THE VTP TEST FOR TRANSIENTS OF EQUAL DETECTABILITY

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ABSTRACT

For detection of a permanent and precisely-modeled change in distribution of iid observations, Page's test is optimal. When employed to detect a transient change between known distributions, Page's test is a GLRT. However, the situation of interest here is of transient of unknown scale parameter: a fixed Page procedure tuned to a "short-and-loud" signal uses heavy biasing and low threshold, a combination ill-suited to a "long-but-quiet" signal. We offer an easy alternative to the standard Page: it uses a constant bias and a time-varying threshold. The idea is that the above *short* signals are detected quickly before post-termination data has a chance to refute them; and that evidence for a *long* signal is allowed to build, rather than being summarily discarded too early. Results show that the approach works quite well.

1. INTRODUCTION

Many signal-processing applications require the detection of a change occurring in a sequence of observed data [2]. Of interest in this paper are transient signals that are of unknown strength and location but with temporal contiguity.

The quickest detection of a change in distributions, occurring at unknown time points, is an old but important problem. The Page (or cusum) test [9] is optimal for the "quickest" permanent change detection in the sense that it minimizes the worst-case average delay to detection given an average distance between false alarms [6]. Now consider further the detection of a *transient* signal, which amounts to the notification of a temporary change in distribution. Mathematically, the independent observation sequence

$$x_n \text{ has density} \begin{cases} f_0(x_n), & 1 \leq n < n_s \text{ and } n_s + n_d \leq n \leq N \\ f_1(x_n), & n_s \leq n < n_s + n_d \end{cases} \quad (1)$$

where N is the sample length, n_s and n_d represent the occurrence of a transient, and f_0 and f_1 are the distributions respectively off and on the change. The Page test forms

$$Z_0 = 0, \\ Z_n = max\{0, Z_{n-1} + g(x_n)\} \quad (2)$$

and declares a detection when Z_n exceeds a threshold h, where the function g is (asymptotically) optimally the log-likelihood ratio (LLR) $log(f_1(x)/f_0(x))$ [6]. Suppose both f_0 and f_1 are known, then the Page test using the LLR as $g(x)$ becomes a GLRT with respect to the unknown parameters n_s and n_d [5].

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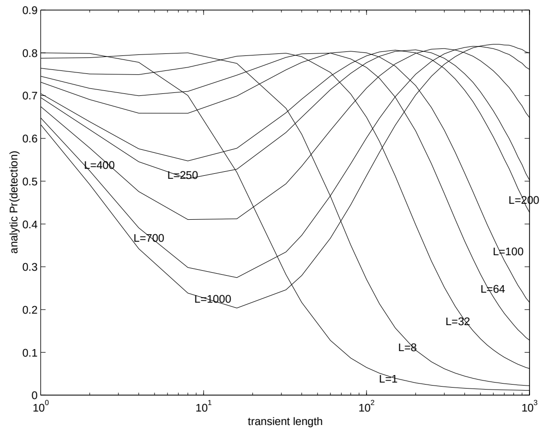

Fig. 1. Detectability of Gaussian shift-in-variance transients using fixed-style Page procedures designed for various but specific transient length L. For each L the transient's aggregate SNR is from figure 2, and provides constant detectability. In all cases, the fixed-style Page procedures are designed to provide $P_d = 0.8$, $\bar{T} = 10^6$ and $N = 10^3$. The update takes the form $g(x) = x^2 - b_L$, where b_L, defined in (4), denotes the bias for the fixed-style Page designed for length L.

If nothing whatever is known about f_1 (noise plus transient), then the situation is probably hopeless. Naturally if appreciable information about the form of the transient signal is available, that information should be used [3, 4]. But if little is known about the transient save that some elevation of the power level is to be expected in a number of contiguous samples, it was found in [10] that a Page procedure based on the Gaussian shift-in-variance model is a simple and robust choice for the detection of a wide variety of transient signals. This is heartening news; however, it must be noted [10] that the variance value of the update $g(x)$ was a tuned value commensurate with the transient's strength. Direct application of Page's test is risky when information of the transient strength (and length) is unknown, as is usually the case in practice. As shown in Figure 1, Page's test provides nowhere near constant detectability, where a Page test designed for a short-and-loud transient provides very poor performance at detecting long-and-quiet transients and vice versa. The reason indicated in Figure 2 is that a long-and-quiet transient is better served by a small negative bias and a high threshold but ill-served by the large bias and low threshold designed for short-and-loud transient signals; and vice versa.

Since it is not unnatural in practice that some transients are short-and-loud, while some are long-and-quiet, we argue that:

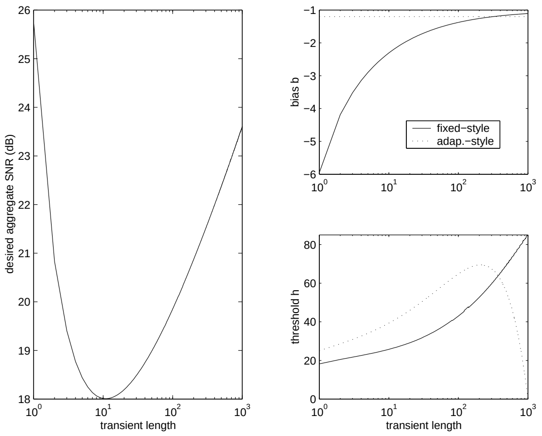

Fig. 2. Gaussian shift-in-variance case: the right plots give comparison of the bias and thresholds used in the new VTP procedure to those of the fixed Page scheme tailored to each specific transient length, where $P_d = 80\%$, $\bar{T} = 10^6$ and $N = 10^3$. In the latter case, note that a Page test designed for a transient length $L = 10$ uses approximate threshold and bias 25.78 and -2.31, respectively, *at all times*; and that a Page test designed for $L = 100$ uses corresponding approximate *fixed* values 42.99 and -1.38. The VTP scheme varies its threshold and bias dynamically as a function of the time since the last reset-to-zero, and thus we wish to stress that although the two pairs of curves are notionally similar, they are functionally completely different. The aggregate SNR S_L necessary to achieve $P_d = 80\%$ and $\bar{T} = 10^6$ in the fixed scheme is plotted in the left figure.

> *A useful scheme should detect any transient signal whose overall detectability P_d exceeds a given value, given the average number of samples between false alarms be at least $\bar{T}$.*

It is reasonable to ask whether the Page test (2) can be improved by appropriate modification. We propose a simple adaptive Page procedure.

2. THE VARIABLE THRESHOLD PAGE TEST

We first consider the Gaussian shift-in-variance case, with

$$f_0(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \qquad (3)$$
$$f_1(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-x^2/2\sigma^2}$$

for which the corresponding LLR is obtained, and thus for simplicity we take the update as $g(x) = x^2 - \frac{\sigma^2 log(\sigma^2)}{\sigma^2-1}$. For transient with length exactly L, the bias is

$$b_L = \frac{(1+S_L/L)log(1+S_L/L)}{S_L/L} \qquad (4)$$

since the variance $\sigma^2 = \frac{S_L}{L} + 1$, where S_L is the aggregate SNR for the L-length transient.

Since we are interested in the detection of time-contiguous transients with unknown location and strength, then the problem is to determine the correct bias and threshold in Page test. Consequently we seek an *adaptive* test with *time-varying* biases and thresholds. However, it is quickly seen to be infeasible, since not only is this a hopelessly complex problem, but it is also a multi-objective optimization. Note that Page's test can be interpreted as consecutive sequential probability ratio tests (SPRT's) with lower threshold zero and upper threshold h. Thus, we propose a heuristic by which the bias and threshold sequence can be set, using the bias and thresholds of the fixed-style Page tests. In essence, the idea is to keep the bias constant, and to adjust the detectability solely through the threshold. We summarize the variable threshold Page (VTP) procedure as follows:

1. Select a minimum performance level that is acceptable. That is, select $\bar{T}$, the average number of samples between false alarms; and select the probability of detection P_d below which an "alert" would be of unacceptably-low fidelity. Example values might be $\bar{T} = 10^6$ and $P_d = 80\%$.
2. It is assumed that the pdf's f_0 and f_1 differ only in a single parameter λ. Determine this parameter λ_k and a corresponding threshold such that a fixed optimal Page test would detect a transient signal (of length k) in which f_1 has parameter λ_k with performance $\bar{T}$ and P_d, $\forall k \in \{1, N\}$.
3. The **implementation** is as a series of sequential tests according to

$$\begin{array}{rcl} Z_n &=& Z_{n-1} + g_0(x_n) - b \\ k &=& k+1 \\ (Z_n \leq 0) &\longrightarrow& \text{set } (k=0) \text{ and } (Z_n = 0) \\ (Z_n \geq h(k)) &\longrightarrow& \text{declare detection} \end{array} \qquad (5)$$

   for $n = 1, ..., N$. Therefore, the update is $g(x) = g_0(x) - b$, with $g_0(\cdot)$ being a fixed memoryless operation without the bias term. For instance, $g_0(x) = x^2$ for the Gaussian shift-in-variance case.
4. In (5) the b and $\{h(k)\}$ are

$$b = b_c; \quad h(k) = h_k + k(b_k - b_c) \qquad (6)$$

   for $k = 1, \ldots, N$, where b_k and h_k are the constant bias and threshold designed for transients with length exactly k, and b_c is the constant bias for the VTP scheme. Here b_k is the bias for an f_1 having parameter exactly λ_k.
5. The bias b and thresholds $\{h(k)\}$ are adjusted recursively to achieve the desirable $\bar{T}$ (see [12] for details).

It is worth emphasizing that we rely on the Markov-chain analysis using the FFT-based procedure [5, 12] to evaluate both P_d and $\bar{T}$, and a subtlety involving the initial condition (etc. the Page statistic can be non-zero at the start point of a change) for the test is particularly important for this paper. The constant bias for the VTP scheme, b_c, needs to be chosen carefully to make the most recent reset happen as near to the actual transient onset as possible and meanwhile keep the threshold always positive,

$$b_c = b_N + \frac{h_N}{N+1} \qquad (7)$$

in which b_N and h_N are respectively the bias and threshold tuned for a fixed Page test designed for a transient of length N. Also, admittedly, Step 5 requires considerable programming and iteration, using the FFT-based analysis procedure. Please refer to the journal version [12] for details.

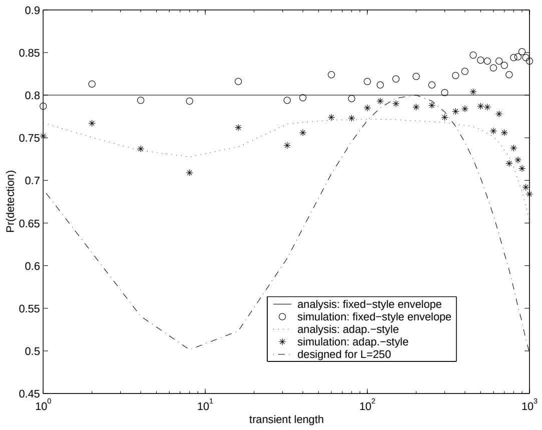

Fig. 3. Performance of the VTP scheme in Gaussian shift-in-variance transient problem. To demonstrate the precision of the FFT-based analytic prediction, the P_d envelope from the normal fixed-style Page procedure and the result of the VTP test are also simulated based on 10^4 runs. The performance of the Page test optimized for transient length $L = 250$ is repeated from figure 1 for comparison only.

3. RESULTS

For the Guassian shift-in-variance case, the bias and thresholds for the variance-based VTP scheme are plotted in figure 2. It is noted in figure 1 that no fixed Page test provides constant detectability of equally-detectable transients. There is, however, a tendency for a Page test designed for an intermediate transient length (e.g. $L = 250$) to be at least *acceptable* for a wide range of signal strength and length; however, it is shown that the VTP scheme offers a nice improvement even over this. From figure 3, a healthy improvement over a wide range of signal strengths is observed. The new VTP scheme achieves nearly the performance of the "envelope" (see figure 1) of all fixed Page tests designed for specific transient signal lengths: basically, it is as good as it can be. The good match between analysis and simulation results indicates that the Markov-chain analysis of provides a good estimate of P_d when the number of quantization levels is 2^{13}. Increasing quantization levels can further increase the accuracy.

3.1. More General Transient Types

That observation from [10] was our motivation in this research: the only apparent degree of non-robustness of the variance-based Page procedure was in its need to know the transient's power level, and its variable threshold (VTP) version does not need that. Thus, we study the performance of the variance-based VTP test as a function of the transient signal's form and strength. The signal model is

$$H_0 : \mathbf{x} = \mathbf{w} \tag{8}$$
$$H_1 : \begin{cases} x(n) = w(n), & n < n_s \text{ and } n_s + M < n \leq N \\ x(n) = s(n) + w(n), & n_s \leq n \leq n_s + M \end{cases}$$

in which $\mathbf{x}$ denotes the observation vector, $\mathbf{w}$ is white Gaussian noise with zero mean and unit variance, $\mathbf{s}$ is the transient signal of interest. We use $N = 128$, $M = 30$, the sampling rate $f_s = 16$ and decaying factor $\lambda = 0.5$ in our simulations. We consider four types of transients:

S_1: - White burst: the transient signal s_1 is white and Gaussian with zero mean;

S_2: - Single exponentially-decaying sinusoid whose frequency is randomly chosen;

S_3: - Exponentially-enveloped white burst;

S_4: - Narrowband burst: s_4 is created by passing white Gaussian noise through a narrowband filter, whose bandwidth is 0.3π, and whose center frequency is chosen randomly.

Certainly this is not an exhaustive menu of transients, but a wide range is covered. We apply the VTP detector (the Gaussian shift-in-variance case) to the above transients' detection, where $P_d = .8$, $\bar{T} = 10^6$ and $N = 128$. The assumptions on which the VTP procedure is built are those of S_1; the detector is weakly suited to S_3, and would seem to be ill-suited for either S_2 or S_4.

To illustrate the performance of our VTP detector, we compare it to the basic and improved "power-law" statistics [8, 11], and the "maximum" detector [7]. The later was found [10] to provides the best performance over a wide range of transients; however, it requires the knowledge of M, and it shows considerable sensitivity as regards this parameter. We plot P_d versus the aggregate SNR in figure 4, in which $P_{fa} = 10^{-4}$. It is noted that the VTP procedure provides very close performance to that of the "maximum" detector in all four situations. This is exciting, as we recall that M is tuned in the "maximum" detector and that our VTP test requires no such prior information. It is additionally noted that the VTP procedure provides performance superior to even the improved power-law detector T_{f2} in most cases, with the exception of S_2 (essentially a tie) in which the transient is highly narrowband.

3.2. The VTP Idea in Other Statistical Situations

The informing application for the VTP test was in the detection of realistic transient signals via a search for elevated variance. However, the approach can be used for other statistical models as well, and here we present the Exponential and the Gaussian shift-in-mean cases in Figures 5 and 6, in which $P_d = 0.8$, $\bar{T} = 10^6$, $N = 10^3$, and the update takes the form $g(x) = x - b$. The bias for exactly length L is $b_L = \frac{(1+S_L/\sqrt{L})log(1+S_L/\sqrt{L})}{S_L/\sqrt{L}}$ for the Exponential case; and $b_L = \frac{S_L}{2\sqrt{L}}$ for the Gaussian shift-in-mean case. Similar to the Gaussian shift-in-variance case, no fixed Page test provides constant detectability of equally-detectable transients, and a gratifying detection improvement of the VTP procedure is observed over a wide range of transient length.

4. SUMMARY

Notionally, a transient signal that is long-and-quiet and one that is short-and-loud ought to have approximately the same detectability. However, these two engender very different Page tests, and, unfortunately, the test designed for one can work quite poorly for the other. Consequently, in this paper, an *variable threshold* Page (VTP) processor has been developed: it uses a constant bias, but has a threshold that adaptively changes with the number of samples since the most recent reset.

The new detector has been studied extensively in the Gaussian shift-in-mean and shift-in-variance and Exponential shift-in-scale cases. It works very well, and in many cases it nearly traces the "envelope" of performances achievable with the best Page processors tuned to each transient length — the proposal is reasonable but ad-hoc, but apparently we could do little better.

Transient detection is interesting because one does not know in advance the sort of transient signal one has to look for. Many transient detectors are tuned to one type of transient and comparatively blind to others. What tends to unite transient signals of practical interest, however, is that they are an organized agglomeration of energy into contiguous (or nearby) time samples. Now, assuming a unit-variance ambient (as would be available after normalization), a transient detector that assumes nothing but this local scale-change — and one that is reasonably insensitive to other characteristics such as spectrum — is that based on the Page structure for Gaussian shift-in-variance. This detector was previously shown to be both robust (against transient type) and very good; its only small disadvantage was the tuning that it needed in terms of the strength (power) of the transient. The VTP test developed here has removed even the need for that knowledge.

5. REFERENCES

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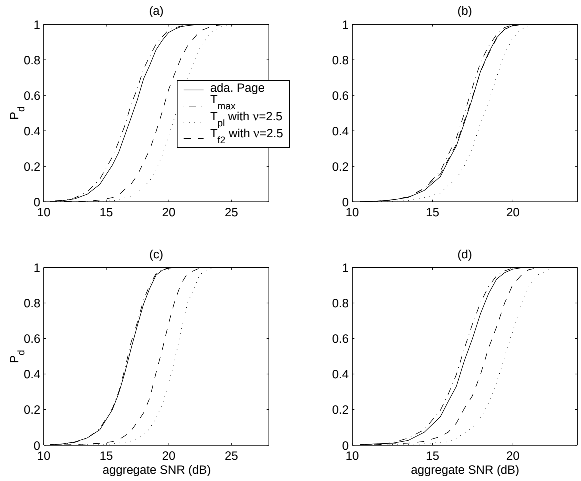

Fig. 4. Detection performances of the VTP scheme. The transient duration is $M = 30$ samples; different panels refer to different transient signals, with (a): transient signal s_1, (b): s_2, (c): s_3, and (d): s_4. The "maximum" detector T_{max} is tuned to the true transient length $M = 30$.

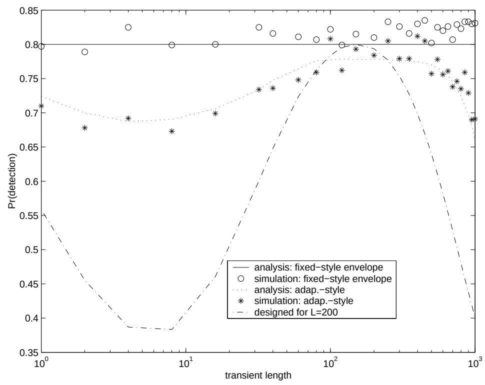

Fig. 5. Performance of the VTP scheme in Exponential transient problem. The P_d envelope from the normal fixed-style Page procedure and the performance of the Page test optimized for transient length L=200 are shown for comparison only.

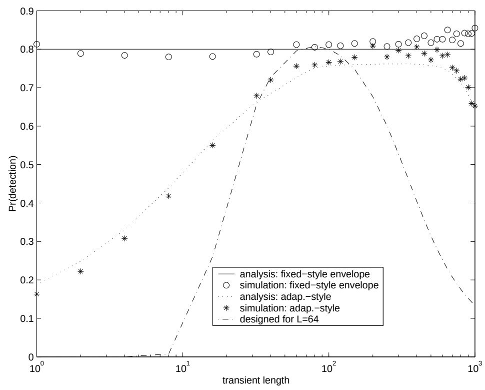

Fig. 6. Performance of the VTP scheme in Gaussian shift-in-mean transient problem. The performance of the Page test optimized for transient length $L = 64$ is used for comparison.