NEW SIGNAL SUBSPACE DIRECTION-OF-ARRIVAL ESTIMATOR FOR WIDEBAND SOURCES

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ABSTRACT

Signal subspace methods in DOA estimation for wideband sources require preprocessing to find initial values which are close enough to the true values or to convert sensor outputs into desired forms. The preprocessing procedure should be carefully done lest it introduce some distortion. Failure to find proper initial values may prevent convergence of the estimator or cause biases in the estimator. The proposed method detects uncorrelated wideband sources using the signal subspace and the noise subspace of decomposed wideband signals. It does not require any initial values. The only preprocessing is narrowband decomposition of the sensor output which is very common in other wideband methods. Computer simulation showed that the proposed method has less bias and comparable variance than CSSM with small focussing errors.

1. INTRODUCTION

Direction-of-arrival (DOA) is one of the main parameters to estimate in array signal processing. There have been many approaches to estimate this parameter more easily and/or more precisely. Among those, signal subspace methods are well known for their attractive performance. Since the MUSIC estimator was published [1], many methods which utilize subspace concepts have been proposed. MUSIC and its descendants are used for narrowband sources whose bandwidth is very narrow relative to the carrier frequency. Many methods have been proposed to apply these narrowband signal subspace methods to wideband sources [2, 3, 4]. The main difficulty in wideband DOA estimation is that the signal subspace is different for different frequency bands. Most of the previous approaches were concerned with how to combine information from different frequency bins into one signal or noise subspace so that we can apply narrowband methods. These methods show better performance than incoherent methods that average independent DOA estimates from several different frequency bins. However, the generated coherent subspace might have a "focussing error" which is generated in the process of focussing. When the focussing angle is not same as the DOA, there is always bias in the coherent signal subspace methods (CSSM) [5]. The beamforming invariance technique, which requires a field of view (FOV) rather than a focussing angle, can not detect as many signals as CSSM [4]. In the following sections, we propose a new method which does not require any preprocessing, focussing, or FOV.

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2. ESTIMATOR

2.1. Array model

A calibrated M-sensor uniform linear array is considered with P far-field wideband sources ($P < M$). We assume that the frequency bands of the sources are known and those bands are overlapped to some extent. The sensor output can be easily decomposed into several frequency bins by a filter bank or the discrete fourier transform (DFT). The sensor output at frequency ω_i is

$$\mathbf{x}(\omega_i) = \mathbf{A_i}(\boldsymbol{\theta})\mathbf{S}(\omega_i) + \boldsymbol{\eta}(\omega_i) \quad (1)$$

where ω_i lies in the source's band, $\boldsymbol{\theta}$ is a vector consist of P DOAs, and $\boldsymbol{\eta}$ is white Gaussian noise. The $M \times P$ matrix $\mathbf{A_i}(\boldsymbol{\theta})$ is

$$\mathbf{A_i}(\boldsymbol{\theta}) = [\ \mathbf{a}(\omega_i, \theta_1)\ \mathbf{a}(\omega_i, \theta_2)\ \ldots\ \mathbf{a}(\omega_i, \theta_P)\ ], \quad (2)$$

and the array manifold $\mathbf{a}(\omega_i, \theta_j)$ is

$$\mathbf{a}(\omega_i, \theta_j) = [\ e^{-j\omega_i\tau_j}\ e^{-j2\omega_i\tau_j}\ \ldots\ e^{-jM\omega_i\tau_j}\ ]^T \quad (3)$$

where $\tau_j = d\sin\theta_j/c$, and c is the speed of propagation. The superscript T denotes transpose. The relation between array manifolds of different frequencies and DOAs is as follows.

$$\mathbf{a}(\omega_x, \theta_x) = \mathbf{\Phi}(\omega_y, \theta_y)\mathbf{a}(\omega_z, \theta_z) \quad (4)$$

where

$$\mathbf{\Phi}(\omega_\mathbf{y}, \theta_\mathbf{y}) = \text{diag}\{e^{-j\omega_y\tau_y}, e^{-j\omega_y 2\tau_y}, \ldots, e^{-j\omega_y M\tau_y}\}. \quad (5)$$

Then, the relations between frequencies and DOAs are

$$\begin{aligned} \omega_x &= \omega_y + \omega_z, & (6)\\ \sin\theta_x &= \frac{\omega_y}{\omega_x}\sin\theta_y + \frac{\omega_z}{\omega_x}\sin\theta_z. & (7) \end{aligned}$$

2.2. Algorithm

Let $\mathbf{x_i} = \mathbf{x}(\omega_i)$ where ω_i for $i = 1, 2, \ldots, K$ is within the frequency bands of all sources. The number of frequency bins, K, is constrained by

$$K \geq \max\left\{\frac{M}{M-P}, 3\right\} \quad (8)$$

The reason for this constraint will be clear at the end of this section. Define the covariance matrix $\mathbf{R_i}$

$$\mathbf{R_i} = E[\mathbf{x_i}\mathbf{x_i^H}] = \mathbf{A_i}\mathbf{R_{s,i}}\mathbf{A_i^H} + \sigma^2\mathbf{I} \quad (9)$$

where $\mathbf{R_{s,i}} = E[\mathbf{s}(\omega_\mathbf{i})\mathbf{s}(\omega_\mathbf{i})^\mathbf{H}]$, and the superscript H denotes transpose conjugate. Assume that $\mathbf{R_{s,i}}$ is full rank. Then, the range of the P largest eigenvectors is the same as range of $\mathbf{A_i}$ [1]. Let $\mathbf{e}_{i,1}, \mathbf{e}_{i,2}, \ldots, \mathbf{e}_{i,P}$ be ordered eigenvectors of $\mathbf{R_i}$ from the largest to the smallest. Define the matrices $\mathbf{F_i}$ and $\mathbf{W_i}$ as

$$\begin{aligned} \mathbf{F_i} &= [\ \mathbf{e}_{i,1}\ \mathbf{e}_{i,2}\ \ldots\ \mathbf{e}_{i,P}\ ], & (10)\\ \mathbf{W_i} &= [\ \mathbf{e}_{i,P+1}\ \mathbf{e}_{i,P+2}\ \ldots\ \mathbf{e}_{i,M}\ ]. & (11) \end{aligned}$$

Then,

$$\begin{aligned} \mathcal{R}\{\mathbf{F_i}\} &= \mathcal{R}\{\mathbf{A_i}\} & (12)\\ \mathcal{R}\{\mathbf{W_i}\} &= \mathcal{N}\{\mathbf{A_i^H}\} & (13) \end{aligned}$$

where $\mathcal{R}\{\cdot\}$ and $\mathcal{N}\{\cdot\}$ denote the range space, and the null space, respectively.

Theorem 1 *Let $\Delta\omega = \omega_j - \omega_i$. Then,*

$$\mathcal{R}\{\mathbf{\Phi}(\Delta\omega, \theta_o)\mathbf{F_i}\} = \mathcal{R}\{\mathbf{A_j}(\hat{\boldsymbol{\theta}})\} \quad (14)$$

where $\hat{\boldsymbol{\theta}} = [\hat{\theta}_1 \ldots \hat{\theta}_P]^T$, and

$$\hat{\theta}_p = \arcsin\left\{\frac{\omega_i}{\omega_j}\sin\theta_p + \frac{\Delta\omega}{\omega_j}\sin\theta_0\right\} \quad (15)$$

Proof: From (12), we know that there exists a full rank $P \times P$ matrix $\mathbf{T_i}$ such that

$$\mathbf{F_i} = \mathbf{A_i}\mathbf{T_i} \quad (16)$$

Therefore,

$$\begin{aligned} \mathbf{\Phi}(\Delta\omega, \theta_o)\mathbf{F_i} &= \mathbf{\Phi}(\Delta\omega, \theta_o)\mathbf{A_i}\mathbf{T_i}\\ &= \mathbf{\Phi}(\Delta\omega, \theta_o)[\mathbf{a_i}(\omega_i)\ldots\mathbf{a_p}(\omega_i)]\mathbf{T_i}\\ &= [\mathbf{a}(\omega_j, \hat{\theta}_1)\ldots\mathbf{a}(\omega_j, \hat{\theta}_P)]\mathbf{T_i}\\ &= \mathbf{A_j}(\hat{\boldsymbol{\theta}})\mathbf{T_i} & (17) \end{aligned}$$

This proves Theorem 1.

Theorem 1 tells us that a signal subspace of one frequency bin can be linearly transformed into that of other frequency with modified DOAs given by (7). If θ_o in (14) is the same as one of the θ_p in the original signal subspace, this DOA is preserved in the new signal subspace. This feature is a key idea of the new DOA estimator.

Theorem 2 *Assume that (8) holds. Let $\mathbf{E_i}$ for $i = 2, \ldots, K$ be $P \times (M-P)$ matrices such that*

$$\mathbf{E_i} = \mathbf{F_1^H}\mathbf{\Phi}(\mathbf{\Delta\omega_i}, \theta)^\mathbf{H}\mathbf{W_i} \quad (18)$$

where $\Delta\omega_i = \omega_i - \omega_1$. Define the $P \times K(M-P)$ matrix $\mathbf{D}$ *such that*

$$\mathbf{D} = [\mathbf{E_2}\ \mathbf{E_3}\ldots\ \mathbf{E_K}] \quad (19)$$

Then,

$$rank\{\mathbf{D}(\theta)\mathbf{D}(\theta)^\mathbf{H}\} = \begin{cases} P-1 & \text{if } \theta = \theta_p\\ P & \text{if } \theta \neq \theta_p \end{cases} \quad (20)$$

Proof: From (13),

$$\mathbf{a_p}(\omega_\mathbf{i})^\mathbf{H}\mathbf{W_i} = \mathbf{0^T} \quad (21)$$

for all $p = 1, 2, \ldots, P$. By Theorem 1, we know that

$$\mathbf{F_1^H}\mathbf{\Phi}(\mathbf{\Delta\omega_i}, \theta)^\mathbf{H} = \mathbf{T_1^H}\mathbf{A_i}(\hat{\boldsymbol{\theta}}_\mathbf{i})^\mathbf{H} \quad (22)$$

where

$$\hat{\theta}_{i,p} = \arcsin\left\{\frac{\omega_1}{\omega_i}\sin\theta_p + \frac{\omega_i - \omega_1}{\omega_i}\sin\theta\right\} \quad (23)$$

If $\theta = \theta_p$,

$$\hat{\theta}_{2,p} = \ldots = \hat{\theta}_{K,p} = \theta_p \quad (24)$$

Therefore,

$$\begin{aligned} \mathbf{E_i} &= \mathbf{T_1^H}\begin{bmatrix} \mathbf{a^H}(\omega_i, \hat{\theta}_1)\mathbf{W_i}\\ \vdots\\ \mathbf{a^H}(\omega_i, \theta_p)\mathbf{W_i}\\ \vdots\\ \mathbf{a^H}(\omega_i, \hat{\theta}_P)\mathbf{W_i} \end{bmatrix} & (25)\\ &= \mathbf{T_1^H}\begin{bmatrix} *\\ \mathbf{0^T}\\ * \end{bmatrix} \leftarrow p^{th} \text{ row} & (26) \end{aligned}$$

When there are multiple sources, even if $\theta \neq \theta_p$, there is some chance that one of the $\hat{\theta}$'s could be the same as θ_j. In other words, it could happen that $\hat{\theta}_{i,p} = \theta_j$. This ambiguity can be removed by using at least three frequency bins. Even if $\hat{\theta}_{i,p} = \theta_j$, $\hat{\theta}_{l,p} \neq \theta_j$ since $\omega_i \neq \omega_l$. This is the reason why K should be at least three. Therefore, only for $\theta = \theta_p$, $\mathbf{D}$ becomes

$$\mathbf{D} = [\mathbf{E_2} \ \dots \ \mathbf{E_K}] = \mathbf{T_1^H} \begin{bmatrix} * \\ \mathbf{0^T} \\ * \end{bmatrix} \leftarrow p^{th} \text{ row} \qquad (27)$$

and it loses rank. Since the rank of $\mathbf{DD^H}$ is p if and only if the rank of $\mathbf{D}$ is p, equation (20) holds [6]. Therefore, we can find DOAs by looking for P different θs that make $\mathbf{DD^H}$ singular.

2.3. Process

In most cases, the covariance matrix is unavailable. Instead, an estimated covariance matrix would be used. Due to errors in estimating the covariance matrix, $\mathbf{DD^H}$ always has full rank. Therefore, the condition number should be used to find a singularity of $\mathbf{DD^H}$ instead of finding its rank. To estimate covariance matrices, the total observation time is divided into several identical blocks.

The estimation process is: 1) Divide the sensor output into J identical blocks. 2) Find the DFT of J blocks. 3) Find $\hat{\mathbf{x}}_\mathbf{i}$ for pre-selected ω_i, $i = 1, \dots, K$. 4) Find the signal subspace $\hat{\mathbf{F}}_\mathbf{1}$ and the noise subspace $\hat{\mathbf{W}}_\mathbf{i}$ for $i = 2, \dots, K$ by eigendecompostion of the covariance matrices, $\hat{\mathbf{R}}_\mathbf{i}$. 5) Find $\hat{\mathbf{E}}_\mathbf{i}$ using (18). 6) Find $\hat{\theta}$ such that

$$\hat{\theta} = \arg\max_{\hat{\theta}} \boldsymbol{\kappa}\{\hat{\mathbf{D}}(\theta)\hat{\mathbf{D}}(\theta)^\mathbf{H}\} \qquad (28)$$

where $\boldsymbol{\kappa}\{\cdot\}$ denotes condition number, and $\hat{\mathbf{D}}$ is from (19). Note that we are looking for P peaks of (28) like MUSIC, not a global maximum.

2.4. Asymptotical results

According to [7], as J goes to infinity, $\hat{\mathbf{R}}$ converges to the true $\mathbf{R}$ and so do the eigenvectors. In that case, $\hat{\mathbf{D}}$ becomes $\mathbf{D}$. The bias of the estimator is asymptotically zero. Finding the variance of the estimator is not easy due to using the condition number.

3. SIMULATION

3.1. Simulation model

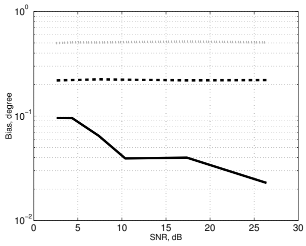
The new algorithm was tested by computer simulation for comparison with CSSM [2]. An eight-sensor uniform linear array was considered in this simulation. The source frequency band was $0 \sim 100$ Hz. The distance between sensors is half the wavelength at a center frequency of 50Hz, and the sampling frequency is three times the highest frequency. Three wideband sources are located at 8°, 33°, and 37°, respectively. The number of blocks is 100 and each of these blocks has 256 samples. For CSSM, 13 frequency bins were processed and the focussing frequency was 47 Hz. A rotational signal-subspace (RSS) focussing matrix was used [8]. The proposed method used 5 frequency bins for processing. In both methods, a 256-point DFT was used and it was assumed that the number of sources is known, or estimated correctly.

3.2. Results

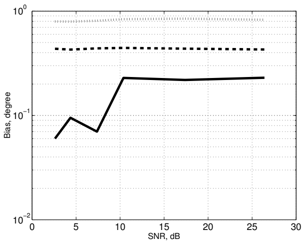
Figure 1 shows the mean of biases of the estimators for three sources after 100 Monte-Carlo simulations. The solid line represents the new method. The dotted line and the dashed line represent CSSM with 0.2° and 0.5° focussing error, respectively. The bias of the new method was the smallest, both at 8° and 33°. For the source at 37°, CSSM with 0.2° focussing error was the best. If the signal-to-noise ratio (SNR) is high and the focussing error small, the CSSM estimator tends to follow the focussing angle. The bias depends on the focussing error. However, if the focussing angle error is large, the estimator can not resolve close sources. This is due to the disagreement between the DOA and the focussing angle. These problems are more apparent when the SNR is large. Variances of the estimator showed different results. The new method showed the highest variance in all three sources. The other two cases showed similar variances. The root mean square (RMS) error would summarize the simulation results since it combines bias and variance together. Figure 2 shows the RMS error of the estimator. Except for the source at 37°, the proposed method has smaller RMS error.

4. CONCLUSION

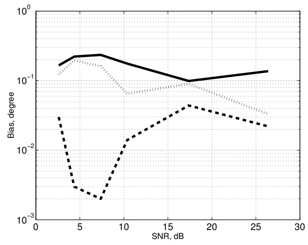
In this paper, we proposed a new DOA estimator for wideband signal sources. The main advantage of this new method is that it does not require any preprocessing or focussing which is a disadvantage of previously proposed methods and it is asymptotically unbiased. Computer simulation showed that the performance of the proposed method is better than or similar to that of CSSM with small focussing errors. Since the focussing error depends on the preprocessing methods as well as the SNR, we can not conclude that the proposed method outperforms CSSM in general. However, the new method would be preferred for applications



(a) At 8°



(b) At 33°



(c) At 37°

Fig. 1. Biases of the estimator at the three signal sources. The solid line, the dashed line, and the dotted line represent the new method, CSSM with 0.5° focussing error, and 0.2° focussing error, respectively.

where avoiding estimation bias is critical, or impinging sources are harmonic signals that have most of their energy at several discrete frequency bins.

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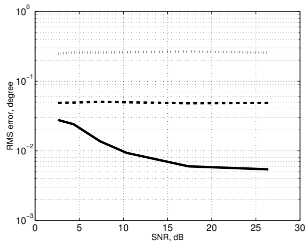
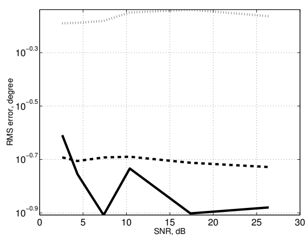
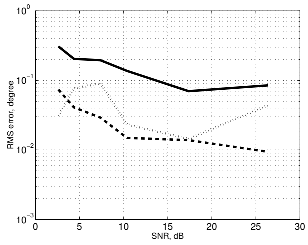


(a) At 8°



(b) At 33°



(c) At 37°

Fig. 2. RMS error of the estimator at the three signal sources. The legend is same as Fig. 1.

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