TDOA-SDOA ESTIMATION WITH MOVING SOURCE AND RECEIVERS

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ABSTRACT

When there is a relative motion (rm) between a signal source and a receiver, the signal arrives at the receiver with a time-scaling. Under this condition, estimating the time-difference-of-arrival (TDOA) of the signal at two receivers will have large errors, if the estimator ignores the effects of the rm. The correct approach estimates both the TDOA and scale-difference-of-arrival (SDOA) between the signals of the two receivers, via the maximization of the cross-ambiguity function (CAF).

This paper first derives the bias of TDOA estimation under rm, when neglecting SDOA. It then gives a new, fast method, based on the Newton root finding algorithm, for determining the maximum of the CAF. Simulation results indicate that the TDOA and SDOA mean square errors attain the Cramer Rao Lower Bound.

1. INTRODUCTION

The measurement of the time-difference-of-arrival (TDOA) of a signal at two separate receivers is the first step to passive localization [1]. This has wide applications in sonar, radar [1, 2], geolocation [3], electronic warfare [4], and more recently the determination of E-911 mobile phone dialers, and vehicle positions in an Intelligent Vehicle Highway Systems [5, 6].

A common TDOA estimator cross-correlates the two receiver outputs and takes the lag (or time shift) at which the correlation function peaks, as the TDOA. This gives satisfactory results when there is no relative motion (rm) between the signal source and the receivers. When there is, the signals arriving at the receivers suffer a time-scaling. This time-scaling creates a scale-difference-of-arrival (SDOA) since the rm between a source and one receiver is different from another receiver, unless the source is traveling perpendicular to and exactly between the stationary receivers. When there is SDOA, the receiver outputs have poor correlation, even though they are from the same source [7]. Estimating TDOA by simple cross-correlation becomes unreliable. It has a large bias, which is proportional to the SDOA and the record length. The bias is the dominant error. To improve accuracy, it is necessary to compensate for the SDOA. This is wideband processing [8].

Let the receiver outputs be

$$\begin{aligned} x(t) &= s(t) + \phi(t) \\ y(t) &= s\left(\frac{t+D}{a}\right) + \psi(t) \end{aligned} \tag{1}$$

where $s(t)$ is the signal, $\phi(t)$ and $\psi(t)$ are the noise sources which are independent of each other and of $s(t)$. In $y(t)$, the signal $s(t)$ has an SDOA "a" and TDOA "D". Wideband processing is the maximization of the cross-ambiguity function

$$\text{CAF}(\alpha, \tau) = \int_0^T x(t) y(\alpha t - \tau) dt \tag{2}$$

over a record length T by the choice of α, τ. In (2), when there is no noise

$$y(\alpha t - \tau) = s\left(\frac{\alpha t - \tau + D}{a}\right) \tag{3}$$

so that (2) is a maximum when $\alpha = a$, $\tau = D$. At present, there is no closed-form solution for α and τ, other than through a 2-D search. This requires computation of (2) for each possible pair of α, τ within a range. This is time consuming especially since $y(t)$ is in general a random signal and there is no fast way to time-scale it by an arbitrary, non-integer α.

It is tempting to consider avoiding the large bias due to a long record length, by partitioning T into several smaller segments and computing the TDOA for each segment [9]. If the TDOA is relatively constant over T, the final estimate is the average of the TDOAs. If the TDOA is approximately a function of time, a least squares fit through the TDOAs will give the TDOA estimate as a function of time [10]. The idea of segmenting T, or noncoherent processing, is that the decorrelating SDOA effects become negligible for short segments, and if there is no rm, the average estimation variance is equal to that of processing with the non-partitioned T [9].

However, as (13) in Section 2 shows, segmentation processing and coherent processing have the same amount of bias of $(1-a)\frac{N}{2}$, where N is the data length in seconds, assuming sampling time is 1 second. Thus wideband processing is necessary when the bias is not acceptable.

This paper presents an iterative search for finding the TDOA and SDOA that maximize the discrete form of (2). The key is in representing $y(n)$ by an analytic expression which has first and second derivatives. Then the maximization is a direct application of Newton's root finding algorithm [11]. To reduce computations at each iteration, the procedure employs a fast time-scaling method that requires only $O(N(2L+1))$, where L is the maximum number of interpolating sinc coefficients. Compared to [12], the savings are significant.

In the remainder of this paper, Section 2 derives the TDOA bias, Section 3 formulates and describes the search procedure, Section 4 contains the simulation results and the conclusions are in Section 5.

2. TDOA BIAS

The relationship between a band limited white noise (BLWN) process $s(t)$ and its 1 Hz samples $s(n)$, $n = 0, 1, ..., N-1$, is

$$s(t) = \sum_{n=0}^{N-1} s(n)\text{sinc}(t-n) \tag{4}$$

where

$$\text{sinc}(\cdot) = \frac{\sin \pi(\cdot)}{\pi(\cdot)} \tag{5}$$

Time-scaling $s(t)$ by a and time-shifting it by D give

$$z(t) = s\left(\frac{t+D}{a}\right) = \sum_k s(k)\text{sinc}\left(\frac{t+D}{a} - k\right) \tag{6}$$

In (6) and in the sequel, all summations are from 0 to $N-1$, unless otherwise stated.

Now $z(t)$ and $s(t)$ are jointly nonstationary due to the time-scaling operation. Thus the crosscorrelation of $s(t)$ and $z(t-\tau)$ will be dependent on both t and τ. From (4) and (6)

$$E\{s(t)z(t-\tau)\} = \sum_n \sum_k E\{s(n)s(k)\} \times$$
$$\text{sinc}(t-n)\text{sinc}\left(\frac{t-\tau+D}{a} - k\right) \tag{7}$$

Let $s(n)$ be zero mean white noise samples of variance σ_s^2, then

$$E\{s(t)z(t-\tau)\} =$$
$$\sigma_s^2 \sum_n \text{sinc}(t-n)\text{sinc}\left(\frac{t-\tau+D}{a} - n\right) \tag{8}$$

From Schwartz's inequality [11], (8) is maximum when

$$\tau = (1-a)t + D \tag{9}$$

which is the estimate of TDOA, when neglecting SDOA. The bias is

$$\tau - D = (1-a)t \tag{10}$$

which, on averaging t from 0 to N, is

$$\frac{1}{N}\int_0^N (1-a)t dt = \frac{(1-a)N}{2} \tag{11}$$

which agrees with the bias obtained in [7] by the frequency domain approach.

Consider next segmentization processing which breaks the N points into M segments of $K = N/M$ points each. The bias of the i-th segment, $i = 1, ..., M$, is from (10)

$$b_i(t) = (1-a)[t + (i-1)K], \quad 0 \le t \le K \tag{12}$$

Since the TDOA estimate is the average of the M estimates, the averaged estimate has bias

$$\begin{aligned} b(t) &= \frac{(1-a)\sum_{i=1}^{M}[t+(i-1)K]}{M} \\ &= (1-a)\left[t + \frac{(M-1)K}{2}\right] \end{aligned} \tag{13}$$

Averaging t over K seconds then gives the bias $(1-a)\frac{N}{2}$, identical to (11). Thus segmentization cannot reduce the TDOA bias and simulation in Section 4 has confirmed this assertion.

3. CAF MAXIMIZATION

When there is rm and the TDOA bias is too large, wideband processing is necessary. It is the maximization of CAF in (2). Let the samples of (1) be $x(n)$ and $y(n)$, then the equivalent discrete form of (2), with minimization replacing maximization, is

$$J(\mathbf{\Theta}) = \sum_n [x(n) - y(\alpha n - \tau)]^2 \tag{14}$$

where

$$\mathbf{\Theta} = [\alpha \quad \tau]^T \tag{15}$$

Following (4) and (6),

$$y(\alpha n - \tau) = \sum_k y(k)\text{sinc}(\alpha n - \tau - k) \tag{16}$$

Minimization of (14) in general requires a 2-D grid search in $\mathbf{\Theta}$, This is time consuming since for each $\mathbf{\Theta}$, it is necessary to compute (16) and then (14).

This paper presents a faster alternative. Putting (16) into (13) and applying Newton's method [11] yields the iteration

$$\mathbf{\Theta}^{i+1} = \mathbf{\Theta}^{i} - \left[\frac{\partial^2 J}{\partial \mathbf{\Theta} \partial \mathbf{\Theta}^T}\right]^{-1} \left[\frac{\partial J}{\partial \mathbf{\Theta}}\right] \tag{17}$$

At the i-th iteration, the value of $\mathbf{\Theta}^i$ is used in the evaluation of the partials

$$\frac{\partial J}{\partial \mathbf{\Theta}} = \left[\frac{\partial J}{\partial \alpha} \quad \frac{\partial J}{\partial \tau}\right]^T \tag{18}$$

and

$$\frac{\partial^2 J}{\partial \mathbf{\Theta} \partial \mathbf{\Theta}^T} = \begin{bmatrix} \frac{\partial^2 J}{\partial \alpha^2} & \frac{\partial^2 J}{\partial \alpha \partial \tau} \\ \frac{\partial^2 J}{\partial \alpha \partial \tau} & \frac{\partial^2 J}{\partial \tau^2} \end{bmatrix} \tag{19}$$

For computational convenience, let

$$e(n) = x(n) - \sum_k y(k)\text{sinc}(\alpha n - \tau - k) \tag{20}$$

$$e_1(n) = -\sum_k y(k)\text{sinc}'(\alpha n - \tau - k) \tag{21}$$

$$e_2(n) = -\sum_k y(k)\text{sinc}''(\alpha n - \tau - k) \tag{22}$$

with $\text{sinc}'(\cdot)$ and $\text{sinc}''(\cdot)$ denoting the first and second derivatives of $\text{sinc}(\cdot)$ with respect to $(\cdot)$. Then it is easy to compute the partials in terms of $e(n)$, $e_1(n)$ and $e_2(n)$.

The iteration stops when $||\mathbf{\Theta}^{i+1} - \mathbf{\Theta}^i|| < TH$, a predetermined threshold value. To ensure that the CAF attain its global minimum within a $\mathbf{\Theta}$-grid, the algorithm takes on several initial conditions, and records the $J(\mathbf{\Theta})$. The $\mathbf{\Theta}$ that gives the minimum is the estimate.

Computing (16) requires $O(N^2)$. However, by noting that the sinc function decays rapidly from its maximum, truncating the summation in (16) to just a few points near the peak of the sinc function can give a quick and accurate approximation of (16). Let (16) become

$$y(\alpha n - \tau) = \sum_{k=l_0-L}^{l_0+L} y(k)\text{sinc}(\alpha n - \tau - k) \tag{23}$$

where

$$l_0 = \lfloor \alpha n - \tau \rfloor \tag{24}$$

is the integer values of $\alpha n - \tau$. This operation makes $0 \leq |\alpha n - \tau - l_0| \leq 1$, so that the largest sinc function samples are present in (23). The number of computations for (16) now reduces to $O(N(2L+1))$, which is a significant saving. By simulation, Section 4 shows that the truncation errors are negligible for $L \geq 5$.

4. SIMULATION EXPERIMENTS

This section describes two experiments to corrobate the developments in Sections 2 and 3. Both signal and noise are zero mean, Gaussian BLWN.

4.1. TDOA bias

This is an evaluation of the TDOA bias when rm is present but the estimator neglects the SDOA.

With no additive noise, Figure 1 plots the TDOA bias against N, for $a = 1.001$ and $D = 0$, at $M = 1$, i.e., no segmentation and $M = 4$ segments. The biases from simulation agree with the formula of (11), and confirm that segmentation cannot reduce the bias.

4.2. TDOA and SDOA estimation

The signal-to-noise-ratio is

$$\text{SNR} = 10 \log \sigma_s^2 / \sigma_\phi^2 \tag{25}$$

where $\sigma_\phi^2 = \sigma_\psi^2$ are the variances of the BLWN processes $\phi(t)$ and $\psi(t)$ in (1). Applying (17) to find the $\mathbf{\Theta}$ that maximizes the CAF of (14), the iterative search starts with $D = 0$ and $a = 1.002$. The true values are $D = 0$ and $a = 1.001$, and $N = 513$. Whenever the convergence is to a $|D| > 0.5$, the search reinitializes with $D = 0$ and $a = 1.001$ plus a random number. Convergence usually occurs in at most 14 iterations.

Figures 2 and 3 are the MSE of the TDOA and the SDOA estimates, as a function of the SNR. The mean square error is

$$MSE(\tau^*) = \frac{\sum_{i=1}^{50} (\tau^*(i) - D)^2}{50} \tag{26}$$

where τ^* is the TDOA estimate of the i-th trial. Also plotted is the Cramer Rao Lower Bound (CRLB) [10] When using the truncation of (23) instead of (16), the MSE attain the CRLB at high SNR, but diverge earlier from the CRLB for smaller L, the number of sinc coefficients. Other simulations with different D and a give similar results.

5. CONCLUSIONS

Estimation of TDOA when there is rm between the source and receivers requires SDOA compensation because the bias is large. An error analysis shows that if the estimator assumes no SDOA but in fact there is, there is a bias that is approximately equal to $\left(\frac{1-a}{2}\right) N$. This bias dominates the errors and is the same even with segmentation processing.

The proper approach is joint TDOA and SDOA estimation. It is through the maximization of the CAF, which requires a 2-D search. Section 3 proposes a faster Newton-based iterative technique. A truncation of the sinc interpolation in time-scaling also helps reduce computations. Simulation results have shown that this estimator attain the CRLB for Gaussian signals and hence is optimum.

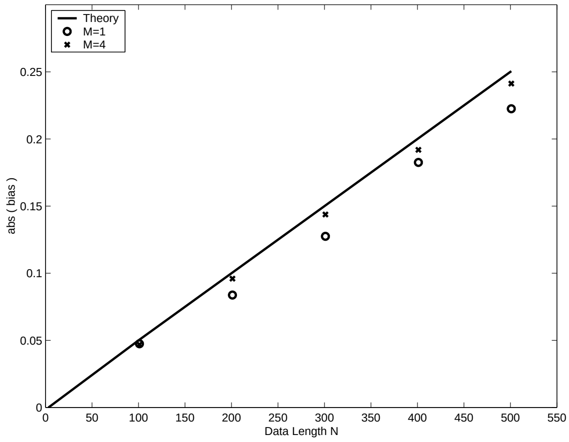

Fig. 1. TDOA Bias.

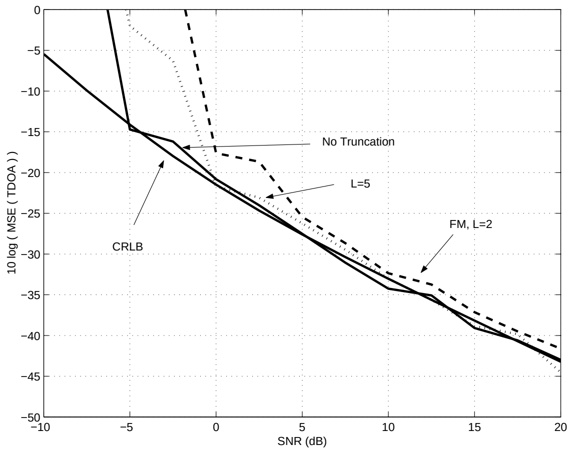

Fig. 2. TDOA MSE.

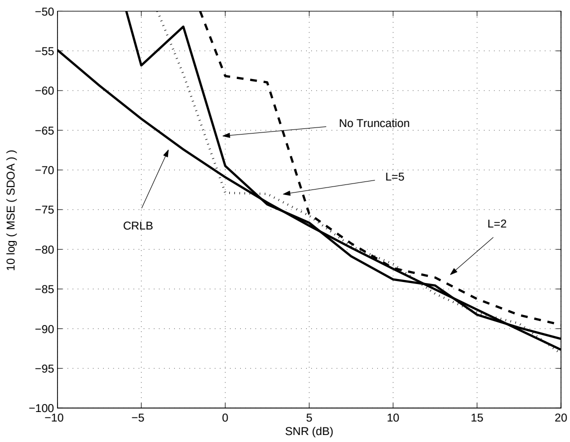

Fig. 3. SDOA MSE.

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