PARAMETER ESTIMATION OF SPIRAL WAVES FROM ATRIAL ELECTROGRAMS

Yuping Xiao[1], *Yibin Zheng*[1], *Donald A. Jordan*[2], *Joseph G. Akar*[3] *and David E. Haines*[3]

[1]Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, VA
[2]Department of Mechanical and Aerospace Engineering, University of Virginia, Charlottesville, VA
[3]School of Medicine, University of Virginia, Charlottesville, VA

ABSTRACT

The problem of retrieving spiral wave parameters (frequency, radial velocity, and center location) using a minimal number (4) of spatial sensors is considered. The problem has the important application of localization of spiral wave sources of atrial fibrillation from basket catheter electrograms. Numerical simulations demonstrate that our algorithm works effectively for a wide range of parameters, and for spiral waves generated by a cellular automaton model of cardiac wave propagation.

1. INTRODUCTION

Atrial fibrillation (AF) is a common cardiovascular disease that affects nearly 1% of the population and up to 5% of the population over 80 years old [1]. A 2.5-fold increase in AF cases is expected by the year 2050. AF is associated with an increased risk of stroke and mortality, impaired exercise tolerance, fatigue and heart failure. AF is characterized by rapid (>400 beats/minute) irregular electrical excitation of the atrial cardiac tissue leading to inefficient pumping of blood from the atria to the ventricles. While the precise physiological mechanisms of initiation and maintenance of AF remain elusive [2] there is increasing evidence that AF is driven by localized organized sources of electrical activity [2-3]. This has been clinically observed and an important therapeutic intervention of AF is tissue ablation [3-4], where a lesion is created with a specialized catheter using radiofrequency energy in order to disrupt the electrical conduction from an organized source of AF. Success of this therapy depends critically on the ability to locate the source, characterize the nature of the source and create a lesion that interrupts electrical conduction from the source. Therefore, significant improvements in AF therapy can be gained by improving the ability to detect such sources.

The localized source hypothesis is but one of several hypothesized mechanisms of AF [5]. It has gained enough acceptance, however, that part of clinical practice is to map the electrical activity of the atria to locate a potential AF source. One method of clinical testing is to insert a basket catheter (figure 1) into the atrium and measure the electrical activity with electrodes at a number of spatial locations (32 locations in the case considered here). While most sources have been shown to originate from the pulmonary veins [3], several other locations have also been implicated [4]. These rotors are thought to represent spiral waves, where the wave front propagation forms a closed loop and electrical activation propagates away from the center of rotation to excite the rest of the atrium. While spiral waves have been observed in some animal studies using optical mapping of excised hearts [6], they are very difficult to identify *in vivo*, since a spiral wave is very likely to be under-sampled with a basket catheter. In order to locate the center of a spiral wave and determine other parameters of the wave for an ablation procedure, parametric models of the wave are required for any estimation algorithm.

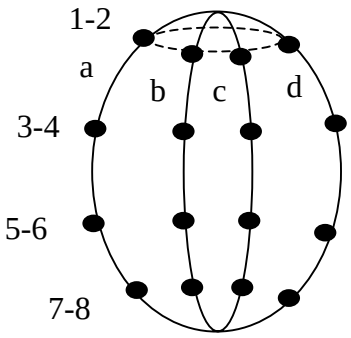

Figure 1: Basket Catheter Electrodes

Our approach to developing signal analysis techniques for better detection of sources of AF is to perform simulations of hypothesized sources on a planar surface, utilizing the cellular automaton (CA) modeling approach. The wave behavior of interest can be sampled as signals at a small number of locations, representative of electrode locations, and then algorithms can be developed to determine the parameters of the source. Validation of the algorithm is conducted by utilizing the full information available from the CA simulation.

2. CELLULAR AUTOMATON MODELS

Cardiac tissue is classified as an excitable medium, where a voltage excitation of sufficient magnitude initiates cardiac wave propagation, a process where individual cells "sense" the state of neighboring cells and "fire" once neighboring cells fire. Once a cardiac cell "fires", it undergoes a sequence of physiological processes that leads to contraction of the cell and ultimately the whole atrium contracts. The idea behind cellular automaton modeling is that an array of "cells" is defined as having a resting state and an excited state and the cells are designed to transition from the resting to the excited state based on the state of nearby cells. Wave propagation is initiated in the array and the resulting wave characteristics (wave speed, dispersion relationship, etc.) emerge from the designated cell properties and the cell-cell coupling. The resulting process very closely mimics the wave behavior in excitable media, such as cardiac tissue. Advantages of CA models are that they are much simpler and faster than models based on differential equations, because the underlying cellular mechanisms need not be computed. CA models of myocardium have actually been in existence for quite some time since the pioneering work of Wiener and Rosenbluth [7]. and by Moe et al. [8]. Over the last several decades, CA models of excitable media have experienced a number of developments and also occur in various forms. In our work, we have adopted the CA algorithm given by Gerhardt, et al. [9] (GST Algorithm). More recently, the GST algorithm has been examined as one of several computational approaches in the framework computational biology [10] and has been implemented for simulation of cardiac excitation pattern of a three-dimensional anatomically correct heart [11].

The cellular automaton model consists of a planar array of cells that contain two integer state variables, the excitation variable, u_t, that can take on the values of 0 or 1 and a recovery variable, v_t, that consists of a refractory period, a time over which the cell can not be re-excited and a relatively refractory period where the cell can be excited only with a sufficiently large excitation. The state variables are

$$\begin{aligned} u_t &\in \{0,1\} \\ v_t &\in \{0,\ldots,V_{\max}\} \end{aligned} \tag{1}$$

The excitation variable, u_t, transitions from 0 to 1 when the cell becomes excited. The recovery variable evolves in time according to the following set of rules, where g_{up} and g_{down} are integer step sizes that can be adjusted to get different CA model behaviors.

$$v_{t+1} = \begin{cases} \min\{v_t + g_{\text{up}}, V_{\max}\}, & \text{if } u_t = 1 \\ \max\{v_t - g_{\text{down}}, 0\}, & \text{if } u_t = 0 \end{cases} \tag{2}$$

At each step in the simulation a defined neighborhood of cells around each cell is examined and a given cell becomes excited at the next time step if it is both sufficiently recovered and if the number of cells in the defined neighborhood exceeds a given threshold, k^0_{exci}. If the cell is in the relatively refractory state then the excitability threshold is larger than k^0_{exci} and is a linear function of the recovery variable, v_t. Finally, we adapt a technique developed by Markus and Hess [12] to eliminate the anisotropy inherent in CA models by superimposing a small random "jitter" on the locations of the center of the cells in the rectangular array.

3. SPIRAL WAVE PARAMETER ESTIMATION

As pointed out in section 1, a spiral wave is spatially under-sampled, therefore parametric methods must be used to estimate its parameters. Let (x_0, y_0) (unknown) be the center of the spiral wave, and (x_i, y_i) (known) be the coordinates of the electrodes. First we introduce the polar coordinate system with respect to (x_0, y_0):

$$\begin{aligned} r &= \sqrt{(x-x_0)^2 + (y-y_0)^2} \\ \phi &= \arctan\left(\frac{y-y_0}{x-x_0}\right) \end{aligned} \tag{3}$$

then we model the spiral wave as Archimedean spiral [13] in polar coordinates as

$$f(r,\phi,t) = h(\phi - \alpha_0 r - \omega_0 t) \tag{4}$$

where $h(\bullet)$ is any 2π periodic function. The angular frequency of the wave is ω_0, the separation between successive turns (radial period) is $2\pi/\alpha_0$, and the radial velocity is ω_0/α_0.

Let $f_n(t)$ be the signal acquired by the *n*'th electrode. Then

$$f_n(t) = f(r_n, \phi_n, t) + w_n(t) \tag{5}$$

where $w_n(t)$ is noise. Note that (r_n, ϕ_n) are parameterized by (r_0, ϕ_0). The angular frequency ω_0 can be easily obtained by doing Fourier analysis of any channel. To estimate (r_0, ϕ_0) and α_0, consider the cross correlation of two channels,

$$R_{mn}(\tau) = \frac{1}{T}\int_0^T f_m(t)\, f_n(t-\tau)dt \qquad (6)$$

The location of the correlation peaks can tell us the relative delays between the two channels, subject to the ambiguity of multiple periods. That is $R_{mn}(\tau)$ will reach maxima at τ_{mn} when

$$(\phi_m - \alpha_0 r_m) - (\phi_n - \alpha_0 r_n) = \omega_0 \tau_{mn} + 2\pi k_{mn} \qquad (7)$$

Here k_{mn} is an integer. Therefore, if we have 4 sensors for a spiral wave, we could measure their relative time delays by cross correlation, and have 3 independent equations from (7). Suppose k_{mn} can be determined through other means and absorbed into τ_{mn}, then

$$\alpha_0 = \frac{\phi_m - \phi_n - \omega_0 \tau_{mn}}{r_m - r_n} \overset{\text{def}}{\equiv} \mathrm{A}_{mn}(x_0, y_0) \qquad (8)$$

From (8) we may obtain the simultaneous nonlinear equations

$$\mathrm{A}_{mn}(x_0, y_0) = \mathrm{A}_{m'n'}(x_0, y_0), \quad (m,n) \neq (m',n') \qquad (9)$$

which could be used to solve (x_0, y_0). We seek the least squares solution by minimizing the cost function

$$C(x_0, y_0) = \sum_{(m,n)\neq(m',n')} \left[\mathrm{A}_{mn}(x_0, y_0) - \mathrm{A}_{m'n'}(x_0, y_0)\right]^2 \qquad (10)$$

The minimization can be carried out by gradient descent or other standard optimization techniques. After (x_0, y_0) have been found, the estimate of α_0 can be obtained through equation (8).

One final issue is the determination of k_{mn}. In principle k_{mn} cannot be determined uniquely from the electrograms alone. This is essentially the aliasing phenomenon due to under-sampling. In our algorithm we use different trial values of k_{mn} to obtain possible solutions. However, in our simulations, we are able to resolve all ambiguous solutions by placing additional restrictions on the spiral wave parameters, such as not allowing the center of the wave to be too far away from the electrodes.

4. SIMULATIONS AND DISCUSSIONS

In this section, we demonstrate the accuracy and effectiveness of the proposed algorithm. Since there would be no practical way to determine the ground truth of spiral wave parameters of *in vivo* AF basket catheter electrograms, we choose to use simulated electrograms to test our algorithm.

4.1. Simulated Spiral Wave

Our first set of results pertains to estimating the angular frequency, radial velocity and center coordinates of a synthesized spiral. Here $h(t)$ is chosen as a periodic triangular pulse with $\omega_0 = 50$ to simulate the rise and fall of the signal voltage due to wave passage. An example configuration of the four electrodes relative to the synthesized spiral wave is shown in figure 2.

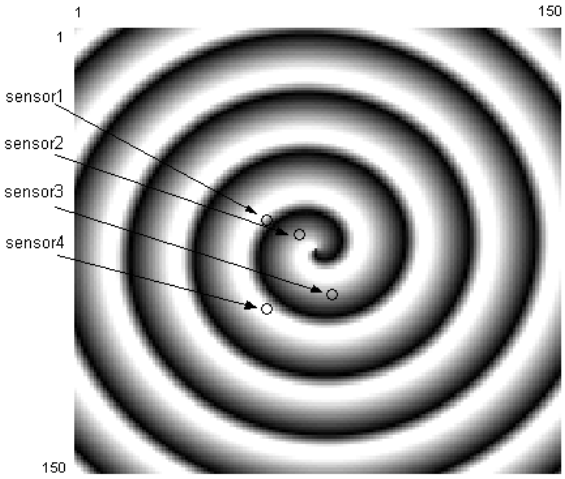

Figure 2: Electrode locations for spiral wave estimation

Table 1 shows a range of synthesized spiral wave parameters and their estimates by the algorithm. We observe that in all cases the parameters have been correctly recovered to within good accuracy.

Table 1: Spiral wave parameters and their estimates

α_0	$[x_0, y_0]$	$\hat{\alpha}_0$	$[\hat{x}_0, \hat{y}_0]$
0.3142 (0.1π)	[75, 75]	0.3179	[74.8541, 74.8198]
	[66, 63]	0.3076	[65.9622, 62.8683]
0.1885 (0.06π)	[75, 75]	0.1844	[75.1447, 75.3469]
	[66, 63]	0.1855	[66.1119, 63.0849]
0.0942 (0.03π)	[75, 75]	0.0916	[74.6622, 75.3036]
	[66, 63]	0.0998	[66.3612, 63.2027]

4.2. Cellular Automaton Model

The second set of our results pertains to spiral waves generated by the cellular automaton model. Here no ground-truth values of the spiral wave parameters are known, however a high-resolution image of the wave can be obtained from the model. Figure 3(a) is a snapshot of the recovery variable, v_t, which clearly displays spiral wave behavior. We position the 4 electrodes at the same

locations as in the first set of results, and estimate the spiral wave parameters. We then reconstruct a spiral wave using the estimated parameters. As shown in figure 3(b), the reconstructed activation distribution looks remarkably consistent with the map generated by the CA model. This result validates our Archimedean spiral model for cardiac wave propagation and demonstrates the effectiveness of our parameter estimation algorithm.

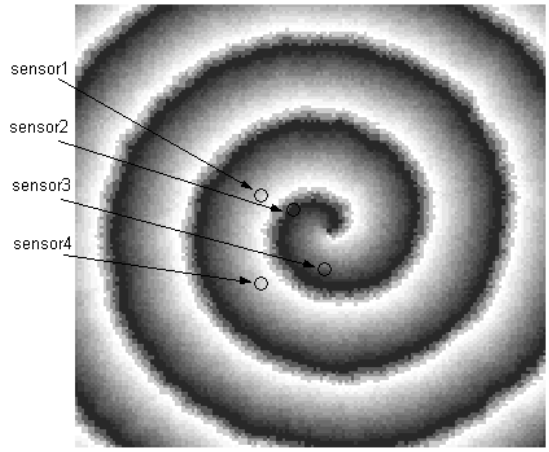

Figure 3(a): Activation map obtained from CA model

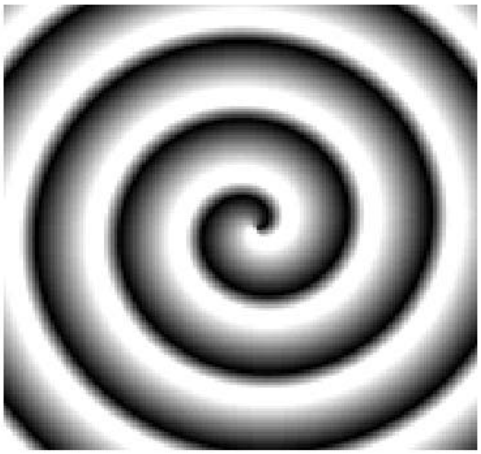

Figure 3(b): Reconstructed spiral wave

In summary, we have developed a robust algorithm to identify parameters of a spiral wave using only 4 sensors. The algorithm has an immediate application in basket catheter electrogram analysis for atrial fibrillation patients. Numerical simulation results have confirmed the algorithm's effectiveness and viability.

We gratefully acknowledge the support of a Harvie Award for Interdisciplinary Research from the Cardiovascular Research Center at UVA.

5. REFERENCES

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