A Single Channel Approach to High Resolution Direction Finding and Beamforming

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I. Abstract

This paper presents an array processing approach that only requires the array outputs to be sampled one at a time by a single channel receiver. We note that the sequential sampling of the array outputs is equivalent to the periodic but non-uniform sampling of the source waveform. The derived array model is structurally similar to the conventional array model. This allows the direct application of most conventional high resolution DOA estimation and beamforming algorithms with the proposed approach. The feasibility of this approach is demonstrated with simulation data.

II. Introduction

Many high resolution direction of arrival (DOA) estimation and adaptive beamforming algorithms require the number of receiver channels to be matched to the number of antennas. This is an expensive requirement to meet in many applications. To reduce the implement cost, a number of methods based on the sequential sampling of the array outputs with a single channel receiver have been proposed. In [3] and [2], the array covariance matrix is estimated using weight perturbation method [6] and multi-output beamforming network, respectively. Due to their slow convergence in estimating the array covariance, they require large number of samples to achieve reliable DOA estimation. The methods devised in [1] and implicitly in [5] sequentially sample the array outputs at very high speed. This is to allow the approximation to the simultaneously sampled array outputs of the conventional approach possible by progressive phase shifting of the sequentially sampled array outputs.

Our approach is based on the recognition that the sequential sampling of the array outputs is equivalent, in its simplest form, to the periodic but non-uniform sampling of the source waveform and that the non-uniform sampling intervals are function of the DOA. Based on this observation, we derive an array model that is structurally similar to the standard array model. This array model allows the direct application of most conventional high resolution DOA estimation and adaptive beamforming algorithms. Unlike the methods devised in [1] [5], the proposed method can operate at lower sampling rate and demand smaller receiver bandwidth.

III. Problem Formulation and Data Model

Consider the single receiver array architecture shown in Figure 1. The single channel receiver sequentially samples the array outputs at a regular interval. The analog-to-digital (ADC) and the RF switches samples synchronously at the same rate[1]. The sampled signal, $x(nT)$, is given by

$$x(nT) = \alpha_i(\theta)\, s(nT - \tau_i(\theta)) + w(nT) \qquad (1)$$

where $i = n \bmod N$, $\{\alpha_i(\theta)\}_{i=0}^{N-1}$ are the complex-valued elemental response of the antenna, $\{\tau_i(\theta)\}_{i=0}^{N-1}$ are the differential time delays with respect to the reference antenna, $s(t)$ and $w(t)$, respectively, are the source waveform and receiver noise. Under noiseless condition, the sequence of sampled output

$$\begin{aligned} \{ \; \cdots \quad & x(0), \quad x(T), \quad \cdots \\ & x((N-1)T), \quad x(NT), \quad \cdots \\ & \cdots \quad x((2N-1)T), \quad \cdots \; \} \end{aligned} \qquad (2)$$

is given by

$$\begin{aligned} \{ \; \cdots \quad & \alpha_0(\theta)\, s(-\tau_0(\theta)), \\ & \alpha_1(\theta)\, s(T - \tau_1(\theta)), \quad \cdots \\ & \cdots \quad \alpha_{N-1}(\theta)\, s((N-1)T - \tau_{N-1}(\theta)), \\ & \alpha_0(\theta)\, s(NT - \tau_0(\theta)), \quad \cdots \\ & \alpha_{N-1}(\theta)\, s((2N-1)T - \tau_{N-1}(\theta)) \quad \cdots \; \} \end{aligned} \qquad (3)$$

[1] In practice, their sampling times may need to be skewed so that there are sufficient time for the RF switch to settle.

From (3), we recognize that the sequential sampling of the antenna array outputs is equivalent to the sampling of the source waveform in a periodic but non-uniform manner. Furthermore, the sampled waveform is periodically modulated. The periodic modulation is due to the antenna elemental response $\boldsymbol{\alpha}(\theta) = \left[\begin{array}{ccc} \alpha_0(\theta) & \cdots & \alpha_{N-1}(\theta) \end{array}\right]^T$ *while the periodic but non-uniform sampling is due to the time difference of arrivals observed by the antenna array,* $\boldsymbol{\tau} = \left[\begin{array}{ccc} \tau_0(\theta) & \cdots & \tau_{N-1}(\theta) \end{array}\right]^T$.

We write the noiseless output of the RF switch as

$$\begin{aligned} x(t) &= s(t) \times \sum_n \alpha_{i(n)}(\theta)\,\delta\left(t - nT - \tau_{i(n)}(\theta)\right) \\ &= \sum_{i=0}^{N-1} x_i(t) \end{aligned}$$

where $x_i(t) = \alpha_i(\theta)\, s(t - \tau_i(\theta)) \sum_k \delta(t - kNT - iT)$, $i(n) = n \bmod N$ and $\delta(t)$ is the Dirac function. Let $s(t) = m(t)\exp(j\omega_c t)$, where $m(t)$ and ω_c, respectively, are the modulation waveform and carrier frequency. We have

$$\begin{aligned} x_i(t) &= \alpha_i(\theta)\, m(t - \tau_i(\theta))\, e^{j\omega_c(t-\tau_i(\theta))} \times \\ & \quad \sum_k \delta(t - kNT - iT). \end{aligned}$$

and the corresponding Fourier transform is given by

$$\begin{aligned} & X_i(\omega) = \alpha_i(\theta) \times \\ & \sum_k M\left(\omega' - \frac{k\omega_s}{N}\right) e^{-j\omega_k \tau_i(\theta)} e^{-j\omega' iT} \end{aligned} \qquad (4)$$

with $\omega' = \omega - \omega_c$, $\omega_s = \frac{2\pi}{T}$ and $\omega_k = \omega - \frac{k\omega_s}{N}$. The spectrum of $x(t)$ is simply given by

$$\begin{aligned} & X(\omega) \\ &= \sum_{i=0}^{N-1} X_i(\omega) \\ &= \sum_{i=0}^{N-1} \alpha_i(\theta) \sum_k M\left(\omega' - \frac{k\omega_s}{N}\right) e^{-j\omega_k \tau_i(\theta)} e^{-j\omega' iT}. \end{aligned} \qquad (5)$$

Assuming that $m(t)$ is bandlimited such that $M(\beta) = 0$ for $|\beta| \geq \frac{\omega_s}{N}$, then for some integer k_0 and ω such that $M(\mu) \neq 0$ where $\mu = \omega - \omega_c - \frac{(k_0+l)\omega_s}{N}$ for $l = \begin{array}{ccc} 0, & 1 & \cdots & N-1 \end{array}$, we have

$$\begin{aligned} & X(\omega_l) \\ &= \sum_{i=0}^{N-1} \alpha_i(\theta)\, M(\mu)\, e^{-j\omega_\mu \tau_i(\theta)} e^{-j\left(\mu + \frac{(k_0+l)\omega_s}{N}\right) iT} \end{aligned}$$

with $\omega_\mu = \mu + \omega_c$. Note that the assumption of $M(\beta) = 0$ for $|\beta| \geq \frac{\omega_s}{N}$ will require the received signal be oversampled by at least N times. One should also note that $X(\omega) = X(\omega + p\omega_s)$, $\forall p \in Z$, and in practice, $X(\omega)$ is bandlimited by the sampling pulse width.

Collecting $X(\omega_0), \cdots, X(\omega_{N-1})$ into a vector, we have

$$\begin{aligned} \mathbf{X}(\omega_0) &= \left[\begin{array}{cccc} X(\omega_0) & X(\omega_1) & \cdots & X(\omega_{N-1}) \end{array}\right]^T \\ &= \mathbf{Z}(k_0)\,\mathbf{P}(\mu)\,\mathbf{a}(\theta, \omega_\mu)\, M(\mu) \end{aligned} \qquad (6)$$

where

$$\begin{aligned} \mathbf{Z}(k_0) &= \left[\begin{array}{cccc} 1 & z^{k_0} & \cdots & z^{(N-1)k_0} \\ 1 & z^{k_0+1} & \cdots & z^{(N-1)(k_0+1)} \\ \vdots & \vdots & & \vdots \\ 1 & z^{k_0+N-1} & \cdots & z^{(N-1)(k_0+N-1)} \end{array}\right], \\ \mathbf{P}(\mu) &= \operatorname{diag}\left[\begin{array}{cccc} 1 & e^{-j2\pi\mu T} & \cdots & e^{-j2\pi\mu(N-1)T} \end{array}\right]^T \end{aligned}$$

with $z = e^{-j\left(\frac{T\omega_s}{N}\right)} = e^{-j\left(\frac{2\pi}{N}\right)}$ and $\mathbf{a}(\theta, \omega_\mu) = \left[\begin{array}{ccc} \alpha_0(\theta)\, e^{-j\omega_\mu \tau_0(\theta)} & \cdots & \alpha_{N-1}(\theta)\, e^{-j\omega_\mu \tau_{N-1}(\theta)} \end{array}\right]^T$ is the conventional array steering vector.

Clearly from (6), we have the following familar array signal model

$$\mathbf{X}(\omega_0) = \mathbf{a}_s(\theta, \omega_\mu)\, M(\mu) + \mathbf{W}(\omega_0)$$

where

$$\mathbf{a}_s(\theta, \omega_\mu) = \mathbf{Z}(k_0)\,\mathbf{P}(\mu)\,\mathbf{a}(\theta, \omega_\mu), \qquad (7)$$

$\mathbf{W}(\omega_0) = \left[\begin{array}{ccc} W(\omega_0) & \cdots & W(\omega_{N-1}) \end{array}\right]^T$ and $W(\omega)$ is the Fourier transform of $w(t)$. Assuming that $w(t)$ is zero mean white noise, then it can be shown that the noise vector $\mathbf{W}(\omega_0)$ is a zero mean Gaussian vector and its covariance is $\sigma^2\mathbf{I}$.

Extending to multi-source scenario, we have

$$\begin{aligned} \mathbf{X}(\omega_0) &= \sum_{k=0}^{d-1} \mathbf{a}_s(\theta_k, \omega_\mu)\, M_k(\mu) + \mathbf{W}(\omega_0) \\ &= \mathbf{A}_s(\boldsymbol{\Theta})\,\mathbf{m}(\mu) + \mathbf{W}(\omega_0) \end{aligned} \qquad (8)$$

where $\mathbf{A}_s(\boldsymbol{\Theta}) = \left[\begin{array}{ccc} \mathbf{a}_s(\theta_0, \omega_\mu) & \cdots & \mathbf{a}_s(\theta_{d-1}, \omega_\mu) \end{array}\right]$ and $\mathbf{m}(\mu) = \left[\begin{array}{ccc} M_0(\mu) & \cdots & M_{d-1}(\mu) \end{array}\right]^T$. We can notice from (8) that one can apply most conventional high resolution DOA estimation and adaptive beamforming algorithms with the proposed approach.

We note from (7) that $\mathbf{a}_s(\theta, \omega_\mu)$ is a linearly transformed version of the conventional array steering vector. The following theorem states the linear independence of $\mathbf{A}_s(\boldsymbol{\Theta})$ in relation to the conventional array manifold.

Theorem 1 *Consider any collection of distinct d DOAs,* $\mathbf{\Theta} = \left[\begin{array}{ccc} \theta_0 & \cdots & \theta_{d-1} \end{array}\right]^T$. *Then the steering vectors* $\mathbf{A}_s(\mathbf{\Theta}) = \left[\begin{array}{ccc} \mathbf{a}_s(\theta_0, \omega_\mu) & \cdots & \mathbf{a}_s(\theta_{d-1}, \omega_\mu) \end{array}\right]$ *defined in (7) are linearly independent if the corresponding conventional array steering vectors* $\mathbf{A}(\mathbf{\Theta}) = \left[\begin{array}{ccc} \mathbf{a}(\theta_0, \omega_\mu) & \cdots & \mathbf{a}(\theta_{d-1}, \omega_\mu) \end{array}\right]$ *from same array geometry are linearly independent.*

Proof: See [4]

IV. Application Examples to DOA Estimation and Beamforming

For all our examples, we use a 4-element uniformly spaced circular array with half-wavelength inter-antenna spacing and consider two independent signal impinging the antenna array from $\theta_1 = 50°$ and $\theta_2 = 60°$. The signal bandwidth is fixed at 4kHz. The complex received signals are generated from first principles according to (1) and the noise samples, $w(nT)$, are drawn from a Gaussian distribution. We divide the time series $\{x(nT)\}$ of (2) into blocks of equal length. We use an 8-channel DFT filterbanks with a 32-tap Blackman-Harris prototype filter and with a decimation factor of 8 to determine $\mathbf{X}(\omega_0, m)$. The index m is included to indicate that it is computed from the m^{th} data block. The steering vectors are generated according to (7) with $\mu = 0$.

Figure 2 and 3 compare the MUSIC spectrum of the proposed approach with "time-shift" compensation approach[2] of [1] [5] at array outputs sampling frequencies of 32kHz and 1MHz. It can be seen that the proposed approach yields better results. The "time-shift" compensation approach could hardly resolve the sources even at high oversampling rate. Figure 4 plots the MUSIC spectrum of 10 independent trials of the proposed approach at 25dB SNR (defined as $\frac{E(|s(t)|^2)}{E(|w(t)|^2)}$). Each trial uses 128 snapshots.

Next we show that the proposed approach is suitable for beamforming with a simple example. We fix the array outputs sampling frequency at 32kHz. Figure 5 plots mixed waveform seen by one of the antenna (top panel) and the sources' waveform. The waveforms are estimated using $\widehat{s}_i(m) = \mathbf{a}_s^H(\theta_i, \omega_c)\, \mathbf{P}^{\perp}_{\mathbf{a}_s(\theta_{j \neq i}, \omega_c)} \mathbf{X}(\omega_0, m)$. As shown in Figure 6, the source waveforms are accurately recovered.

[2] "Time-shifting" compensation is achieved by phase-shifting the i^{th} antenna outputs by multiplying $\exp(-j\omega i T)$. T is the sampling interval.

V. Concluding Remarks

An important aspect of the proposed single channel based array processing approach is that it allows current high resolution direction finding and beamforming algorithms to be readily applied with little or no modifications. While it shares similar architecture as the methods in [1] [5], the proposed approach can operate at a much lower array sampling rate. As the sequential sampling of the array outputs spreads the signal over the sampling bandwidth (c.f. (5)), the receiver bandwidth will need to be as large as the array outputs sampling rate. As shown in simulation examples, the array outputs sampling rate needed by [1] [5] is many times larger than the signal bandwidth. This will impose a large operating bandwidth requirement on the receiver which may negate the advantages offered by a single channel solution. Further results of this development will be reported in [4].

VI. Acknowledgement

The author is grateful to Mr Quek Gim Pew, Defence Science and Technology Agency, Singapore, for providing the idea to the proposed array processing approach. A US Patent of the proposed approach is pending.

References

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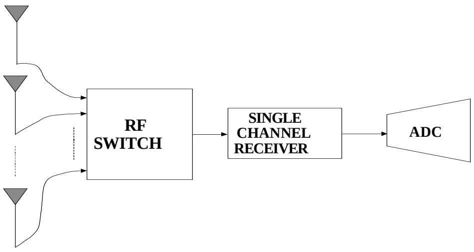

Figure 1 –

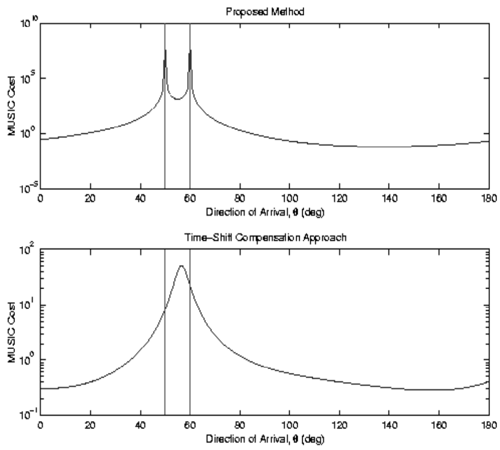

Figure 2 – f_s = 32kHz. SNR = ∞

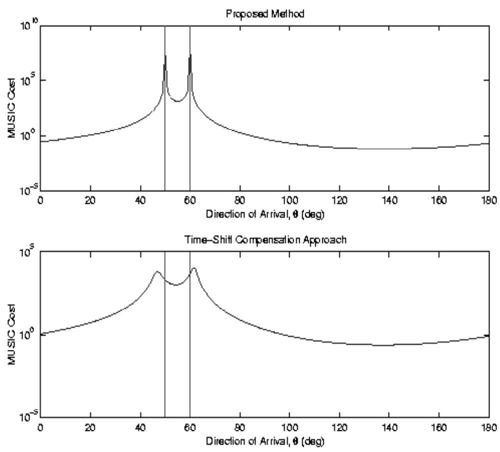

Figure 3 – f_s =1MHz. SNR = ∞

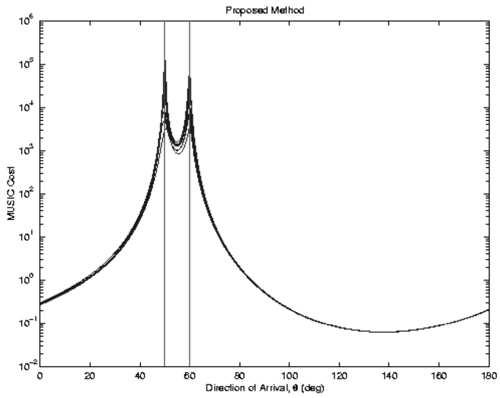

Figure 4 – f_s =32kHz. SNR = $25dB$.

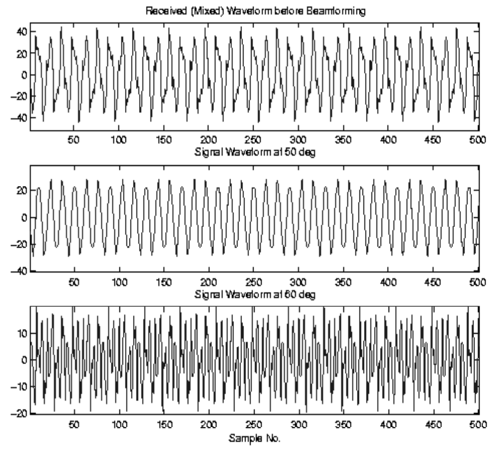

Figure 5 –

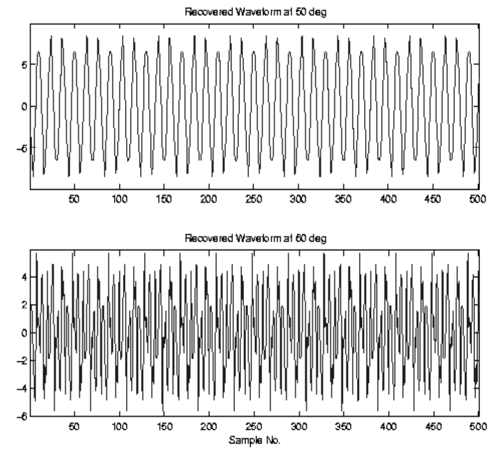

Figure 6 –