TOWARD INTELLIGENT SENSORS — RELIABILITY FOR TIME DELAY BASED DIRECTION OF ARRIVAL ESTIMATES

Tuomo Pirinen, Pasi Pertilä and Ari Visa

Tampere University of Technology, Institute of Signal Processing
Korkeakoulunkatu 1, 33720 Tampere, Finland
{tuomo.pirinen, pasi.pertila, ari.visa}@tut.fi

ABSTRACT

Increasing demand for automatic large area surveillance has made methods estimating direction of arrival (DOA), especially acoustic methods, an interesting topic. In these systems false alarms and faults are a disturbing factor. This paper presents a reliability criterion and a new method to diminish these errors. In very low signal-to-noise ratio conditions there are seldom any means to improve the performance of a DOA estimator. In these cases it is important to have some information about the estimation reliability. Our focus is on wideband signals propagating as planar waves in three dimensional space. If the estimation method makes no assumptions on the signal propagation speed, it is possible to compute a reliability measure for obtained estimates. With this reliability measure we are able to decide whether the data produced by a time delay based DOA estimation system is usable or not. Indeed, with this criterion an estimation system can state its own reliability.

1. INTRODUCTION

In automation there are some trends. Firstly, the number of sensors is increasing. Secondly, the sensors are equipped with Internet addresses and they are connected Local Area Networks. Thirdly, there is a need to monitor processes and restricted areas.

In monitoring of restricted areas or in surveillance one of the main approaches is to estimate the direction of arrival (DOA) of acoustic waves. The goal of DOA estimation is often to determine a bearing for a source of waveforms. Practically, DOA estimation is done through determination of the direction of propagation for source emitted signals. This direction of propagation is opposite to the DOA, and under certain circumstances DOA approximates the actual source bearing with sufficient precision. Therefore, applications of DOA estimation are related to localization of signal sources, e.g. speakers as in [1]. Acoustic DOA applications can also be used in automatic surveillance of large areas and they are robust to visibility conditions, which explains their use in various military applications, as in [2].

There exist several sophisticated methods for estimating the DOA of propagating planar wavefronts efficiently and precisely [3, 4]. However, many of these methods become unusable in low SNR conditions, as noise corrupts the estimation results and observation reliability is compromised. Traditionally, methods have been divided into two main categories: conventional and parametric. Conventional methods (e.g. beamforming and Fourier-based methods) are often preferred over parametric methods (e.g. MUSIC and Capon methods), because they are not as sensitive to variations in noise and signal parameters as parametric methods [5]. With conventional methods the performance and accuracy of estimation is not as good as with parametric methods, but this inaccuracy is often acceptable — especially when considering wideband signals — due to robustness and simplicity of conventional approaches.

In moderate noise conditions there exist some pre- and post-processing methods [6] to enhance and partially separate source signals from unwanted noise. When noise is dominant in observed signals and its properties are unknown and time varying, improvement of SNR to an acceptable level is seldom possible. If estimation reliability can not be improved, it is important to be able to measure the reliability of obtained estimates. The ability to measure the reliability of obtained estimates makes it possible to a sensor system to self diagnose, which is a new and desired property.

This research is aimed at finding a reliability measure for DOA estimates. Among many conventional approaches to the DOA problem we wanted to utilize one which gives unambiguous estimates into three dimensional space without assumptions on source signals. Moreover, we wanted to keep the number of sensors as small as possible. This paper focuses on sound waves propagating in air, but the introduced reliability measure is independent of type of waves or medium provided that the direction and speed of propagation are constant within the sensor array.

2. ESTIMATION METHOD

The above constraints for estimator can be met, for example, by a simple four sensor system. Time delays between sensors are estimated and from these delays the DOA can be solved. One example of such a robust estimator for wideband signals and its variations are described in [7, 8, 9, 10]. Observed signals are assumed to obey the signal model given in [7]

$$s(\boldsymbol{x}, t) = \sum_l A_l \, e^{i\omega_l \left(t - \boldsymbol{k}^T \boldsymbol{x}\right)} \tag{1}$$

where $\boldsymbol{x}$ is the observation location in three dimensional space, t is time and A_l is the amplitude of frequency component ω_l. Vector $\boldsymbol{k}$ is the propagation vector and it is closely related to the wavenumber vector $\boldsymbol{\kappa}$ defined in [11]

$$\boldsymbol{k} = [k_x \; k_y \; k_z]^T = \frac{\boldsymbol{\kappa}_l}{\omega_l} \tag{2}$$

An estimate for propagation vector $\boldsymbol{k}$ can be obtained using four sensors as follows. First, estimate time delays between all sensor pairs to form the time delay vector $\hat{\boldsymbol{t}}$. Time delays can be estimated with several methods and the choice of method is a compromise between computational complexity and performance. If SNR is high, a low-complexity and accurate estimator can be used [12]. As we are focused on low SNR, we utilize the traditional direct correlation estimator, because it has the best performance in noisy conditions [13]. Our choice of a simple time delay estimator for time delays restricts us to observe only the most dominant signal source. An estimate for DOA results from the time delays as a solution of a linear equation [7]

$$\boldsymbol{X}\hat{\boldsymbol{k}} = \hat{\boldsymbol{t}} \quad \Leftrightarrow \quad \hat{\boldsymbol{k}} = \boldsymbol{X}^{-1}\hat{\boldsymbol{t}} \tag{3}$$

where $\boldsymbol{X}$ is a matrix describing the sensor vectors, i.e. vectors between utilized sensors. Inverse solution in (3) requires the sensor matrix $\boldsymbol{X}$ to be full rank.

3. ERROR SOURCES

Errors can result into estimate $\hat{\boldsymbol{k}}$ in (3) for two reasons: there are errors in sensor locations or in time delays. If the sensor vector matrix is not exactly known, we have only an estimate with some error $\boldsymbol{E_X}$

$$\hat{\boldsymbol{X}} = \boldsymbol{X} + \boldsymbol{E_X} \tag{4}$$

If time delays are known precisely, this error propagates to the DOA estimate through equation (3)

$$\hat{\boldsymbol{k}} = \boldsymbol{k} + \boldsymbol{e_k} \tag{5}$$

where the actual error can be shown to be of the form

$$\boldsymbol{e_k} = -\boldsymbol{X}^{-1}\boldsymbol{E_X}\left(\boldsymbol{X} + \boldsymbol{E_X}\right)^{-1}\boldsymbol{t} \tag{6}$$

In most cases the sensor array and individual sensors are stationary, and therefore the matrix $\boldsymbol{E_X}$ is constant and error becomes only a function of time delays. Errors in sensor vector matrix should be easily detected and avoided if sensor locations are measured with sufficient precision. Thus, we assume from now on that the sensor vector matrix is exactly known. Errors in time delays are more difficult to deal with. Consider an erroneous time delay estimate and the resulting error in DOA estimate

$$\hat{\boldsymbol{t}} = \boldsymbol{t} + \boldsymbol{e_t} \quad \Rightarrow \quad \boldsymbol{e_k} = \boldsymbol{X}^{-1}\boldsymbol{e_t} \tag{7}$$

which is now a random process. Errors in time delays occur when noise corrupts the individual sensor signals and their coherence. An extreme source of errors in time delays are hardware malfunctions, e.g. broken sensors. In practical situations estimates become erroneous also when signals form additional non-dominant sources disturb the estimation. Small errors in time delays, such as quantization, can usually be compensated by some method. A simple option is to use all possible time delays and a least squares solution [7]

$$\hat{\boldsymbol{k}} = \left(\boldsymbol{X}^T\boldsymbol{X}\right)^{-1}\boldsymbol{X}^T\hat{\boldsymbol{t}} \tag{8}$$

If one or more of time delay estimates contain large errors, then some form of selection is required to form the best possible set of time delays. These selection methods and their performance are demonstrated in [8] and [9].

If an optimum set cannot be constructed, e.g. all of the time delays are largely erroneous, the DOA estimate cannot be reliably computed. To obtain DOA estimates at a constant rate in all situations, the unreliability must be accepted and DOA computed with a non-optimum set of time delays. This is a feasible solution, if we can compute the amount of unreliability in the estimate, rather than simply discard the estimate. A criterion for the reliability can be computed as follows.

4. RELIABILITY CRITERION

Suppose the DOA estimation method has produced a three-dimensional estimate $\hat{\boldsymbol{k}}$ for the actual propagation vector $\boldsymbol{k}$ with error $\boldsymbol{e_k}$

$$\hat{\boldsymbol{k}} = \boldsymbol{k} + \boldsymbol{e_k} \quad \Leftrightarrow \quad \boldsymbol{e_k} = \hat{\boldsymbol{k}} - \boldsymbol{k} \tag{9}$$

Taking the norms of both sides of equation (9) and utilizing the triangle inequality we have

$$\begin{aligned} \|\boldsymbol{e_k}\| &= \|\hat{\boldsymbol{k}} - \boldsymbol{k}\| && (10) \\ &\geq \left| \|\hat{\boldsymbol{k}}\| - \|\boldsymbol{k}\| \right| && (11) \\ &= \left| \|\hat{\boldsymbol{k}}\| - \frac{1}{c} \right| && (12) \end{aligned}$$

since by definition $\|\boldsymbol{k}\| = \frac{1}{c}$. Clearly, the left side of equation (12) gives a lower bound for the norm of estimation error $\boldsymbol{e_k}$. Multiplication by propagation speed c gives the lower bound in the form

$$c\|\boldsymbol{e_k}\| \geq c\left| \|\hat{\boldsymbol{k}}\| - \frac{1}{c} \right| \tag{13}$$

We define the left side of equation (13) as the reliability criterion for DOA estimate, in simplified form

$$\epsilon = \left| c\|\hat{\boldsymbol{k}}\| - 1 \right| \tag{14}$$

The numerical value of defined criterion is ϵ independent of the wave propagation speed, due to normalization in equation (13). If the wave propagation speed c is known, the criterion ϵ can be computed.

With the criterion we can evaluate the reliability of our estimates. It is possible to analyze if our observations are error-free or corrupted by noise. In addition, we can extend our analysis to system functionality and use the reliability criterion to detect malfunctions in our sensor array. A fault in the sensor system will cause a systematic increase in the reliability criterion. For example, consider a physical disturbance which causes a displacement to at least one sensor in our array. In section 3 we assumed that the sensor locations are exactly known when we compute the DOA estimates. This does not restrict the use of reliability criterion in a displacement situation. It should be obvious that in case of sensor displacement the estimates will become erroneous, which can also be seen in the reliability criterion. Thus, the reliability criterion can be used to detect configuration errors in the estimation system.

5. SIMULATIVE RESULTS

In order to test the reliability criterion (14) a simulation was carried out. The simulation consisted of one point-like spherical source emitting white noise in free field. Source signal was received with a four microphone tetrahedron array. Propagation of signals was modeled as a linear path from source to each sensor, with $c = 343\ \frac{\mathrm{m}}{\mathrm{s}}$. This was done in order to show that the DOA estimation method is not strictly dependent on the planar wave assumption, and DOA can be estimated also for signals approximating the planar wavefront. Source was located at a distance of 200 times the

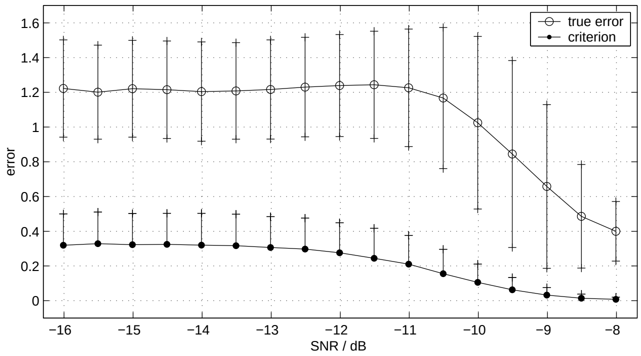

Fig. 1. Average values with mean absolute deviations for norm of true error and reliability criterion as a function of SNR from 3150 estimates per one SNR value. (Distribution of criterion values is not symmetric and thus we give the mean absolute deviations only for the positive side of averages.) The lower bound property or reliability criterion is clearly visible. An increase in true error results in an increase in the reliability criterion.

length of tetrahedron base. Tetrahedron shaped microphone array was chosen because it is the optimal array configuration for omnidirectional estimation with four microphones [14]. The number of sensors was desired to be as small as possible, and four sensors is the absolute minimum when performing three dimensional DOA estimation. Presence of noise was modeled through adding spatially uncorrelated white noise to each sensor signal. Figure 1 shows the behavior of reliability criterion (14) and the norm of true error (10) as a function of SNR. Values were averaged from 3150 estimates. Time delays were computed with the direct cross correlation method in frequency domain and DOA estimated with the least squares solution (8). No pre- or post-processing methods were used to optimize the results.

Figures 2 and 3 show an example of one five second simulative realization with SNR$= -10.5$ dB. The actual DOA estimates are shown in figure 2. For clarity, these DOA estimates are projected into two dimensional angles of arrival (AOA), azimuth and elevation. In this realization there are estimates which are correct up to precision of one degree and also clearly erroneous estimates. Figure 3 illustrates the values of reliability criterion (14) for estimates in figure 2. As is clearly seen, correct DOA estimates give a small value for reliability criterion and we can distinguish the erroneous ones. In figure (3) this discriminative property of reliability criterion is illustrated with a constant threshold level.

6. REAL DATA RESULTS

The simulation results on the reliability criterion were also validated with real data from outdoor measurements in a forested location. Sound source was a subwoofer emitting white noise at a distance of 1.0 km. Signals were recorded with a four sensor tetrahedron array, as in simulations. Figures 4 and 5 present five seconds of data from one measurement. In figure 4 are the DOA estimates projected into AOAs and values of reliability criterion for these estimates are in figure 5. This situation is much more complicated than the one presented in simulations, due to various error sources. These include natural background noise and reverberation. Also some form of reflections and multi-path propagation are present

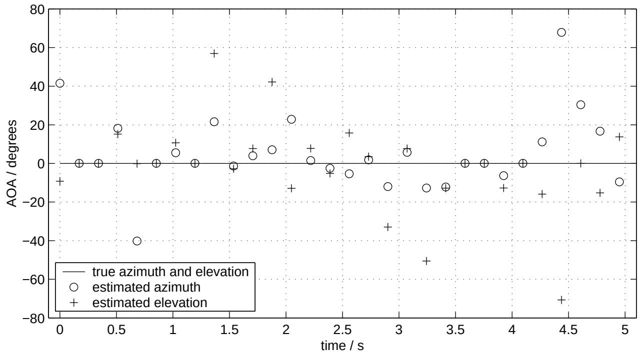

Fig. 2. Example of DOA estimates in one realization with SNR$= -10.5$ dB. Estimates are projected into two dimensional AOA estimates (azimuth and elevation) for better visualization. Source is located at $0°$.

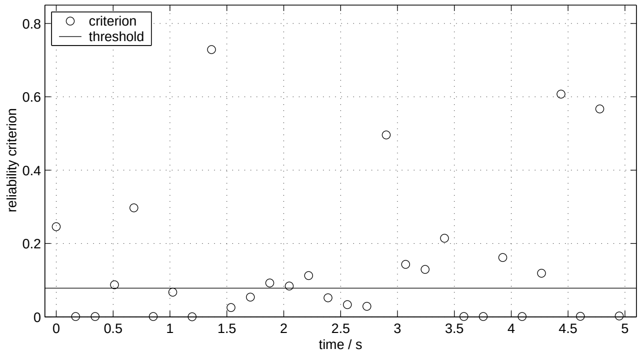

Fig. 3. Values of reliability criterion for estimates in figure 2, with $c = 343\ \frac{\text{m}}{\text{s}}$. Erroneous estimates have large values and can be separated from correct ones which have criterion value very close to zero. Here we have simply set a constant threshold level for error detection.

and disturb the time delay estimation. In addition, neither the sensor locations nor sound propagation speed are known with ideal precision as in simulations. The effect of these factors can be seen as the absence of perfectly correct estimates. It should be noted that it is not possible to use the reliability criterion to distinguish between different signal sources, such as our subwoofer and some background noise source. The criterion does not give any information on whether we are observing the desired signal source or some unwanted noise source. Instead, it provides us with a measure on how erroneous our estimates are and also information on the status of our sensor system.

Despite the various error sources, it is possible to separate out largely erroneous estimates from those with a smaller error. The discriminative behavior of reliability criterion observed in simulations is also present in real data results. This is illustrated with a level of threshold in figure 5.

7. DISCUSSION

What is the purpose with this kind of research? There have earlier been calculations on maximum errors. We have introduced a reliability criterion to time delay based direction of arrival estimates.

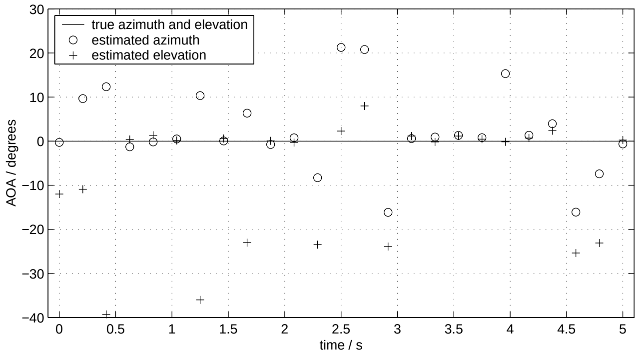

Fig. 4. DOA estimates projected into AOAs (azimuth and elevation) from a real data experiment. Some form of error is always present in real situations and therefore there are not perfectly correct estimates.

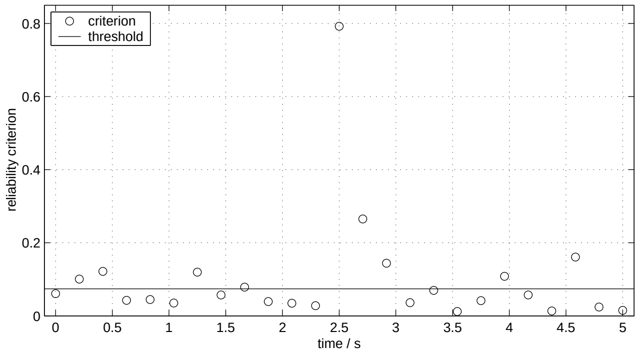

Fig. 5. Values of reliability criterion for estimates in figure 4, with $c = 335\ \frac{\mathrm{m}}{\mathrm{s}}$. Distinction between reliable and unreliable estimates is not as easy as in the simulative case, but we can still utilize a simple constant threshold level.

This means that it is not only possible to consider a single estimate but to consider how well the whole sensor system is behaving. This is something new and desirable. The contrast is large with sensor systems using mean and standard deviation to attempt to do the same. Our reliability measure can be computed directly from estimates utilizing wave propagation speed, without restricting assumptions on signals or sources. For the most common physical mediums, wave propagation speeds are known and they can be obtained from tables. For example, the speed of sound in air can be somewhat accurately estimated when temperature is known.

What is the prize of this progress? It is true that some extra calculations are needed. However, these calculations are not exhaustive. With the recent micro and signal processors it is possible to calculate this reliability measure to audio signals. This should be considered in contrast to the knowledge that we know our sensor system is working properly.

8. CONCLUSIONS

In this research we derived a reliability criterion for time delay based DOA estimate from wave propagation speed and demonstrated its behavior with simulated and real data. This reliability criterion is also a lower bound for the norm of estimation error and therefore with the criterion it is possible to state the minimum amount of error in a DOA estimate. Thus, an estimation system can automatically give a measure for its estimates and furthermore indicate when the produced data is unreliable or the sensor system is working in an improper way.

9. REFERENCES

[1] C. Liu, B.C. Wheeler, W. D. O'Brien Jr., R. C. Bilger, C. R. Lansign, and A. S. Feng, "Localization of multiple sound sources with two microphones," *Journal of Acoustical Society of America*, vol. 108, no. 3, pp. 1888–1905, 2000.

[2] R. Blumrich and J. Altmann, "Medium range localisation of aircraft via triangulation," *Applied Acoustics*, vol. 61, no. 1, pp. 65–82, 2000.

[3] "Special issue on time delay estimation," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 29, no. 3, 1981.

[4] S.P. Applebaum, "Adaptive arrays," *IEEE Transactions on Antennas and Propagation*, vol. 24, no. 5, pp. 585–598, 1976.

[5] P. T. Karttunen, S. J. Ovaska, and T. I. Laakso, "Comparison of direction of arrival estimation methods: A case study," in *Proceedings of the IEEE Nordic Signal Processing Symposium (NORSIG)*, 1996, pp. 223–226.

[6] K.U. Simmer, J. Bitzer, and C. Marro, "Post-filtering techniques," in *Microphone Arrays*, M. Brandstein and D. Ward, Eds., pp. 39–60. Springer-Verlag, 2001.

[7] J. Yli-Hietanen, K. Kalliojärvi, and J. Astola, "Low-complexity angle of arrival estimation of wideband signals using small arrays," in *Proceedings of the 8th IEEE Signal Processing Workshop on Statistical Signal and Array Signal Processing*, 1996, pp. 109–112.

[8] J. Yli-Hietanen, K. Kalliojärvi, and J. Astola, "Robust time-delay based angle of arrival estimation," in *Proceedings of the IEEE Nordic Signal Processing Symposium*, 1996, pp. 219–222.

[9] J. Yli-Hietanen, K. Koppinen, and J. Astola, "Time-delay selection for robust angle of arrival estimation," in *Proceedings of the IASTED International Conference on Signal and Image Processing*, 1999, pp. 81–83.

[10] J. Yli-Hietanen, T. Saarelainen, and J. Routakangas, "Robust angle of arrival estimation of transient signals," in *Proceedings of the IASTED International Conference on Signal and Image Processing*, 2000.

[11] D. E. Dudgeon and R. M. Merserau, *Multidimensional digital signal processing*, Prentice-Hall, 1984.

[12] G. Jacovitti and G. Scarano, "Discrete time techniques for time delay estimation," *IEEE Transactions on Signal Processing*, vol. 41, no. 2, pp. 525–533, 1993.

[13] A. Fertner and A. Sjölund, "Comparison of various time delay estimation methods by computer simulation," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 34, no. 5, pp. 1329–1330, 1986.

[14] T.Saarelainen and J.Yli-Hietanen, "A design method for small sensor arrays in angle of arrival estimation," in *Proceedings of the X European Signal Processing Conference (EUSIPCO)*, 2000, pp. 1589–1592.