

# SENSING FIELD: COVERAGE CHARACTERIZATION IN DISTRIBUTED SENSOR NETWORKS

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## ABSTRACT

The ability to characterize sensing quality is central to the design and deployment of practical distributed sensor networks. This paper introduces the concept of a sensing field defining, for each point in the physical space of a phenomenon of interest, a measure of how well a sensor network can sense the phenomenon at that point. Using target localization and tracking as examples, the paper derives an upper bound for this measure of goodness measure, using the Cramer-Rao bound and models of sensor observation and network layout. It then evaluates the validity of statistical observation models used by a family of estimators. Simulation results of applying the analytical analysis to a randomly spaced network are presented.

## 1. INTRODUCTION

Sensor networks are emerging as a field with potential impact on many applications such as intelligent infrastructure, environmental monitoring, and battlefield monitoring. A distributed sensor network can be flexibly deployed in an area where there is no a priori sensing infrastructure. In contrast to traditional centralized sensor array processing where a central processing unit processes the measurements collected from all sensors and make decisions, sensor networks distribute the computation among sensor units. Such distributed processing enjoys better flexibility, scalability, and survivability.

The ability to cover a large area is important for tracking events of a significant spatial extent, as in tracking a large number of events simultaneously, or for tracking dynamic events traversing the sensing ranges of many individual sensors, as in tracking a moving vehicle. The characterization of a sensor network's ability to perform a sensing task is a fundamental problems in the deployment and use of such systems. Note that traditional sensors are characterized by specifications such as range, resolution, and accuracy. These specifications are used to decide if a sensor is suitable for the task and system at hand. The simplicity and elegance of these specifications makes the use of individual sensors a very easy task. Similar specifications for sensor networks currently do not exist and extensive testing is necessary.

This paper introduces the notion of *sensing field* for characterizing the pointwise ability of a sensor network to measure a physical phenomenon of interest. The domain a sensing field is

the physical space where the phenomenon is to be estimated. For each point in that space, a measure is defined to characterize how well the sensor network can sense the phenomenon at that point. We will present a statistical derivation of the sensing field given a sensor layout and observation models. The sensing field can be computed at design time, or evaluated at run-time using a distributed algorithm and even updated dynamically, for example, when nodes move.

Consider vehicle tracking as an example for sensor network applications. The designer of the sensor network would like to deploy sensors at specific locations so as to maximize the probability of detection if there is a vehicle in the sensor field, and obtain good estimation accuracy of the vehicle position. For an already deployed sensor network, it is often important to characterize well-covered areas, weakly-covered regions, or blind spots in order to adjust algorithms and cope with resource constraints. On the other hand, from the perspective of an adversary, the notion of well- or weakly- covered regions are also important, for example, in planning minimal-exposure "stealthy" routes.

Though coverage characterization of sensing systems is of great significance, existing work in sensor network literature is very limited. The approaches in [1, 2] tackle the coverage problems using distance from the sensor nodes to characterize the sensing ability. While there is often a correlation between distance and sensing ability due to physical signal attenuation, using distance as the only metric is crude and inherently limiting. These approaches do not take into consideration any sensor characteristics and therefore, results in the same coverage characterization independent of the type and capabilities of the individual sensors.

## 2. SENSING FIELDS IN TARGET LOCALIZATION

Target localization is a canonical application for array signal processing, extensively discussed since the sixties. In the recent decade, due to the benefit of their easy deployment and scalability, sensor networks have become increasingly important for target localization. Elaborate systems have been developed; see [3, 4] for example descriptions. In this section we use target localization as the context to illustrate the approach of sensor coverage characterization.

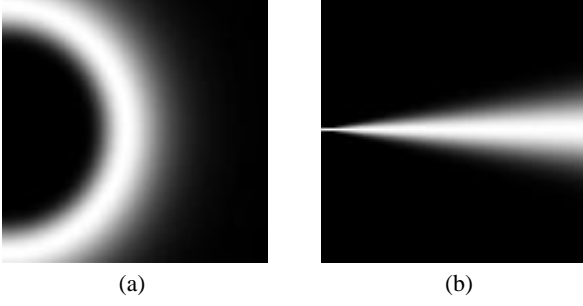
In target localization problems, we assume that the target is present in a location domain, for example, a 2-D terrain. The goal of localization is to locate the target in the domain to some accuracy based on the observed data. To illustrate the basic idea, we consider the simple sensing model as follows:

- Acoustic amplitude sensors. They calculate sound ampli-

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**Fig. 1.** Observational model: a)  $p(\text{amp}|x)$  for acoustic amplitude sensors, and b)  $p(\theta|x)$  for DOA sensors.

tude measured at each microphone, and estimate the distance of the vehicle based on the physics of sound attenuation. Assuming that the sound source is a point source, and sound propagation is lossless and isotropic, a root-mean-squared (rms) amplitude measurement  $\text{amp}$  is related to the sound source position  $x$  as

$$\text{amp} = \frac{A}{r(x, \zeta)} + w, \quad (1)$$

where  $A$  is the rms amplitude of the sound source,  $\zeta$  is the location of the sensor,  $r(x, \zeta)$  measures the Euclidean distance between the sound source and the sensor, and  $w$  is rms measurement noise [5]. For simplicity, we model  $w$  as zero mean Gaussian. The observational likelihood function  $p(\text{amp}|x)$  is plotted in Fig. 1a. Given the amplitude measurement, the sensor can infer the target location as a ring around itself with some uncertainty.

- The direction-of-arrival (DOA) sensors are small microphone arrays. Using beamforming techniques, they determine the direction where the sound comes from, i.e., the bearing of the vehicle. We use the maximum likelihood (ML) DOA estimation algorithm proposed by Chen *et al.* [6]. Due to the presence of noise, the DOA estimate of angles is often an approximation, i.e.,

$$\theta = \text{atan}(x_h - \zeta_h, x_v - \zeta_v) + n, \quad (2)$$

where the subscripts  $h$  and  $v$  denote the horizontal and vertical coordinates respectively, and  $n$  is the perturbation noise. Repeated tests have shown that  $n$  can be well-modeled by a zero mean Gaussian [4]. For this type of sensor, the observation likelihood function  $p(\theta|x)$  is shaped like a cone, as plotted in Fig. 1b.

It is clear that the two types of sensors have quite different characteristics, and under both sensing models, the likelihood function is non-Gaussian in the Cartesian space, as shown in Fig. 1. Target localization algorithms need to incorporate the sensor measurement through proper data fusion mechanisms, and the sensing ability of such a heterogeneous sensor network depends on the estimation algorithm itself, the sensor layout, and the specifics of individual sensors.

### 3. MEASURE OF ESTIMATION ACCURACY

In target localization context, sensing field characterizes the sensor network's ability to localize target in a physical location do-

main. In this section, we compute an instantiation of sensing field based on the accuracy of estimation, which is a major focus of performance. In practice, the estimation accuracy can be obtained by extensive testing. Estimation bounds are of interest because they can be obtained analytically, and are often insightful approximations of reality. In this section, we explore the Cramer-Rao bound (CRB) of estimation accuracy (or equivalently the Fisher information) in sensor networks.

Given the observation model  $p(z|x)$ , one can estimate the underlying parameter  $x$  from the observation  $z$ . The Fisher information matrix

$$I = E_{p(z|x)} \left[ (\nabla_x \log p(z|x)) \cdot (\nabla_x \log p(z|x))^T \right] \quad (3)$$

measures how informative the measurement  $z$  is about  $x$ . The variance of any unbiased estimator is lower-bounded by the inverse of diagonal elements of Fisher information matrix, i.e.,

$$\text{Cov}(\hat{x}(z)) \geq I^{-1}. \quad (4)$$

The notation of  $A \geq B$  means that the matrix  $A - B$  is positive semi-definite. The right hand side of (4) is known as the CRB. In this sense, CRB characterizes the best achievable performance in the family of unbiased estimators, and is asymptotically tight [7].

We use the sensing model described in Sec. 2 as an example. Here we present the main results without detailed derivations. Let  $d_k = (d_{k,h}, d_{k,v})$  be the displacement vector (in the two-D location domain) from the target to the  $k$ -th sensor, i.e.,  $d_k = x - \zeta_k$ . We have the following results:

- The Fisher information of estimating the target location based on an amplitude sensor is

$$I_{k, \text{amp}} = \frac{A^2}{\sigma_{k, \text{amp}}^2 |d_k|^6} \begin{pmatrix} d_{k,h} \\ d_{k,v} \end{pmatrix} \cdot \begin{pmatrix} d_{k,h} & d_{k,v} \end{pmatrix}, \quad (5)$$

where  $\sigma_{k, \text{amp}}$  is the standard deviation of noise in sensor  $k$ 's amplitude measurement.

- The Fisher information of estimating the target location based on a DOA sensor is

$$I_{k, \text{DOA}} = \frac{1}{\sigma_{k, \text{DOA}}^2 |d_k|^4} \begin{pmatrix} d_{k,v} \\ d_{k,h} \end{pmatrix} \cdot \begin{pmatrix} d_{k,v} & d_{k,h} \end{pmatrix}, \quad (6)$$

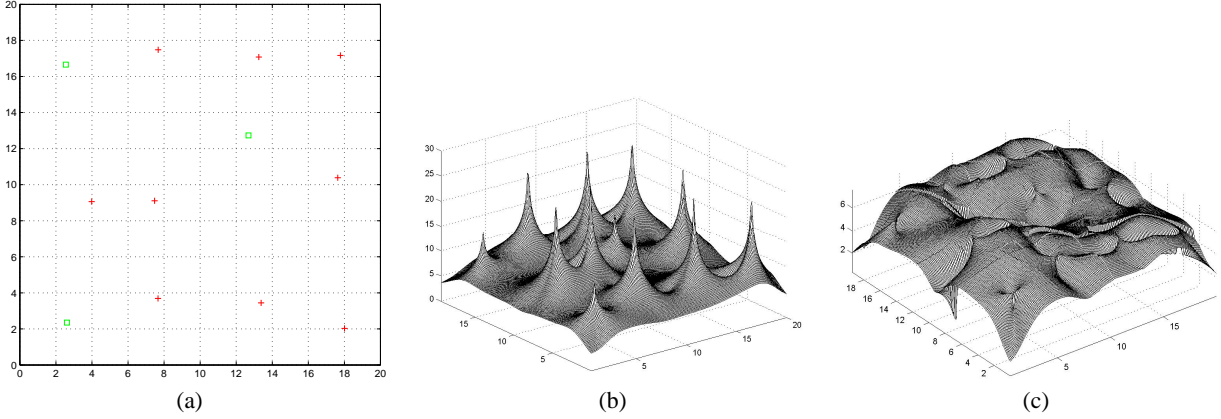
where  $\sigma_{k, \text{DOA}}$  is the standard deviation of noise in the sensor's direction measurement.

- Assume that given the target location  $x$ , the measurement across different sensors are statistically independent, i.e.,  $p(z_1, \dots, z_k|x) = \prod_{i=1, \dots, k} p(z_i|x)$ . The Fisher information of estimating the target location based on the *simultaneous* measurements of a collection of sensors is the sum of the CRB of individual sensors, i.e.,

$$I_{\text{total}} = \sum_k I_k. \quad (7)$$

This corresponds to the limit of an unbiased centralized localization system.

Fig. 2a shows a randomly deployed sensor network, in which the sensor locations are generated by perturbing a uniform  $3 \times 4$  grid over a  $20 \times 20$  meter<sup>2</sup> field by Gaussian noise. For this sensor layout, Fig. 2b plots the determinant of Fisher



**Fig. 2.** The Fisher information: (a) the sensor network layout. The points marked with “+” denote amplitude sensors, assuming  $A = 40$  and the Gaussian amplitude noise of standard deviation  $\sigma_{k,amp} = 0.5$ . The points marked with squares denote DOA sensors, assuming that the noise to angle measurements have  $\sigma_{k,DOA} = 3^\circ$ . (b) The determinant of the centralized Fisher information ( $|I_{total}|$ ) as in (7). (c) The determinant of the distributed Fisher information as in (8). Subfigures (b) and (c) are plotted in log scale.

information matrix. High value implies good estimation accuracy. It reaches local maxima at sensor locations. This is expected, since the SNR is high if the target is close by. Note that amplitude and DOA sensors have a different impact on Fisher information, since their sensing characteristics are distinct. In this configuration of model parameters, DOA sensors produces lower peak values for Fisher information.

- The Fisher information of estimating target location with some prior knowledge is the Fisher information without prior knowledge plus the Fisher information for the prior knowledge itself. This is a well-known result in estimation theory [7].

In sensor networks, the constraints on computation and communication prohibit the use of all sensor measurements simultaneously, especially when the underlying state is time-varying, e.g., the target is moving. Sensor network systems often implement estimation in a recursive way. At time  $t$ , only a small number (one or several) of simultaneous sensor measurements are incorporated to estimate the target location. The estimated location  $x(t)$ , combined with target dynamics, becomes the prior knowledge for the next time step  $t + 1$ . The Kalman filter is a well-known instance of this type of recursive algorithm [8].

The Fisher information and CRB for recursive estimation algorithms can also be computed recursively, as described in [9]. We assume the vehicle dynamics take the form  $x_{t+1} = x_t + s_t$ . For simplicity, we assume that the displacement  $s_t$  is a stationary independent Gaussian random process with zero mean and covariance  $R_s$ . Under these conditions, we have

$$I_{t+1} = I_{p(z_{t+1}|x_{t+1})} + R_s^{-1}(R_s^{-1} + I_t)^{-1}I_t \quad (8)$$

The result is actually intuitive. The first term,  $I_{p(z_{t+1}|x_{t+1})}$ , is the new information brought in by the new measurement  $z_{t+1}$ . The second term is the Fisher information  $I_t$ , but discounted by the dynamics using the factor  $R_s^{-1}(R_s^{-1} + I_t)^{-1}$ . Prior knowledge is smeared and weakened by the target dynamics. The faster the random walk, the smaller  $R_s^{-1}$  is, meaning that the less what we

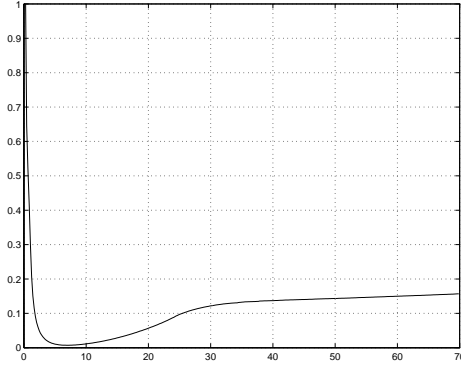
know at time  $t$  can help estimation at  $t + 1$ . Hence the effect of  $I_t$  will be greatly discounted when computing  $I_{t+1}$ . On the other hand, if the target dynamics are deterministic and known, then  $R_s = 0$ , and (8) is simply  $I_{t+1} = I_{p(z_{t+1}|x_{t+1})} + I_t$ , meaning that the Fisher information from different sensors can be added directly. In this special case, the target is equivalent to a stationary one, hence delay between readings does not matter, and we get the same result as in the centralized case.

For the sensor network plotted in Fig. 2a, we compute the Fisher information (8) for a distributed estimation scheme which uses measurements from four sensors nearest to the target location. The measurement of the nearest sensor is applied at time  $t = 0$ , the measurement of the second nearest sensor is applied at time  $t = 1$ , and so on. Fig. 2c plots the determinant of Fisher information (8). Compared to Fig. 2b, the Fisher information for the distributed system is lower, since the distributed system trades off optimality with computational and communication efficiency. Nevertheless it better describes the actual performance of distributed sensor networks.

The recursive computation of Fisher information is not limited to the Gaussian assumption of the displacement  $s_t$ . With some modification, the above analysis still applies. In particular, if the displacement  $s_t$  is independent of  $x_t$ , (8) holds with  $R_s$  substituted by the Fisher information of the vehicle dynamics model  $p(x_{t+1}|x_t)$ .

#### 4. VALIDITY OF STATISTICAL MODELS

Sec. 3 explores the theoretical estimation accuracy bound, which is independent of the specifics of any particular estimation algorithm. On the other hand, the actual estimation performance depends to a large extent on the estimation scheme itself. In particular, statistical estimation algorithms often use parametric models to approximate the actual sensing physics. For example, the well-known Kalman filter and many of its variants are based on fundamental assumptions that the observation model is linear and Gaussian. In general, the performance of the sensor network estimation depends on the validity of the assumed model.



**Fig. 3.** Kullback-Leibler divergence as a function of distance  $r$  between the target and the sensor.

To quantify the model validity, we use the distance between the assumed model  $q(z|x)$  and the actual model  $p(z|x)$ , measured by information-theoretic Kullback-Leibler divergence:

$$D_{KL}(p, q) = \int \log(p/q) dp. \quad (9)$$

It measures the number of bits to describe the discrepancy between  $p$  and  $q$  [10]. This metric reflects how well  $q$  fits  $p$ . A good fit would justify the use of a tracking algorithm built on the assumption  $q$ . A bad fit would suggest otherwise.

In conjunction with estimation accuracy, we consider a different instantiation of sensing field based on model validity, measuring the goodness of the fit of the assumed statistical model. For a single sensor, the field is computed as the discrepancy between the actual observational probability  $p(z|x)$  and its assumed model  $q(z|x)$  as a function of  $x$ , and calculate the value for each  $x$ .

For example, consider a Kalman filter in a homogeneous sensor network consisting of sensors which have an amplitude measurement (hence range  $r$ ) and a direction measurement  $\theta$ . Let  $z$  be the sensor measurement about target location, i.e.,  $z = (z_h, z_v) = \zeta + (r \cos \theta, r \sin \theta)$ . The likelihood  $p(z|x)$  is simply the product of the two likelihoods in Fig. 1, multiplied by the Jacobian of  $r$  with respect to coordinate transformation. The likelihood is a comet shape around  $x$ , and Kalman filter approximates it with a Gaussian fit. For the sensing model described here, the validity measured as the Kullback-Leibler divergence is only dependent on the distance  $r$  between the sensor and the target. The computed model validity is thus radially symmetric. Fig. 3 plots its value along a radius. For small  $r$ , since the target is very close by, the signal-to-noise ratio (SNR) is high, therefore  $p(z|x)$  is a very thin arc which cannot be well approximated by a Gaussian fit. For very large  $r$ , the tail of the comet becomes longer, which makes the Gaussian fit not very accurate. Similar techniques can be applied to calculate model validity for more complicated sensing models.

The model validity characterization for single sensors can be generalized to sensor networks consisting of multiple sensors. The Kullback-Leibler divergence of each sensor can be superimposed. This corresponds to the goodness of fit of the joint observational model  $p(z_1, \dots, z_k|x)$  assuming conditional independence. The regions with low divergence values are considered as well-modeled and closer-to-theory performance is expected.

## 5. DISCUSSION

We have presented two examples of the sensing field in the target localization context. The estimation accuracy characterizes the theoretically achievable performance of sensor networks. The model validity reflects our “confidence” of achieving the theoretical bound. Such information could be very useful to sensor network users. In particular, it can be used as a mechanism for evaluating the sensor network, and provides reference to sensor deployment and estimation and routing algorithms. For example, for a randomly deployed sensor network, once the sensor locations are calibrated, the user can have a good sense of strength and weakness of the sensor network’s sensing capabilities. Algorithms can be adapted accordingly. For example, the system’s awareness of the fact that the target is approaching a sensor hole is valuable. It can trigger algorithms to query sensors further away, call for more extensive sensor collaboration, and planning for limited resources with sufficient look-ahead. Another example is in mobile sensor network, where one may pose the problem of adjusting sensor layout as a high-dimensional optimization problem in an average or minimax sense. Given the layout of local neighborhood, individual sensors can be adjusted to provide even coverage across the field, or to focus coverage on specific regions of interest to satisfy user requirements.

Although the framework presented here is illustrated in the context of target localization, the basic idea of characterizing a sensor network’s ability is not limited to that problem. For example, in the target detection problem, where only the detection performance (detection probability and/or false alarm probability) is of interest, one can explore the possibility of using a Chernoff bound to characterize performance. The techniques to compute model validity are rather general and can be easily adapted to different sensing models and different phenomena of interest.

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