ON THE ESTIMATION OF INTERFEROMETRIC PHASES FOR MULTIBASELINE SAR INTERFEROMETRY USING A RELAXATION-BASED TECHNIQUE

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ABSTRACT

In this paper, we examine how one can exploit baseline diversity of a multichannel interferometric SAR system to overcome the layover problem. The problem arises when different height contributions collapse in the same range-azimuth resolution cell, due to the presence of strong terrain slopes or discontinuities in the sensed scene. We propose a multilook approach to counteract the presence of the time-and-space varying amplitude distortion which is due to the extended nature of natural targets; to this purpose we extend a recent relaxation based approach by estimating the interferometric phases using a nonlinear least squares estimation technique that is based on a deterministic modelling of the amplitude distortion.

1. INTRODUCTION

Interferometric synthetic aperture radar (InSAR) is a powerful and increasingly expanding technique allowing estimation of three dimensional terrain images, with high spatial resolution and height accuracy. The technique suffers from the layover phenomenon that shows up when the imaged scene contains highly sloping areas or discontinuous surfaces [1, 2]. In these conditions, the received signal is the superposition of the echoes backscattered from the various patches of terrain that are mapped in the same range-azimuth resolution cell but have different elevation angles. The result is that the height map produced by the InSAR system is affected by strong distortions. Given the extended nature of the backscattering sources, the received signal is affected by a time-and-space varying amplitude distortion (often modelled as multiplicative noise), termed *speckle* in the radar imaging jargon. A multibaseline InSAR system has the ability to resolve the multiple sources along the elevation angle, and several approaches have been suggested in the literature. In [3], a beamforming approach was suggested to solve this problem. However, beamforming suffers from well-known problems of resolution and leakage. Generally, it is not possible to obtain a large overall baseline with enough coherent samples to attain the desired resolution in elevation [4, 5]. Subsequently, superresolution techniques, not taking account of the extended nature of the backscattering sources, were considered [6, 7]. In [8], the APES algorithm was extended to handle multilook data. Recently, the Capon, least-squares, root-MUSIC and RELAX algorithms have also been applied to the multibaseline problem [9, 10]. Of these, the multilook version of RELAX, termed M-RELAX, was the one often to be preferred. A mathematically closely related problem is encountered in wireless communication due to the presence of scatterers in the vicinity of the mobile, a problem that has received a lot of attention in the recent literature (see, e.g., [11, 12] and the references therein). In this work, we propose using the nonlinear least squares (NLS) estimator of a single backscattered signal as derived in [11] in the M-RELAX algorithm [9, 10]. The RELAX algorithm [13] is a relaxation based technique that recursively estimates the coefficients of a multicomponent complex exponential signal. The M-RELAX estimator is derived under the assumption that the speckle can be modelled as a spatially stationary, temporally white Gaussian random vectors. This common (and rather restrictive) assumption is also employed in [12] and in many other recent papers. The proposed estimator (termed DM-RELAX) differs in the sense that it allow an arbitrary distortion of the wavefront, with the only restriction (to ensure identifiability) being that the propagation of the wavefront along the array is not distorted in phase. This much less restrictive model also allows the gains of the sensors in the array to be unknown [11]. The numerical simulations in [9, 10] indicates that the M-RELAX algorithm is basically efficient for a modest number of looks. However, the deterministic Cramér-Rao bound (CRB) is lower than the stochastic CRB [11], suggesting room for further improved performance. Our numerical simulations verify this, showing that DM-RELAX yields significantly improved estimates of the interferometric phases.

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2. MODEL DESCRIPTION

We consider a multibaseline cross-track interferometer system with K phase centers aligned to form a uniform linear array (see [8, 9, 10] for further background on the model assumptions). To increase the accuracy in presence of speckle, multiple looks are collected from homogeneous adjacent pixels or from multiple observations obtained by partitioning the synthetic aperture [1, 2]. The complex amplitudes of the pixels of the nth look corresponding to the same imaged area on the ground, collected at the K phase centers of the antenna array in the presence of layover, are modelled as

$$\mathbf{y}(n) = \sum_{m=1}^{N_s} \sqrt{\tau_m}\mathbf{x}_m(n) \odot \mathbf{a}(\varphi_m) + \mathbf{v}(n), \quad (1)$$

for $n = 1, \ldots, N$, where $\mathbf{y}(n)$, $\mathbf{x}_m(n)$, $\mathbf{a}(\varphi_m)$, and $\mathbf{v}(n)$ are K-dimensional complex vectors, $\odot$ is the Hadamard product, N is the number of available looks, and N_s is the number of extended backscattering sources, i.e., the number of laid over terrain patches located in the same range-azimuth resolution cell, having different elevations. To ensure identifiability, we assume that $N_s \leq K-1$, where N_s is assumed to be known [14]. The term τ_m is the texture, or radar reflectivity, of the mth terrain patch; it does not change from one look to the other, but is different for different sources. Under the uniform linear array and far field assumptions, the steering vector, $\mathbf{a}(\varphi_m)$, can be written as

$$\mathbf{a}(\varphi_m) = \left[\begin{array}{cccc} 1 & e^{j\varphi_m/(K-1)} & \ldots & e^{j\varphi_m} \end{array}\right]^T, \quad (2)$$

where $(\cdot)^T$ denotes the transpose and the interferometric phases, $\{\varphi_m\}$, are defined as the phase difference between the two furthest phase centers. They are related in a one-to-one mapping with the elevation angle of the mth terrain patch as well as to the spatial frequency ω_m as [1, 9, 10]

$$\varphi_m = (K-1)\omega_k. \quad (3)$$

The speckle vectors, $\{\mathbf{x}_m(n)\}$, are some form of time-and-space varying amplitude distortion. In the following, we will at first model the speckle as stationary complex Gaussian distributed vectors in the derivation of the M-RELAX estimator. We will then proceed to loosen this assumption, allowing an (almost) arbitrary distortion, in the derivation of the DM-RELAX estimator.

Finally, $\{\mathbf{v}(n)\}$ are N independent and identically distributed (i.i.d.) complex white Gaussian distributed vectors whose components have power σ_v^2.

3. THE MULTILOOK RELAX ESTIMATOR

The multilook RELAX (M-RELAX) algorithm [9, 10] is an extension to the multilook scenario of the RELAX algorithm. It is derived under the assumption that the amplitude distorting vectors, $\{\mathbf{x}_m(n)\}$, are modelled as stationary complex Gaussian distributed vectors with zero-mean, unit variance and covariance matrix

$$\mathbf{C}_m = \mathrm{E}\left\{\mathbf{x}_m(n)\mathbf{x}_m^H(n)\right\}, \quad m = 1, \ldots, N_s \quad (4)$$

where $\mathrm{E}\{\cdot\}$ denotes the expectation operator and $(\cdot)^H$ the conjugate transpose. In shorthand notation, we write

$$\mathbf{x}_m(n) \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_m). \quad (5)$$

Assuming that both the looks and the backscattering sources are independent, the vectors $\mathbf{x}_i(n_1)$ and $\mathbf{x}_l(n_2)$ are mutually independent for $i \neq l$, or $n_1 \neq n_2$. We also assume that the speckle correlation sequences, $r_{xm}(k)$, are real-valued. Given these assumptions, the M-RELAX estimates are obtained by minimizing [9, 10]

$$Q_N(\boldsymbol{\alpha}_n, \tilde{\mathbf{u}}) = \sum_{n=1}^{N} \left\| \mathbf{y}(n) - \sum_{m=1}^{N_s} \alpha_m(n)\mathbf{a}(\omega_m) \right\|^2 \quad (6)$$

where

$$\tilde{\mathbf{u}} = \left[\begin{array}{ccc} \omega_1 & \ldots & \omega_{N_s} \end{array}\right]^T \quad (7)$$

$$\boldsymbol{\alpha}_n = \left[\begin{array}{ccc} \alpha_1(n) & \ldots & \alpha_{N_s}(n) \end{array}\right]^T \quad (8)$$

The M-RELAX estimate is obtained by estimating the dominant component, removing it from the data, and then repeating the procedure for each of the N_s components to be estimated. The algorithm estimates recursively all the components until the convergence condition has been satisfied. In this way, the multidimensional non-linear minimization is transformed into a sequence of simpler one-dimensional problems. For a signal consisting of multiple complex sinusoids in additive white Gaussian noise, the RELAX algorithm provides frequency and amplitude estimates that are asymptotically Gaussian distributed, unbiased and efficient [13]. Define the residual data

$$\mathbf{y}_l(n) = \mathbf{y}(n) - \sum_{m=1, m\neq l}^{N_s} \hat{\alpha}_m(n)\mathbf{a}(\hat{\omega}_m), \quad (9)$$

where $\{\hat{\alpha}_m(n), \hat{\omega}_m\}_{m=1,m\neq l}^{N_s}$ are the estimates already available, obtained in the previous steps. By replacing $\mathbf{y}(n)$ with $\mathbf{y}_l(n)$ in the cost function (6), minimizing with respect to ω_l and $\alpha_l(n)$, we obtain

$$\hat{\omega}_l = \arg\max_{\omega} \mathbf{a}^H(\omega)\hat{\mathbf{R}}_l\mathbf{a}(\omega), \quad (10)$$

where $\hat{\mathbf{R}}_l$ is the sample covariance matrix estimated using $\{\mathbf{y}_l(n)\}_{n=1}^N$, and

$$\hat{\alpha}_l(n) = \frac{1}{K}\mathbf{a}^H(\omega)\mathbf{y}_l(n). \quad (11)$$

It is worth noting that the data model in (6) is mismatched with the actual data model in (1), due to the presence of the amplitude distortion. As a result, the cancellation in (9) will never completely remove the signal components, not even if the parameters are perfectly estimated. Also note that the maximization in (10) is the NLS estimate of a single non-extended source (single scatterer) in the spatial-domain, constituting of nothing but the beamforming spectrum of the data $\{\mathbf{y}_l(n)\}$.

4. THE DM-RELAX ESTIMATOR

We proceed to allow an arbitrary deterministic distortion of the wavefront, with the only restriction being that the propagation of the wavefront along the array is not distorted in phase, i.e., under the assumption that the speckle vectors, $\{\mathbf{x}_m(n)\}$, are real-valued. However, we do allow an unknown time-varying phase component as long as it does not change over the array [11]. We note that results reported in [11] indicate that the estimator will be robust and perform reasonably well even in the presence of phase fluctuations of the wavefront, i.e., when $\{\mathbf{x}_m(n)\}$ is complex-valued, a result verified in our numerical simulations.

Under these assumptions, the NLS estimate of the spatial frequency, $\hat{\omega}$, in the case of a single source, can be shown to be [11]

$$\hat{\omega} \quad = \quad \arg\max_{\omega} \sum_{n=1}^{N} \left| \sum_{k=0}^{K-1} a_k^{2H}(\omega) y_k^2(n) \right|, \qquad (12)$$

where $y_k(n)$ and $a_k(\omega)$ denote the kth index of $\mathbf{y}(n)$ and $\mathbf{a}(\omega)$, respectively. Note that the unambiguous estimation range of ω using (12) will be half of the estimation range obtained using (10), and the spatial frequency estimate must thus be scaled accordingly. The inner sum in (12) can be evaluated using the FFT algorithm with zero padding applied to the squared data samples $\left\{y_k^2(n)\right\}_{k=0}^{K-1}$ (for each t).

For multiple sources, the number of unknowns is larger than the number of available observation, and the NLS estimate is not identifiable [11]. Furthermore, the single source estimator in (12) would fail if applied as an alternative to (10) in the M-RELAX recursion, as the squaring of the samples in (12) will introduce undesired cross terms yielding erroneous spatial frequency estimates. However, this problem can be avoided by initializing the estimation using the M-RELAX algorithm, and then applying the single source estimator in (12) on the residual data sequences as obtained from (9).

In an effort to refine the interferometric phase estimation, we thus propose to replace the single source phase estimate in (10) with the estimate in (12), after obtaining a set of initial estimates using M-RELAX.

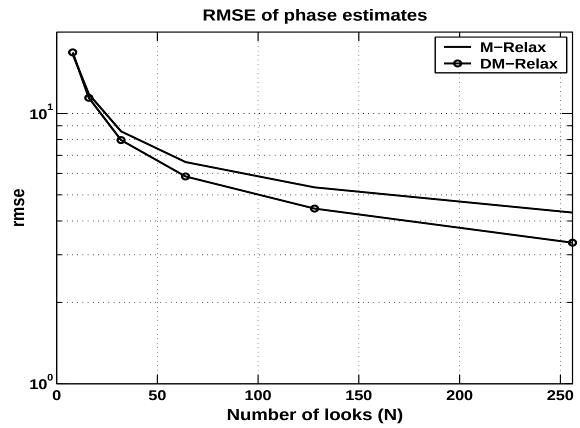

Fig. 1. Root mean square error of φ_1.

5. NUMERICAL EXAMPLE

Performance analysis has been carried out assuming that the vectors $\{\mathbf{x}_m(n)\}_{n=1}^{N}$ are i.i.d. with zero mean and spatial autocorrelation sequence given by

$$\begin{aligned} r_{xm}(k) &= \mathrm{E}\left\{[\mathbf{x}_m(n)]_l \cdot [\mathbf{x}_m(n)]_{l+k}^H\right\} \\ &= \begin{cases} 1 - \frac{|k|}{K-1} b_m, & \text{for } |k| \leq \frac{K-1}{b_m} \\ 0, & \text{otherwise} \end{cases} \end{aligned} \qquad (13)$$

where K is the number of antennas in the InSAR system and b_m is the normalized baseline relative to the mth patch [9, 10]. This triangular correlation sequence is the basic speckle model used in SAR interferometry for flat extended targets [1, 2]. Note that it will yield complex valued speckle vectors, testing the robustness of the DM-RELAX estimator. As was shown in [15], one often obtain improved spectral estimates by substituting the forward-only sample covariance matrix estimate, $\hat{\mathbf{R}}_l$, with the forward-backward averaged sample covariance matrix,

$$\hat{\mathbf{R}}_l^{fb} = \frac{1}{2}\left(\hat{\mathbf{R}}_l + \mathbf{J}\hat{\mathbf{R}}_l^T\mathbf{J}\right), \qquad (14)$$

where $\mathbf{J}$ is the $K \times K$ exchange matrix. This is also our experience, and we thus replace $\hat{\mathbf{R}}_l$ in (10) with $\hat{\mathbf{R}}_l^{fb}$. As is done in [9, 10], we assume a system with $K = 8$ phase centers with two sources present ($N_s = 2$). Furthermore, we assume that the values of the parameters $\varphi_1 = 0°$, $\varphi_2 = 360°$, $b_1 = b_2 = 0.2$, $SNR_1 = SNR_2 = 12$ dB, where SNR_m is the signal to noise ratio of the mth source, defined as $SNR_m = \tau_m/\sigma_v^2$. Two sources with normalized baselines b_1 and b_2 are considered adjacent when $\Delta\varphi = |\varphi_1 - \varphi_2|$ is equal to $\Delta\varphi_{ad} = 2\pi(b_1 + b_2)$. When separated less than $\Delta\varphi_{ad}$, the sources collapse into only one [16]. In our case, the Rayleigh limit is $\Delta\varphi_B = 2\pi(K-1)/K = 315°$, and $\Delta\varphi_{ad} = 144°$, so $\Delta\varphi > \Delta\varphi_B > \Delta\varphi_{ad}$.

The performance of the estimators are analyzed in terms of root mean square error (rmse),

$$\text{rmse}(\hat{\varphi}_m) = \sqrt{E\{(\hat{\varphi}_m - \varphi_m)^2\}}, \qquad (15)$$

evaluated by means of 10^4 Monte-Carlo simulations, where the estimators are implemented using the FFT zero padded to 256 points. Figures 1 and 2 shows the rmse of the first and second interferometric phase estimates, clearly indicating the improved performance of DM-RELAX.

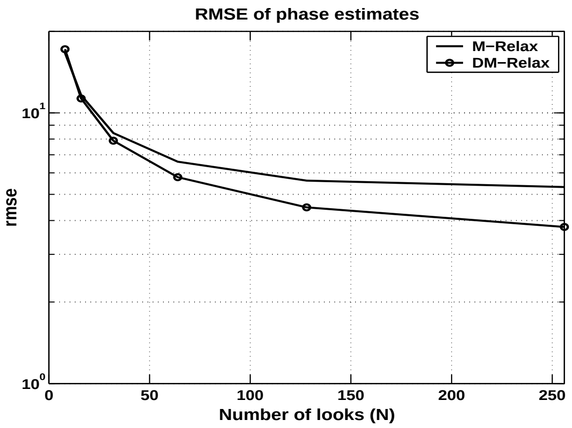

Fig. 2. Root mean square error of φ_2.

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