



# A UNIFORM OVERSAMPLED FILTER BANK APPROACH TO INDEPENDENT COMPONENT ANALYSIS

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## ABSTRACT

We present a new approach to perform independent component analysis (ICA) for convolved mixtures. This approach is based on filter banks, and a simplified network efficiently performs ICA with decimated signals in each subband. Decimation provides much less computational complexity and faster convergence speed than the time domain approach. Furthermore, the approach does not have a performance limitation of the frequency domain approach, and it is able to select the number of filters in the filter bank regardless of reverberation. With an oversampled filter bank, adaptive parameters can be adjusted without any information of other subbands, and the approach is suitable for parallel processing. We verify the effectiveness of the filter bank approach through simulations on adaptive noise cancelling.

## 1. INTRODUCTION

Independent component analysis (ICA) is a signal processing method to express multivariate data as linear combinations of statistically independent random variables [1]. Especially, performing ICA for convolved mixtures has become important because it has prospective signal processing applications such as speech enhancement, telecommunications, and medical signal processing in real-world situations [1]. There are several different approaches to ICA, and a simple and biologically plausible adaptive learning algorithm has been proposed with entropy maximization by Bell and Sejnowski [2]. To deal with convolved mixtures, the algorithm has been extended to deconvolution of mixtures in the time domain [3] and the frequency domain [4].

The time domain approach requires intensive computations with a long reverberation, and it shows slow convergence speed especially for colored input signals [5]. The computational load can be reduced by the frequency domain approach, in which multiplication at each frequency bin replaces convolution operation in the time domain. However, the performance of the frequency domain approach is limited because a long frame size is required to

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cover a long reverberation whereas the number of learning data in each frequency bin decreases as the frame size increases [6]. Additionally, delay difference among mixing filters may cause a severe error in the block processing of the frequency domain approach.

With an intention to overcome these disadvantages of the time domain and the frequency domain approaches, we propose a filter bank approach to ICA. In this approach, input signals are split into a number of subbands. Then, each subband signal is decimated and used for ICA. Since the ICA algorithm in each subband is basically same as the time domain approach, the filter bank approach does not have any performance limitation of the frequency domain approach. In addition, decimation of the subband signals saves some computations and makes convergence speed faster than the time domain approach.

Many researchers have studied adaptive filtering in subbands mostly with the least-mean-square (LMS) type algorithm [7]. If input signals are decomposed by critically sampled filter banks, cross adaptive filters between adjacent bands are required to compensate for the distortion caused by aliasing, or spectral gaps are required in order not to have aliasing. However, the cross adaptive filters introduce additional adaptive parameters and may cause slow convergence speed or a poor performance. On the other hand, the spectral gaps distort reconstructed signals. These problems are resolved by oversampled filter banks in which aliasing is negligible by using filters with high stopband attenuation [5].

For verification, we apply the filter bank approach to adaptive noise cancelling and compare it with other approaches.

## 2. INDEPENDENT COMPONENT ANALYSIS

ICA is a linear transform of multivariate data to make the resulting random vector as statistically independent as possible [1]. Let us consider a set of unknown independent components,  $\{s_i(n)\}$ , such that the components are zero-mean and mutually independent. If the mixing of independent components involves convolution and time-delays,

$$x_i(n) = \sum_{j=1}^N \sum_{k=0}^{K-1} a_{ij}(k) s_j(n-k), \quad (1)$$

where  $x_i(n)$  is an observation, and  $a_{ij}(k)$  denotes a mixing filter coefficient.

To obtain independent components, a feedforward architecture can be considered as

$$u_i(n) = \sum_{j=1}^N \sum_{k=0}^{K-1} w_{ij}(k) x_j(n-k), \quad (2)$$

where adaptive filters  $w_{ij}(k)$  supposedly make outputs  $u_i(n)$  reproduce the original independent components  $s_i(n)$ . Entropy maximization algorithm provides learning rules of the adaptive filter coefficients as follows [3]:

$$\begin{aligned} \Delta \mathbf{W}(0) &\propto [\mathbf{W}^T(0)]^{-1} - \varphi(\mathbf{u}(n)) \mathbf{x}^T(n), \\ \Delta w_{ij}(k) &\propto -\varphi(u_i(n)) x_j(n-k), \quad k \neq 0, \\ \varphi(u_i(n)) &= -\frac{\frac{\partial p(u_i(n))}{\partial u_i(n)}}{p(u_i(n))}, \end{aligned} \quad (3)$$

where  $\mathbf{W}(0)$  is the matrix composed of zero-delay weights, and  $\mathbf{u}(n)$  and  $\mathbf{x}(n)$  denote a set of estimated independent components and the observation vector, respectively.  $\varphi(\cdot)$  is called a score function, and  $p(u_i)$  denotes the probability density function of  $u_i$ .

In the time domain approach, the computational load is large with a long reverberation to compute the convolution of long filters, and the convergence is slow especially for colored input signals such as speech signals [5].

Instead of the time domain approach, one can consider the frequency domain approach [4]. Here, the convolved mixtures can be expressed as

$$\mathbf{X}_f(n) = \mathbf{A}_f \mathbf{S}_f(n), \quad \forall f \quad (4)$$

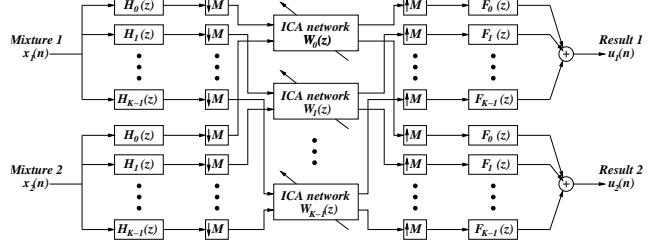
where  $\mathbf{X}_f(n)$  and  $\mathbf{S}_f(n)$  are vectors, each of which is a frequency component of the mixtures and the original independent components at frequency  $f$ , respectively, and  $\mathbf{A}_f$  denotes a matrix containing the elements of the frequency transforms of the mixing filters at frequency  $f$ , respectively. From eq. (4), the original independent components can be recovered by applying ICA to instantaneous mixtures at each frequency bin. In order to deal with complex-valued data, a score function was proposed as

$$\varphi(u_i) = -\frac{\frac{\partial p(|u_i|)}{\partial |u_i|}}{p(|u_i|)} \exp(j \cdot \angle u_i). \quad (5)$$

Applying natural gradient by Amari *et al.* [8], the learning rule becomes

$$\Delta \mathbf{W} \propto [\mathbf{I} - \varphi(\mathbf{u}) \mathbf{u}^H] \mathbf{W}. \quad (6)$$

The frequency domain approach is computationally more efficient because convolution operation in the time domain can be replaced by multiplication at each frequency bin. In addition, the parameters of unmixing networks can be adapted in an orthogonal domain. Since the adaptation of one parameter does not interfere with other parameters, the frequency domain approach can improve convergence. However, a long frame size is required to cover a long reverberation. To maintain computational efficiency and obtain data which are independent of those from adjacent frames, the frame shift has to increase as the frame size increases. Therefore, the number of data in each frequency bin decreases. Since this causes insufficiency of data to learn adaptive parameters, the performance of ICA will be poor [6]. In addition, different time



**Fig. 1.** A  $2 \times 2$  network for the oversampled filter bank approach to ICA

ranges of independent components are combined to form mixtures if the delay difference among mixing filters is large. Therefore, when one performs ICA with blocks which contain different time ranges of the independent components, an inferior performance is obtained from the block processing of the frequency domain approach.

### 3. A FILTER BANK APPROACH TO INDEPENDENT COMPONENT ANALYSIS

With oversampled filter banks in which the decimation factor is smaller than the number of analysis filters, aliasing can be neglected with each filter which has a high stopband attenuation. To implement oversampled filter banks, we consider uniform complex-valued filter banks [5]. In these filter banks, the analysis filters  $h_k(n)$  are obtained from a real-valued low-pass prototype filter  $q(n)$  by a generalized discrete Fourier transform (GDFT),

$$h_k(n) = e^{j \frac{2\pi}{K} (k+1/2)(n-(L_q-1)/2)} \cdot q(n), \\ k = 0, 1, \dots, K, \quad n = 0, 1, \dots, L_q - 1, \quad (7)$$

where  $L_q$  is the length of  $q(n)$ . Complex-conjugate and time-reversed versions of the analysis filters are selected for the synthesis filters

$$f_k(n) = \tilde{h}_k(n) = h_k^*(L_q - n - 1). \quad (8)$$

The prototype filter can be designed by iterative least-squares algorithm with a cost function which considers reconstructiveness and stopband attenuation. In addition, we can implement these filter banks efficiently by employing polyphase representation of the analysis and synthesis filters, and using properties of the GDFT [5].

When we perform ICA in the oversampled filter banks, adaptive filter coefficients in each subband can be adjusted without any information of other subbands because of negligible aliasing of the filter banks [5]. Fig. 1 shows a  $2 \times 2$  network for the oversampled filter bank approach to ICA. The input signals which are mixtures of unknown independent components are split into subband signals by analysis filters. Then, each subband signal is subsampled by factor  $M$ . In each subband, a usual ICA algorithm for convolved mixtures independently processes the subsampled signals. Each output signal from the ICA network is expanded, and independent components can be reconstructed from the subband output signals through synthesis filters after fixing permutation and scale. In the frequency domain approach, several methods have been proposed for fixing permutation and scale [4, 9]. Some methods can be also used in the filter bank approach such as filter coefficient

normalization for fixing scale and envelope correlation method for fixing permutation [9].

The ICA algorithm in each subband is basically the time domain approach. To perform ICA with a complex-valued filter bank, we use the polar-coordinate based score function of eq. (5) in each subband. The learning rules of the adaptive filter coefficients are changed to deal with complex-valued data. Using a feedforward network in each subband, the learning rules are

$$\begin{aligned}\Delta \mathbf{W}(0) &\propto [\mathbf{W}^H(0)]^{-1} - \varphi(\mathbf{u}(n))\mathbf{x}^H(n), \\ \Delta w_{ij}(k) &\propto -\varphi(u_i(n))x_j^*(n-k), \quad k \neq 0.\end{aligned}\quad (9)$$

Since ICA in each subband is based on the time domain approach, the filter bank approach to ICA does not have problems with the frequency domain approach such as a performance limitation and a mismatch of the block processing. Since a simplified ICA network can be used to process decimated input signals at the subsampled rate in each subband, computational complexity is considerably reduced for a long adaptive filter length. Let's assume that we have  $L_a$  adaptive filter coefficients for the fullband time domain approach. Then, approximately  $2L_a$  multiplications are required to compute outputs of the ICA network and update the filter coefficients. On the other hand, approximately  $L_a/M$  filter coefficients in each subband for the filter bank approach are sufficient to span corresponding time ranges of the fullband time domain approach with decimation factor  $M$ . For a large number of the adaptive filter coefficients, the computational load to compute subband signals and reconstructed signals is negligible because the adaptive filter length is much greater than the analysis filter lengths and the synthesis filter lengths. For real input signals, we need to keep only lower half subbands of a uniform GDFT filter bank since the remaining parts become complex conjugates. With a  $K$ -channel oversampled filter bank, approximate total number of multiplications,  $N_{fb}$ , is

$$N_{fb} \approx \frac{1}{M} \cdot 4 \cdot \frac{K}{2} \cdot 2 \cdot \frac{L_a}{M} \quad (10)$$

because one complex multiplication equals to four real multiplications, and the filter bank approach processes signals at the subsampled rate in each subband. Therefore, the number of multiplications for the filter bank approach is approximately  $2K/M^2$  times as large as that of the fullband time domain approach.

Each subband can independently compute subband output signals and adapt filter coefficients of the ICA network without other subbands. So, the filter bank approach is apt to parallel processing. Additionally, the approach is able to choose the number of subbands regardless of reverberation. Because it uses input signals more whitened by decimation than the fullband time domain approach, it improves convergence of the adaptive filter coefficients of the ICA network.

#### 4. ADAPTIVE NOISE CANCELLING BASED ON THE FILTER BANK APPROACH

Adaptive noise cancelling is an approach to reduce noise based on reference signals [10]. In conventional adaptive noise cancelling systems, the primary input signal is a combination of signal  $s(n)$  and noise  $r_0(n)$  in which the signal  $s(n)$  is transmitted over a channel from a signal source to a sensor and a noise  $r_0(n)$  is added in the sensor from a noise source. Another sensor receives a noise signal  $r_1(n)$  through another channel from the same noise source,

and this sensor acts as the reference input. The goal is to get a system output  $u(n)$  in which noise components are removed as much as possible.

Although the most popular algorithm for noise cancellation is LMS algorithm, the performance of the adaptive noise cancelling systems can be improved by ICA which can consider higher-order statistics [10]. By the entropy maximization, learning rules of adaptive filter coefficients  $w(k)$  can be derived as

$$\Delta w(k) \propto \varphi(u(n))r_1(n-k), \quad (11)$$

where the output  $u(n)$  is

$$u(n) = s(n) + r_0(n) - \sum_{k=1}^K w(k)r_1(n-k). \quad (12)$$

The filter bank approach to ICA can be applied to the adaptive noise cancelling system. The primary and the reference input signals are split into subband signals by analysis filters, and all input subband signals are decimated by factor  $M$ . In each subband, an adaptive filter is independently adjusted without any information of other subbands. Then, the system output  $u(n)$  is reconstructed from noise-cancelled subband output signals via synthesis filters after expansion. The adaptation algorithm in each subband is essentially same as the learning rule in eq. (11). In addition, noise components in the primary input signal are cancelled using the reference input signal, and the desired signal is observed in the output without distortion. Therefore, the filter bank approach to the adaptive noise cancelling does not have permutation and scale indeterminacy which ICA generally has.

#### 5. EXPERIMENTAL RESULTS

We have performed experiments on the adaptive noise cancelling with the proposed filter bank approach. Two real-recorded speech data were used as the signal and the noise sources. Each signal had 10 second length at 16kHz sampling rate. It is known that speech signal approximately follows Laplacian distribution. Therefore,  $\text{sgn}(\cdot)$  was used as the score function  $\varphi(\cdot)$ . Experimental results were compared in terms of signal-to-noise ratio (SNR), which we define as the power of components caused by the signal source versus that caused by the noise source at the output  $u(n)$  in the typical adaptive noise cancelling system,

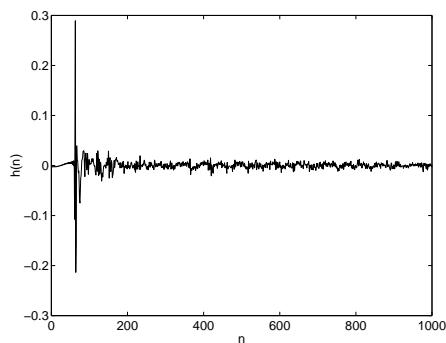
$$\text{SNR} = \frac{\langle (s(n))^2 \rangle}{\langle (r_0(n) - \sum_{k=1}^K w(k)r_1(n-k))^2 \rangle}. \quad (13)$$

The mixing filters from the signal source to the primary input and from the noise source to the reference input were simple linear scales. The scale values were chosen to obtain desired initial SNRs. For the mixing filter from the noise source to the primary input, we have used a measured filter in a normal office room as shown in Fig. 2. Assuming that the primary and the reference inputs receive signals with appropriate powers, we have normalized mixture powers properly (generally to 1), and this prevents severe mismatching between recovered signal levels and the nonlinear function. (We have not normalized mixture powers to match output levels exactly with the nonlinear function.) All experiments were conducted with several step sizes, and the best performance is shown.

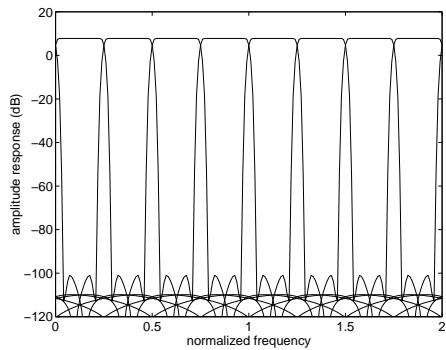
Fig. 3 shows the frequency response of analysis filters of an eight-channel oversampled filter bank using the GDFT. The filter

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**Fig. 2.** The mixing filter measured in a normal office room



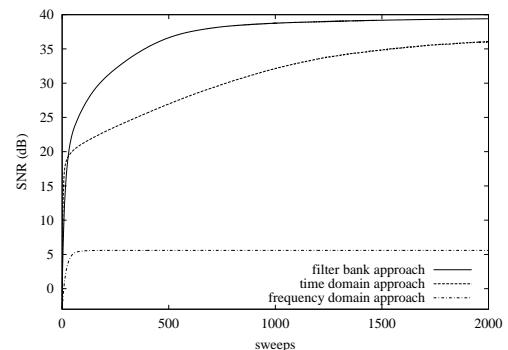
**Fig. 3.** Frequency response of analysis filters of a uniform eight-channel oversampled filter bank

bank was designed for alias-free decimation by factor  $M = 6$ , and it was constructed from a prototype filter with 192 taps.

Fig. 4 shows a learning curve of the proposed filter bank approach to adaptive noise cancelling with the oversampled filter bank above. The number of taps of the adaptive filter coefficients was  $\lceil \frac{1024}{M} \rceil$  in each subband. For comparison, we have also applied the time domain and the frequency domain approaches to the adaptive noise cancelling and displayed learning curves in Fig. 4. We have used 1024 taps of the adaptive filter coefficients for the time domain approach. In the frequency domain approach, the frame size was 8192, and the frame shift was a half of the frame size. SNRs of the frequency domain approach are much lower than those of the other two approaches. This is because the frequency domain approach has a performance limitation which comes from the severe block processing error and the contradiction between the long reverberation covering and the insufficient learning data. The learning curves in Fig. 4 show that the filter bank approach has much faster convergence speed than the time domain approach. Experiments for a car and a music noise showed the same tendency.

## 6. CONCLUSION

In this paper, we proposed a filter bank approach to perform ICA for convolved mixtures. The approach provides a much better performance than the frequency domain approach. Additionally, it enables us to select the number of filters of the filter bank independent of reverberation. It also has much less computational



**Fig. 4.** Learning curves of the three different approaches to adaptive noise cancelling

complexity and faster convergence speed than the time domain approach. Adaptive parameters can be adjusted without any information of other subbands by performing the filter bank approach with an oversampled filter bank. Therefore, the approach is suitable for parallel processing. Simulation results on adaptive noise cancelling showed the effectiveness of the filter bank approach.

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