *reality* of the associated QTr.

3. SOURCES SEPARATION WITH SQTFD

3.1. Correlated sources: effect of pre-whitening

In the context of correlated sources, their correlation matrix is no more diagonal for $\tau = 0$ but writes $\boldsymbol{C_s}(0) = \boldsymbol{I}_N + \boldsymbol{\varepsilon}_N$, where $\boldsymbol{I}_N$ is the $N \times N$ identity matrix and $\boldsymbol{\varepsilon}_N$ the symmetric matrix of inter-correlations between sources, having zeros on its diagonal. By Singular Value Decomposition (SVD), the mixing matrix $\boldsymbol{A}$ can be decomposed [4] as $\boldsymbol{A} = \boldsymbol{V}\boldsymbol{\Delta}^{\frac{1}{2}}\boldsymbol{U}$ with $\boldsymbol{V}$ and $\boldsymbol{U}$, $N \times N$ orthogonal matrices and $\boldsymbol{\Delta}$ a $N \times N$ diagonal matrix. $\boldsymbol{C_x}(0) = \boldsymbol{A}\boldsymbol{C_s}(0)\boldsymbol{A}^T = \boldsymbol{A}(\boldsymbol{I}_N + \boldsymbol{\varepsilon}_N)\boldsymbol{A}^T = \boldsymbol{V}(\boldsymbol{\Delta} + \boldsymbol{\Delta}^{\frac{1}{2}}\boldsymbol{U}\boldsymbol{\varepsilon}_N\boldsymbol{U}^T\boldsymbol{\Delta}^{\frac{1}{2}})\boldsymbol{V}^T = \boldsymbol{V}\boldsymbol{M}\boldsymbol{V}^T$ where $\boldsymbol{M} = \boldsymbol{\Delta} + \boldsymbol{\Delta}^{\frac{1}{2}}\boldsymbol{U}\boldsymbol{\varepsilon}_N\boldsymbol{U}^T\boldsymbol{\Delta}^{\frac{1}{2}}$ is also a symmetric matrix (in the case of decorrelated sources $\boldsymbol{M} = \boldsymbol{\Delta}$). Its eigenvalue decomposition writes: $\boldsymbol{M} = \boldsymbol{C}\boldsymbol{D}\boldsymbol{C}^T$ with $\boldsymbol{C}$ an orthogonal matrix and $\boldsymbol{D}$ a diagonal matrix. Finally, $\boldsymbol{C_x}(0) = \boldsymbol{V}\boldsymbol{C}\boldsymbol{D}\boldsymbol{C}^T\boldsymbol{V}^T$ which corresponds to its eigenvalue decomposition since $\boldsymbol{V}\boldsymbol{C}$ is an orthogonal matrix. The whitening matrix is then defined as $\boldsymbol{W} = \boldsymbol{D}^{-1/2}\boldsymbol{C}^T\boldsymbol{V}^T$, leading to whitened observations: $\boldsymbol{z}(t) = \boldsymbol{W}\boldsymbol{A}\boldsymbol{s}(t) = \boldsymbol{D}^{-1/2}\boldsymbol{C}^T\boldsymbol{\Delta}^{1/2}\boldsymbol{U}\boldsymbol{s}(t) = \boldsymbol{E}\boldsymbol{s}(t)$. One can check that $\boldsymbol{C_z}(0) = \boldsymbol{I}_N$, however, $\boldsymbol{E}$ is no more an orthogonal matrix since:
$\boldsymbol{E}^T\boldsymbol{E} = \boldsymbol{U}^T\boldsymbol{\Delta}^{1/2}\underbrace{\boldsymbol{C}\boldsymbol{D}^{-1}\boldsymbol{C}^T}_{\boldsymbol{M}^{-1}}\boldsymbol{\Delta}^{1/2}\boldsymbol{U} = \boldsymbol{C_s}(0)^{-1} \neq \boldsymbol{I}_N$
and the eigen-decomposition of $\boldsymbol{C_x}(0)$ does not allow any more to estimate two of the three matrices involved in the decomposition of the mixing matrix. In the case of decorrelated sources, on the contrary, the whitening matrix is simply defined as $\boldsymbol{W} = \boldsymbol{\Delta}^{-1/2}\boldsymbol{V}^T$, leading to $\boldsymbol{z}(t) = \boldsymbol{U}\boldsymbol{s}(t)$.

3.2. Sources separation without pre-whitening

Under our assumptions, the SQTFD of the observed signals writes $\boldsymbol{D_{xx}}(t,\nu) = \boldsymbol{A}\boldsymbol{D_{ss}}(t,\nu)\boldsymbol{A}^T$ with $D_{x_k x_p}(t,\nu) = \sum_{i,j=1}^{n} a_{ki}a_{pj} D_{s_i s_j}(t,\nu)$ and $\boldsymbol{A} = (a_{ij})$.

3.2.1. Building of the set of matrices to be JD

The real part $\Re(.)$ and the imaginary part $\Im(.)$ of auto-terms $D_{x_p x_p}$ and cross-terms $D_{x_k x_p}$ are equal to:

$$\begin{aligned}
\Re(D_{x_p x_p}(t,\nu)) &= \sum_{g=1}^{N} a_{pg}^2 D_{s_g s_g}(t,\nu) \\
&+ 2\sum_{h=1}^{N}\sum_{g=1(g<h)} a_{pg}a_{ph}\Re(D_{s_g s_h}(t,\nu)) \\
\Im(D_{x_p x_p}(t,\nu)) &= 0 \\
\Re(D_{x_k x_p}(t,\nu)) &= \sum_{g=1}^{N} a_{pg}a_{kg} D_{s_g s_g}(t,\nu) \\
&+ \sum_{h=1}^{N}\sum_{g=1(g\neq h)}^{N} a_{pg}a_{qh}\Re(D_{s_g s_h}(t,\nu)) \\
\Im(D_{x_k x_p}(t,\nu)) &= \sum_{h=1}^{N}\sum_{g=1(g\neq h)}^{N} a_{pg}a_{qh}\Im(D_{s_g s_h}(t,\nu))
\end{aligned}$$

The purpose is to determine which *t-f* points correspond to sources auto-terms only ($D_{s_i s_i}$ terms only and no $D_{s_i s_j}$), because in such *t-f* points the SQTFD should be diagonal. The same kind of ideas as those developed in [3] can then be used, except that it is worked

on SQTFD matrices related to the observations instead of those of the pre-whitened observations. Moreover, the JD procedure used, in [3], on the resulting set to identify the orthogonal matrix $\boldsymbol{U}$, will be replaced by the non-orthogonal JD procedure proposed by [7].
Two thresholds S_1 and S_2 are defined. They happen to be small positive constants. Using the two previous equations, one can notice that when at least one signal is present, without any sources cross-terms (no $D_{s_is_j}$) then the absolute value of the imaginary part of the SQTFD is inferior to S_2 whereas the absolute value of its real part is superior to S_1. Note that such a decision criterium is not restrictive enough, in fact a problem may occur when interferences between sources do exist (their real parts are not null) but with null imaginary parts (in such *t-f* points, if there are sources cross-terms only, the corresponding SQTFD should be joint anti-diagonalized, and if there are both sources auto and cross-terms, nothing should be done). To eliminate this ambiguity, an additional condition is introduced thanks to a Sobel 2D-filter applied on each imaginary parts of the cross-terms of the SQTFDs, whose effect is to differentiate the *t-f* images. A third threshold S_3 is introduced: if the derivative in this *t-f* point is smaller than S_3, there is no sources interferences but only sources auto-terms and the corresponding SQTFD has to be JD, in the other case nothing is done. The resulting decision algorithm is displayed on Fig.1.

3.2.2. Procedure of non-orthogonal JD

In [7], A. Yeredor demonstrates how to JD a set of non-orthogonal matrices according to a least square criterion. Two distinct algorithms have been developed to consider either the case of *hermitian* or *symmetrical* matrices. They follow the same stages. First, a set $\mathcal{A}$ of K non-orthogonal matrices $\boldsymbol{A}_1, \boldsymbol{A}_2, \ldots, \boldsymbol{A}_K \in \mathbb{C}^{N\times N}$ is considered. The approximate JD problem consists in searching a diagonalizing matrix $\boldsymbol{B} \in \mathbb{C}^{N\times N}$ and K associated diagonal matrices $\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_K \in \mathbb{C}^{N\times N}$ such that $C_{LS}(\boldsymbol{B}, \boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_K) = \sum_{k=1}^{K} w_k \|\boldsymbol{A}_k - \boldsymbol{B}\boldsymbol{\Lambda}_k\boldsymbol{B}^H\|_F^2$ is minimized. $w_1, w_2, \ldots, w_K \in \mathbb{R}_+^*$ are positive weights, $\|\cdot\|_F^2$ is the square Frobenius norm and ^{H} is either the transpose or the conjugate transpose. For the minimization of this weighted least-square criterion with respect to (w.r.t.) a *general* diagonalizing matrix, an iterative alternating-direction algorithm is proposed:
• The "Alternating Columns" phase minimizes $C_{LS}(\boldsymbol{B}, \boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_K)$ w.r.t. a selected column of $\boldsymbol{B}$ while keeping its other columns, as well as $\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_K$, fixed.
• The "Diagonal Centers" phase minimizes $C_{LS}(\boldsymbol{B}, \boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_K)$ w.r.t. all the diagonal matrices $\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \ldots, \boldsymbol{\Lambda}_K$, while keeping $\boldsymbol{B}$ fixed.
Furthermore, this algorithm may be adapted to non square diagonalizing matrix $\boldsymbol{B}$ (this problem occurs in blind sources separation when there are more sensors than sources.)

4. COMPUTER SIMULATIONS

In this section, we compare the results obtained using a pre-whitening of the observations followed by a classical orthogonal JD procedure with those obtained without pre-whitening but with a non-orthogonal JD procedure. The experiment is realized on $N = 3$ correlated sources signals received on $N = 3$ sensors, there is no noise. The SQTFD are computed over 64 frequency bins and 128 time samples using Pseudo Wigner-Ville (PWV) distribution. The trace of the spatial PWV distribution of sources is given Fig.2. The estimated correlation matrix of the sources and the mixing matrix are respectively:

$$\boldsymbol{C_s}(0) = \begin{pmatrix} 1 & 0.12 & 0.18 \\ 0.12 & 1 & 0.17 \\ 0.18 & 0.17 & 1 \end{pmatrix} \quad \boldsymbol{A} = \begin{pmatrix} 1 & 0.6 & 0.2 \\ 0.4 & 1 & 0.6 \\ 0.7 & 0.3 & 1 \end{pmatrix}$$

To establish a comparison between the algorithms, the following performance index [5] is used:

$$\begin{aligned} I_2(\hat{\boldsymbol{A}}^{-1}\boldsymbol{A}) &= \frac{1}{2}\Big[\sum_i\Big(\sum_j \frac{|(\hat{A}^{-1}A)_{ij}|^2}{\max_l |(\hat{A}^{-1}A)_{il}|^2} - 1\Big) \\ &+ \sum_j\Big(\sum_i \frac{|(\hat{A}^{-1}A)_{ij}|^2}{\max_l |(\hat{A}^{-1}A)_{lj}|^2} - 1\Big)\Big] \end{aligned}$$

where $\hat{A}$ stands for the estimated mixing matrix. Applying this performance index, we find $I_{2_o} = 0.0377$ for JD after pre-whitening and $I_{2_{no}} = 5.7e^{-6}$ for non-orthogonal JD without pre-whitening. In Fig.3, are pictured the *t-f* points used in the building of the matrices set to be joint-diagonalized thanks to the two different techniques we have used. Fig.4 represents the estimated sources and Fig.5 displays a comparison between the theoretical results and those obtained with both orthogonal and non-orthogonal JD.

5. DISCUSSION & CONCLUSION

In this communication, we have shown that blind sources separation based on spatial quadratic time-frequency distribution exhibiting hermitian symmetry can be performed without a preliminary whitening of the observations. This is made possible thanks to a generalization of *t-f* points selection criteria and to the non-orthogonal JD procedure developed by A. Yeredor. The main advantage of this approach is that the classical hypothesis of independence of the sources can be given up: even correlated sources can be treated now.

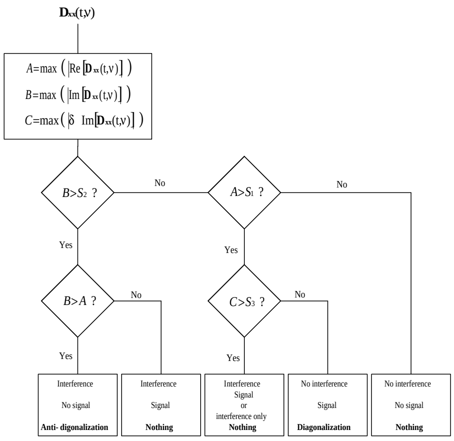

Figure 1: Decision algorithm, with S_1, S_2, S_3 three thresholds (3 small constants in the noiseless case)

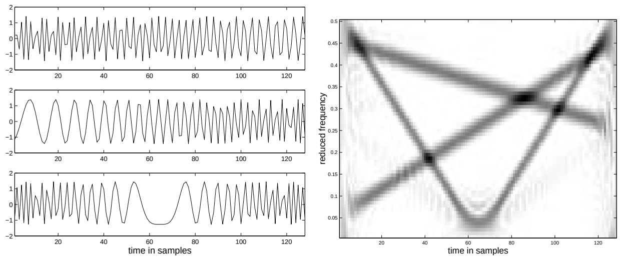

Figure 2: The 3 sources (left), the trace of the spatial PWV distribution of sources (right)

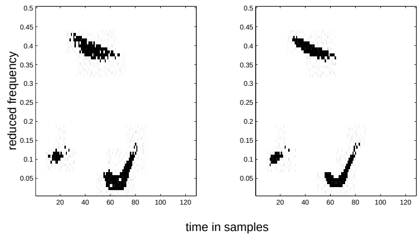

Figure 3: t-f points retained in the building of matrices set to be JD: orthogonal JD (left), non-orthogonal JD (right)

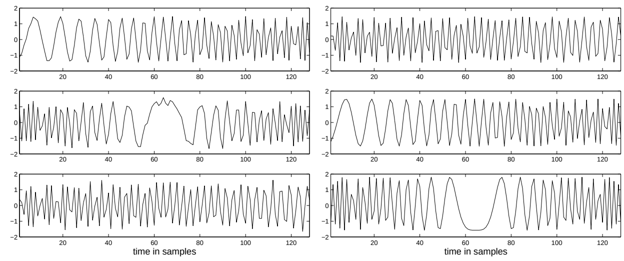

Figure 4: Sources estimated thanks to orthogonal JD (left), non-orthogonal JD (right)

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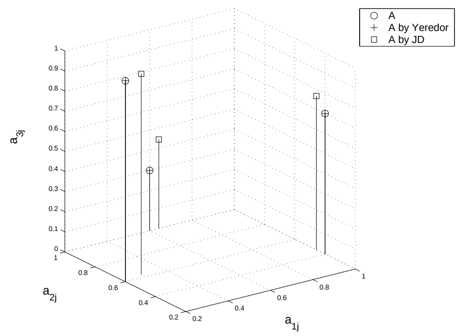

Figure 5: A comparison of results