SOURCE DETECTION IN CORRELATED MULTICHANNEL SIGNAL AND NOISE FIELDS

Karim G. Oweiss

Electrical & Computer Eng. Dept., Michigan State University, East Lansing 48824, USA

ABSTRACT

The problem of detecting the number of sources impinging on an array of sensors has received wide interest in many research problems. In particular, the detection of the number of distinct neural sources using a recording array of closely spaced sensors in the brain is one such application. The special case of transient source signals of unknown waveforms corrupted by Gaussian noise is the focus of this paper. We propose a new approach for solving this problem when no apriori knowledge is given about the neural sources and/or the noise processes. By extending our previous array multiresolution analysis framework for noise suppression, signal detection and identification [1-4], we show that it is feasible to achieve reasonable source detection performance in moderate to low SNR scenarios. Comparison to traditional detection schemes is presented.

1. INTRODUCTION

The problem of determining the number of sources impinging on an array of sensors is a traditional problem in array processing often referred to "source detection". Termed as a model order selection problem, Akaike's Information Criterion (AIC) and Rissanen's Minimum Description Length (MDL) criterion are the most widely known information theoretic methods to solve this problem [5-6]. Variants of these two methods led researchers to develop numerous techniques to cope with specific cases where these methods have yielded overestimation or underestimation of the number of sources [7-11]. The original hypothesis testing approach to the problem derived from the statistical literature is a sphericity test [12], that sequentially tests for equality of the smallest eigenvalues, presumably attributed to the noise subspace.

The challenge in detecting the number of sources becomes greatly complicated when the source signals are either correlated, weak, or closely spaced and the sample size is limited. Moreover, if the noise corrupting the observations is colored or inherently correlated with the signals of interest, then the detection task becomes cumbersome and almost all existing techniques yield significant performance degradation. As an example, consider detecting the number of neural cells in a neural cell population using an array of closely spaced electrodes [13]. The main characteristics of such neurophysiological signal environment can be summarized as follows:

1- Multiple neural sources are present with various strengths, and their waveforms exhibit significant correlation and coherence structures as illustrated in Fig.1.
2- If the recording array is closely spaced, which is the typical case for the purpose of understanding interaction in small neural cell populations, the noise process exhibit significant spatial correlation.
3- Since a major component of the noise process is attributed to the background activity of neural sources far from the array, the noise tends to be temporally correlated and cross-correlated with the signal of interest.

In a neurophysiological context, one needs to know how many cells constitute the mixture recorded by the array of electrodes, which eventually leads to better understanding of neural function and connectivity. In almost all existing source detection algorithms, it is always assumed on one hand that source signals are statistically independent and have a positive definite covariance matrix, i.e., none of the signals are perfectly coherent. On the other hand, the noise is assumed white and statistically independent of the source signals. Clearly, these assumptions render existing algorithms inapplicable to the application at hand and to other array processing applications where similar characteristics are encountered.

We adopted a novel application driven framework in [1-4] for performing typical array signal processing tasks such as noise suppression, signal detection and source separation in the multiresolution domain obtained by means of the Discrete Wavelet Transform (DWT). The work we present here is a natural extension to this framework. The only assumption under which we carry out analysis is the Gaussian nature of the noise.

2. THEORY

2.1. Problem Statement

The array model in the time domain assumes the presence of P sources impinging on an M-channel array according to

$$\boldsymbol{Y} = \boldsymbol{AS} + \boldsymbol{Z} \tag{1}$$

where $\boldsymbol{Y} \in \Re^{M\times N}$ denotes the multichannel observations in the sampled time interval $t = nT$, where n is an integer, $\boldsymbol{A} \in \Re^{MxP}$ denotes the mixing (steering) matrix, $\boldsymbol{S} \in \Re^{P\times N}$ denotes the source signal matrix, and $\boldsymbol{Z} \in \Re^{M\times N}$ denotes an additive Gaussian noise component with nontrivial temporal and spatial covariance matrices. It will be assumed that within the N snapshots consisting the analysis window, no more than $P \le M$ sources are present.

The matrix $\boldsymbol{S}$ is full rank when all the P sources are independent. When $\boldsymbol{S}$ is rank-deficient, this usually means that either the source signals have correlated waveforms, or a subset of the signals are perfectly coherent, i.e., at least one of the signals is just a scaled and delayed version of another signal. This type of situation arises when the multipath phenomenon occurs, i.e., a direct signal path and one or more indirect paths are received by the array in which case the signals are not independent. On the other hand, the columns of the matrix $\boldsymbol{A}$ represent the array response due to each of the P signals impinging on the array. Each column depends only on the geometrical construction of the array and the directional response of the sensors. Generally speaking, if the array is properly designed and the sources are independent and treated as point sources, $\boldsymbol{A}$ will be full rank. $\boldsymbol{A}$ can also be rank deficient when the propagating medium has nonstationary characteristics, or in some sense anisotropic. This situation can very likely arise in a neurophysiological experiment [14].

First, consider the *noise free* observations given by the product matrix $\boldsymbol{X}=\boldsymbol{AS}$. If $\boldsymbol{A}$ or $\boldsymbol{S}$ has rank less than P, $\boldsymbol{X}$ will also have rank less than P. If there are $P \le M \le N$ independent rows in $\boldsymbol{X}$, then this matrix is said to have a P-dimensional range or row space, which is a subspace of the M-dimensional Euclidean space $\Re^M$. The rank of this matrix is the dimension of this subspace. The spatial covariance of $\boldsymbol{X}$ when spectrally factored yields

$$\boldsymbol{R}_X = E\left[\boldsymbol{X}\boldsymbol{X}^T\right] = \boldsymbol{U}_X.\boldsymbol{D}_X.\boldsymbol{U}_X{}^T \tag{2}$$

where $\boldsymbol{D}_X \in \Re^{MxM}$ is a diagonal matrix containing the rank ordered eigenvalues $\delta_1 > \delta_2 > \ldots > \delta_P > \delta_{P+1} = \ldots = \delta_M = 0$ of $\boldsymbol{R_X}$. If there are P signal sources, the largest P eigenvalues correspond to the P sources and the first P columns of the unitary matrix $\boldsymbol{U}_X$ span the signal subspace. The remaining M-P eigenvalues are equal to zero with probability one. In practice, the finite sample size and the presence of noise amount to estimating the sample eigenvalues $\lambda_1 > \ldots > \lambda_P > \ldots > \lambda_M > 0$ from the sample covariance matrix

$$\boldsymbol{R}_Y \cong \frac{1}{N}\sum_{n=0}^{N-1} \boldsymbol{y}[n]\boldsymbol{y}^T[n] \tag{3}$$

The remaining M-P eigenvalues no longer equal to zero and the corresponding M-P eigenvectors span the noise subspace. When the noise is cross correlated with the signal of interest, $\boldsymbol{R_Y}$ can be expressed as

$$\boldsymbol{R}_Y = \boldsymbol{A}\boldsymbol{R}_S\boldsymbol{A}^T + \boldsymbol{R}_Z + \boldsymbol{A}\boldsymbol{R}_{SZ} + \boldsymbol{R}_{ZS}\boldsymbol{A}^T \tag{4}$$

The presence of the cross covariance terms shrinks the separation distance that should be observed to determine the subset of eigenvalues belonging to the signal subspace because of the mutual correlation of the sample eigenvalues, making it practically impossible to determine P.

2.2. Source Detection

In good SNR conditions, the separation between the signal and noise subspaces is easily obtained. However, in low SNR conditions, this separation do not yield easily and the source detection problem amounts to the so-called sphericity test [15] to determine the multiplicity of the smallest M-P eigenvalues using the likelihood function

$$\Lambda = \frac{\left(\prod_{i=p}^{M}\lambda_i\right)^{(1/M-p+1)}}{\frac{1}{M-p+1}\sum_{i=p}^{M}\lambda_i} \qquad p = 1, \ldots, M\text{-}1 \tag{5}$$

which is the ratio of the geometric mean to the arithmetic mean of the smallest M-p+1 sample eigenvalues. The sphericity test evaluated in the time domain tends to underestimate the number of sources when the source signals are correlated due to the presence of small signal eigenvalues that are indistinguishable from the noise eigenvalues (multipath situation for instance).

2.3. Source Detection in the Multiresolution Domain

Exploring the array model in the multiresolution domain has shown to provide significant advantages over the time domain analysis, especially in the correlated signal and noise environments, which are the main focus of this paper [13]. Denoting by $\boldsymbol{Y}_j \in \Re^{MxN}$ the multichannel coefficient matrix describing the stationary Discrete Wavelet Packet Transform (DWPT) in the j^{th} node of the wavelet binary tree up to L levels[1] [16], then by the linearity of the transform

$$\boldsymbol{Y}_j = \boldsymbol{X}_j + \boldsymbol{Z}_j \tag{6}$$

The covariance of $\boldsymbol{Y}_j$ can be spectrally factored yielding

$$\boldsymbol{R}_{\boldsymbol{Y}}^{\boldsymbol{j}} = E[\boldsymbol{Y}_j\boldsymbol{Y}_j^T] = \boldsymbol{U}_Y^j\boldsymbol{D}_Y^j\boldsymbol{U}_Y^{j^T} \tag{7}$$

In the wavelet domain, it was shown that each source can be characterized by a *pruned* binary tree describing the *best basis* for that source [3-4], by searching the full DWPT tree for the eigenvalue/eigenvector pair that remains *invariant* throughout the full wavelet decomposition, i.e., an invariant $\boldsymbol{U}_{\boldsymbol{S}}^{\boldsymbol{j}}$ that spans the same subspace spanned by the columns of the steering matrix $\boldsymbol{A}$. Depending on the span of wavelet basis in the j^{th} subband, *at most* the first P columns of $\boldsymbol{U}_{\boldsymbol{Y}}^{\boldsymbol{j}}$ span the signal subspace, while *at least* the smallest M-P eigenvalues of $\boldsymbol{D}_{\boldsymbol{Y}}^{\boldsymbol{j}}$ are nonzero with probability one and the corresponding M-P eigenvectors of $\boldsymbol{U}_{\boldsymbol{Y}}^{\boldsymbol{j}}$ span the noise subspace. Let p_j denotes the dimension of the signal subspace in the j^{th} node, then the i^{th} eigenvalue/eigenvector pair in the spectral factorization of $\boldsymbol{R}_{\boldsymbol{Y}}^{\boldsymbol{j}}$ in (7) will be the *dominant* mode in the j^{th} node if and only if

$$i = \arg\max(<\boldsymbol{s}_k\boldsymbol{b}_j>) \qquad k = 1, \ldots, P \tag{8}$$

where $<\cdot>$ denotes a dot product and $\boldsymbol{b}_j$ denotes the wavelet basis spanning the j^{th} subband. The Multiresolution Sphericity Test (MRST) for equality of the smallest eigenvalues can be formulated as

$$\Lambda_j = \frac{\left(\prod_{i=p_j}^{M}\lambda_i\right)^{(1/M-p_j+1)}}{\frac{1}{M-p_j+1}\sum_{i=p_j}^{M}\lambda_i} \quad , \quad p_j = 1, \ldots, M\text{-}1 \tag{9}$$

[1] For an L level DWPT binary tree, there is a total of $2^{(L+1)}$-1 nodes

where now the likelihood function is indexed by j, the subband index. The sphericity test formulated in (9) can be used to determine how many sources $p_j \le P$ are projected onto the subspace spanned by the wavelet basis in the j^{th} node. The test takes the form of a series of nested hypothesis tests, testing $M - p_j$ eigenvalues for equality; the hypotheses are of the form

$$H_0(p_j): \lambda_1^j \ge \lambda_2^j \ge \ldots \ge \lambda_{p_j+1}^j = \lambda_{p_j+2}^j = \ldots = \lambda_M^j$$
$$H_1(p_j): \lambda_1^j \ge \lambda_2^j \ge \ldots \lambda_{p_j}^j \ge \lambda_{p_j+1}^j \ldots > \lambda_M^j \quad (10)$$

We are interested in finding the smallest value of p_j for which $H_0(p_j)$ is true, which is done by testing $p_j = 0, p_j = 1$ … until $p_j = M-1$ or the test does not fail, i.e., $H_0(p_j)$ is determined to be true. Using a desired performance threshold for the probability of false alarm (overdetermination of p_j), a set of $p_j \le P$ dominant modes is obtained for each node and these are described by their corresponding rank ordered p_j eigenvectors. To estimate P, we form the augmented matrix $\overline{A}$ by concatenating all the eigenvectors from all the sphericity tests in (9), $\forall\ j = 0, 1, \ldots, J$ where J is the total number of nodes, in the columns of $\overline{A}$, i.e.,

$$\overline{A} = \left[u_1^0\ u_2^0 \ldots u_{p_0}^0\ u_1^1\ u_2^1 \ldots u_{p_1}^1 \ldots u_1^J\ u_2^J \ldots u_{p_J}^J\right] \quad (11)$$

where u_i^j denotes the i^{th} dominant eigenvector in the j^{th} node. The final step to estimate P is to test for dependent columns, which is equivalent to determine the rank of $\overline{A}$, i.e.,

$$\hat{P} = rank(\overline{A}) \quad (12)$$

The estimate of P is shown to be consistent from the results we'll present in the next section. Due to the lack of space, a formal proof of consistency is reported in [18].

3. RESULTS

For the purpose of performance evaluation, the aforementioned technique was applied to simulated multichannel neural recordings. The signals used in the simulations consisted of template neural spike waveforms extracted from experimental data. The multiple events for each source were sorted using the technique reported in [2] and averaged to reduce noise. We used up to five distinct spike waveforms as illustrated in Fig.1. The noise used was extracted from *signal free* observations from the same experiments from which spike templates were extracted. Typically, we had M = 16 channels, N is in the order of a few thousands snapshots. Each analysis window was chosen so that the number of distinct sources may not exceed the number of channels processed simultaneously. A subset of the number of snapshots of noisy 16 channel data is illustrated in Fig.2. From these illustrations, it is clear that the source signals possess a high degree of temporal correlation and coherence across channels.

Performance was assessed using Monte Carlo simulation and compared to MDL and AIC. Fig. 3 illustrates the performance for the AIC criterion, MDL criterion, and our proposed Multi-Resolution Sphericity Test (MRST) method for two SNR conditions. Clearly, the AIC has the lowest performance compared to all the other methods. The MRST outperforms both the MDL and the AIC. Moreover, the estimates are consistent as the number of snapshots becomes large and the superiority of the proposed MRST remains intact despite degradation in SNR.

4. CONCLUSION

We have introduced a new technique to determine the number of sources in a multichannel correlated signal and noise environments. The technique is based on estimating the signal subspace dimension in each subband of the wavelet decomposition using a sphericity test for the equality of the smallest rank ordered eigenvalues. The source detection was accordingly performed in each subband separately whereby allowing each source signal to be projected onto the subspace spanned by the corresponding wavelet basis in each subband. This enabled each source to have a variable *mode strength* in each node, thereby permitting extra degrees of freedom for the sphericity test to asymptotically attain a desired performance by minimizing the probability of miss (underestimation of the number of sources). The probability of false alarm (overestimation of P) was minimized by rank reduction of an augmented matrix containing all the dominant eigenvectors in all subbands, thus eliminating redundant eigenvectors from the signal subspace estimates.

One additional advantage gained is the tolerance allowed for erroneous decisions in the sphericity tests. If a noise component predominates in a certain subband to the extent of masking a weak source, then the sphericity test may underestimate P in that subband. In this case, the overall estimate of P is minimally affected due to the fact that this source may be better represented in another subband where the noise has less masking effect, thereby amounting to a stronger mode that will depend on how much correlation exists between that source and the corresponding wavelet basis. The thresholds for the sphericity tests are determined using the fact that the likelihood function in eq.(5) statistically approaches a *Chi-square* distribution with $(M-p)^2 - 1$ degrees of freedom. One limitation currently under investigation is the perturbations to the eigenvectors under *severely* low SNR (< 4 dB in neural environment) composing the augmented matrix $\overline{A}$ that may cause overestimation of the rank. Nevertheless, the performance was significantly improved in moderate to low SNR over existing methodologies.

To sum up, the source detection technique introduced in this work has some clear advantages over existing source detection techniques in correlated signal and noise environments. Existing source detection schemes in the case of correlated noise assume the noise covariance to be either known, or can be estimated reliably, or have certain banded structure [8]. Other schemes developed for coherent signals [7,9] assume that the mixing matrix is known and that it is full rank. In our case, these assumptions are invalid due to the nature of the neural signal environment discussed earlier. We've also demonstrated the robustness to degradation in SNR, which is a desirable feature in case of weak source presence. Current work is aimed to assess the performance using sub-array processing to speed up the computational load.

5. REFERENCES

[1] Oweiss K.G., Anderson D.J. "A New Approach to Array Denoising," *Proc. of the 34th Asilomar Conf. on SSC*, pp. 1407-1411, October 2000.

[2] Oweiss K.G., Anderson D. J., " A Multiresolution Generalized Maximum Likelihood Approach for the Detection of Unknown Transient Multichannel Signals in Colored Noise with Unknown Covariance," *Proc. of ICASSP'2002*, vol. 3, pp.2993-2996, May 2002

[3] Oweiss K.G., Anderson D.J., "A New Technique for Blind Source Separation using Subband Subspace Analysis in Correlated Multichannel Signal Environments," *Proc. of ICASSP'01*, pp. 2813-2816., May 2001

[4] Oweiss K. G., Anderson D.J., " MASSIT-Multiresolution Analysis of Signal Subspace Invariance Technique: A Novel Algorithm for Blind Source Separation," *Proc. of the 35th Asilomar Conf. on SSC*, pp. 819-823, November 2001

[5] Akaike H., "A New Look at the Statistical Model Identification," *IEEE Trans. On Automatic Control*, vol. AC-19, pp.716-723, 1974.

[6] Rissanen J., "Modeling by Shortest Data Description," *Automatica*, vol. 14, pp. 465-471, 1978.

[7] Wax M., Ziskind I., "Detection of the Number of Coherent Signals by the MDL principle," *IEEE Trans. on ASSP*, vol. 37, No.8, pp. 1190-1196, Aug. 1989.

[8] Fuchs J.J., "Estimation of the Number of Signals in the Presence of Unknown Correlated Sensor Noise," *IEEE Trans. On SP,* vol. 40, No.5, pp. 1053-1061, May 1992

[9] Wu Y., Tam K. and Li F., " Determination of the Number of Sources with Multiple Arrays in Correlated Noise Fields," *IEEE Trans. On SP,* vol.50, No.6, pp. 1257-1260, June 2002

[10] Costa P., Grouffaud J., Larzabal P. and Clergeot H., "Estimation of the Number of Signals from Features of the Covariance Matrix: A Supervised Approach," *IEEE Trans. On SP*, vol. 47, No. 11, pp. 3108-3115, Nov. 1999

[11] Brcich R., Zoubir A. and Pelin P., "*IEEE Trans. On SP*, vol.50, No.2, pp. 206-215, Feb. 2002

[12] Lawley D., "Tests of Significance for the Latent Roots of Covariance and Correlation Matrices," *Biometrika*, vol. 43, pp. 128-136, 1956

[13] Oweiss K.G., "Multiresolution Analysis of Multichannel Neural Recordings in the Context of Signal Detection, Estimation, Classification and Noise Suppression," *Ph.D. Dissertation*, Univ. of Michigan, Ann Arbor, 2002

[14] Fee M.S. *et. al.,* "Automatic Sorting of Multiple Unit Neuronal Signals in the Presence of Anisotropic and Non-Gaussian Variability," *J. Neuroscience methods*, vol. 69, pp. 175-188, 1996

[15] James A.T., "Tests of Equality of Latent Roots of the Covariance Matrix," *Multivariate Analysis-II,* P.R. Krishnaiah, Ed. New York: Academic, pp. 205-218, 1969

[16] Coifman R. and Wickerhauser M., "Entropy Based Algorithms for Best Basis Selection," *IEEE Trans. on IT*, vol. 38: pp. 713-718, March 1992

[17] Mauchley J., "Significance Test for Sphericity of a Normal *n*-variate Distribution," *Ann. Math. Statistics,* vol. 11, pp. 204-209, 1940

[18] Oweiss K.G., "On the Determination of the Signal Subspace Dimension in Multichannel Correlated Signal and Noise Environments," *in preparation*.

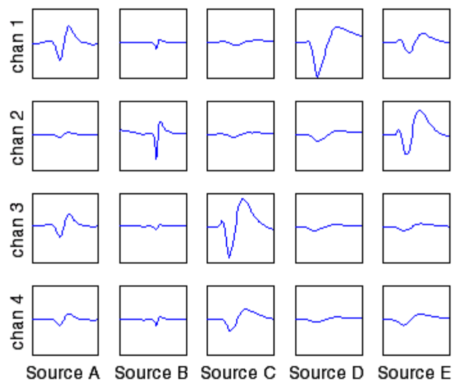

Fig. 1. Source spatial distribution for five distinct neural sources extracted from the same neural recording experiment at different occurrence times from a subarray of 4 channels of a 16 channel array of electrodes.

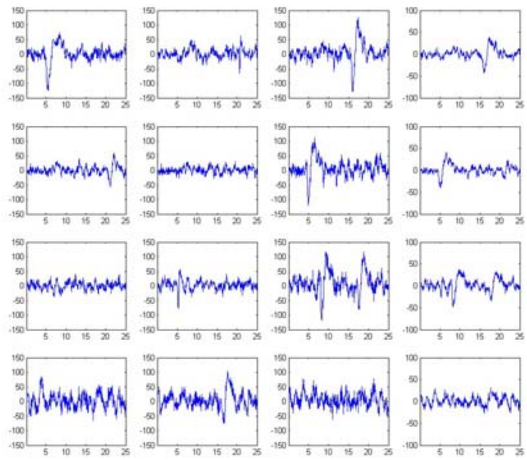

Fig. 2. A noisy subset of array snapshots (25 msec. at a sampling rate of 20 KHz) for a 16 channel array

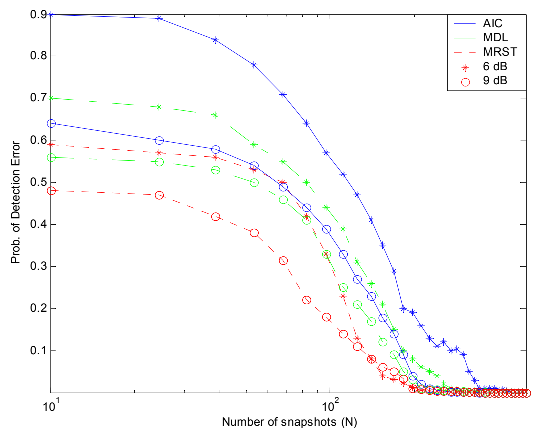

Fig. 3. Probability of Detection error vs. the number of array snapshots for the AIC, MDL and MRST.