COHERENT SOURCE LOCALIZATION USING VECTOR SENSOR ARRAYS

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ABSTRACT

This paper addresses the problem of coherent/fully correlated source localization using vector sensor arrays. A novel method for "decorrelating" the incident signals is presented. The method is based on vector sensor smoothing (VSS) and enables the use of eigenstructure-based techniques, which require uncorrelated or partially correlated signals. The method is implemented as a preprocessing stage before applying eigenstructure-based techniques, such as MUSIC. The performance of the proposed VSS preprocessing combined with MUSIC is evaluated and it is shown that it asymptotically achieves the Cramer-Rao bound.

1. INTRODUCTION

Vector sensors enable estimation of the angle of arrival and polarization of impinging electromagnetic waves with arbitrary polarization. In the last decade, many array processing techniques for source localization and polarization estimation using vector sensors have been developed. Nehorai and Paldi [1], [2] developed the Cramer-Rao bound (CRB) for this problem and the vector cross-product direction-of-arrival (DOA) estimator.

Eigenstructure-based source localization techniques, such as ESPRIT and MUSIC using vector sensors have been extensively investigated. Li [3], applied the ESPRIT algorithm to a vector sensor array. ESPRIT-based direction finding algorithms using vector sensors have been further investigated in several papers [4], [5]. MUSIC-based algorithms for this problem have been applied in [6]. These techniques yield high-resolution and asymptotically efficient estimates in case of uncorrelated or partially correlated signals. However, since these techniques assume non-singular signal correlation matrix, they encounter difficulties in cases of coherent/fully correlated signals like in multipath scenarios.

In order to "decorrelate" the signals in the data covariance matrix, Evans et al. [7] proposed a preprocessing technique referred to as spatial smoothing. The drawback of this approach is the reduction of the effective array aperture length, resulting in lower resolution and accuracy. An alternative spatial averaging method is redundancy averaging [8]. In [9], it is shown that this preprocessing method induces bias in the DOA estimates.

In this paper, a novel preprocessing method is proposed to remove the singularity in the signal correlation matrix in scenarios with fully correlated sources. The method is based on Vector Sensor Smoothing (VSS), which enables the use of eigenstructure-based algorithms, such as MUSIC and ESPRIT for DOA estimation in these scenarios.

This paper is organized as follows. Section 2 describes the measurement model using a vector sensor array. Section 3 presents the proposed VSS method as a preprocessing stage for eigenstructure-based source localization. The performance of the proposed algorithms is evaluated via computer simulation and described in section 4. Section 5 summarizes our conclusions.

2. PROBLEM FORMULATION AND MODELING

Consider a vector sensor containing 3 electric and 3 magnetic orthogonal sensors, azimuthally rotated by an angle δ, as depicted in Fig. 1.

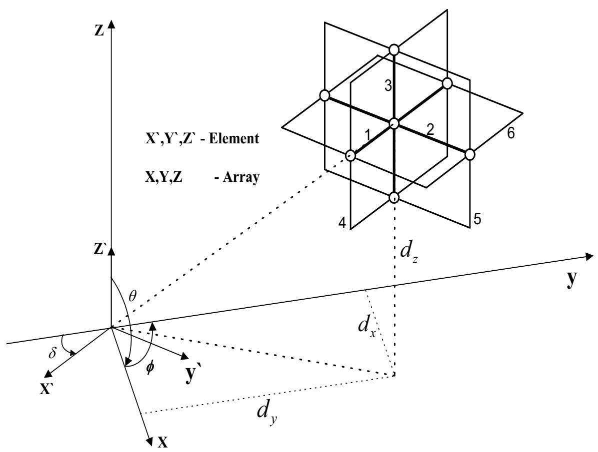

Figure 1: Vector sensor geometry

The spatial response in matrix notation of the vector sensor for the particular case, $\delta = 0$, can be expressed by

$$\mathbf{g}_0(\theta,\phi,\mathbf{p}) = \underbrace{\begin{bmatrix} -\cos\theta\sin\phi & -\cos\phi \\ \cos\theta\cos\phi & -\sin\phi \\ -\sin\theta & 0 \\ -w\sin\phi & -w\cos\theta\cos\phi \\ -w\cos\phi & w\cos\theta\sin\phi \\ 0 & w\sin\theta \end{bmatrix}}_{\mathbf{A}(\theta,\phi)} \underbrace{\begin{bmatrix} p_\theta \\ p_\phi \end{bmatrix}}_{\mathbf{p}} \tag{1}$$

where w denotes the ratio between the induced voltage in an electric sensor to the corresponding induced voltage in a magnetic sensor. The polarization vector, $\mathbf{p}$, is determined by two real parameters, γ and η: $\mathbf{p}_\theta = \sin\gamma e^{j\eta}$ and $\mathbf{p}_\phi = \cos\gamma$. In general, a vector sensor may contain part of the 6 sensor shown in Fig. 1, and therefore the corresponding spatial response vector size is given by $1 \leq L \leq 6$.

For the general case of a 3D array with N vector sensors, the spatial response in matrix notation of the array vector sensors is expressed by

$$\mathbf{g}(\theta,\phi,\mathbf{p}) = \mathbf{q}(\theta,\phi) \otimes \mathbf{g}_0(\theta,\phi,\mathbf{p}) = \underbrace{[\mathbf{q}(\theta,\phi) \otimes \mathbf{A}(\theta,\phi)]}_{\mathbf{F}(\theta,\phi)}\mathbf{p} \tag{2}$$

where $\otimes$ denotes the Kronecker product. The size of the vector $\mathbf{q}(\theta,\phi)$ is $N \times 1$ and its elements represent the phase delay associated with each vector sensor in the array due to its relative location for an incident plane wave from the direction (θ,ϕ):

$$q_n(\theta,\phi) = e^{jk_0[x_n\sin\theta\cos\phi + y_n\sin\theta\sin\phi + z_n\cos\theta]},\ n = 1,\cdots,N \tag{3}$$

and k_0 is the wavenumber in the medium.

Consider the scenario of M signals, impinging on the array from directions (θ_m,ϕ_m) and polarization vectors $\mathbf{p}_m$, where $m = 1,\cdots,M$. Then, the spatial response of the array to the mth signal is denoted by $\mathbf{g}(\theta_m,\phi_m,\mathbf{p}_m)$, and the data model is given by

$$\mathbf{y}_k = \sum_{m=1}^{M} \mathbf{F}(\theta_m,\phi_m)\mathbf{p}_m s_{mk} + \mathbf{n}_k,\ k = 1,\cdots,K, \tag{4}$$

where K is the number of independent samples collected by the array and $\mathbf{n}_k$ represents the kth sample of the additive noise and interference vector.

The measurement and noise vectors, $\mathbf{y}_k$ and $\mathbf{n}_k$ are each of size LN, the matrix $\mathbf{F}(\theta_m,\phi_m)$ is of size $LN \times 2$ whose columns denote the spatial transfer functions for both polarization components of the mth signal, and $\mathbf{p}_m$ is a complex vector of size 2 describing the corresponding signal polarization state.

We assume that the noise vector, $\{\mathbf{n}_k\}_{k=1}^{K}$, is an i.i.d. sequence with zero-mean, complex Gaussian distribution, $\mathbf{n}_k \sim CN(\mathbf{0},\mathbf{R_n})$, and independent of the signals. The signals samples, s_{mk}, are assumed to be unknown deterministic. In case of fully correlated signals, they can be decomposed as $s_{mk} = \mu_m s_k,\ \ m = 1,\cdots,M\ , k = 1,\cdots,K$, where μ_m denotes the relative amplitude and phase of the mth signal. Thus, Eq. (4) can be rewritten in the form:

$$\mathbf{y}_k = \sum_{m=1}^{M} \mathbf{F}(\theta_m,\phi_m)\underbrace{\mathbf{p}_m\mu_m}_{\boldsymbol{\zeta}_m} s_k + \mathbf{n}_k\ ,\ \ k = 1,\cdots,K\ . \tag{5}$$

The unknown parameters space can now be reduced to the source directions $\{\theta_m,\phi_m\}_{m=1}^{M}$, the modified signal polarization vector, $\{\boldsymbol{\zeta}_m\}_{m=1}^{M}$, and the signal $\mathbf{s} \triangleq (s_1,\cdots,s_K)^T$.

The problem addressed in this paper is to estimate the directions of arrival, $(\boldsymbol{\theta},\boldsymbol{\phi})$, while the signal vector $\mathbf{s}$ and the vector of the modified signal polarizations of the M arrivals, $(\zeta_1,\cdots,\zeta_M)$, are unknown complex vector parameters.

3. THE VECTOR SENSOR SMOOTHING (VSS) ALGORITHM

The eigenstructure-based techniques for source localization such as MUSIC, rely on identification of the signal and noise subspaces. In the presence of fully correlated signals the dimension of the signal subspace is smaller than the number of signals, M, and therefore, the signal subspace does not span the M-dimensional subspace of the spatial transfer functions, $\mathbf{g}(\theta_1,\phi_1,\mathbf{p}_1),\cdots,\mathbf{g}(\theta_M,\phi_M,\mathbf{p}_M)$. In this case, it is required to employ the information on the structure of the spatial transfer function, $\mathbf{g}(\theta,\phi,\mathbf{p})$, in order to determine its subspace. Spatial smoothing [7], redundancy averaging [8] and forward-backward averaging techniques utilize the information on the structure of the spatial transfer function in order to estimate this subspace or part of it. The deficiency of the spatial smoothing method is the reduction of the effective array aperture length resulting in lower resolution and accuracy, while the deficiency of the redundancy averaging method is that its estimation errors bias does not vanish asymptotically for large number of measurements. In addition, both approaches are limited to the case of a linear equally spaced (LES) sensor array with far-field approximation. The forward-backward averaging method assumes a symmetric array, far-field approximation and unequal signal phases at the center of the array.

In the proposed method, the vector sensor information is used in order to determine the subspace spanned by the steering vectors $\mathbf{q}(\theta_1,\phi_1),\cdots,\mathbf{q}(\theta_M,\phi_M)$, which enables estimation of direction of arrivals using eigenstructure-based methods, such as MUSIC. This objective can be obtained by Vector Sensor Smoothing (VSS) method as described below.

By substitution of (2) into (5), the measurement model at the array can be written in the form

$$\mathbf{y}_k = \sum_{m=1}^{M} [\mathbf{q}(\theta_m,\phi_m) \otimes \mathbf{A}(\theta_m,\phi_m)]\,\boldsymbol{\zeta}_m s_k + \mathbf{n}_k\ . \tag{6}$$

If we consider only the sensors of type l $(1 \leq l \leq L)$, then the corresponding measurement vector, $\mathbf{y}_{lk}$, can be expressed as

$$\mathbf{y}_{lk} = \sum_{m=1}^{M} [\mathbf{q}(\theta_m,\phi_m) \otimes \mathbf{A}_l(\theta_m,\phi_m)]\,\boldsymbol{\zeta}_m s_k + \mathbf{n}_{lk}\ , \tag{7}$$

which can be simplified to

$$\mathbf{y}_{lk} = \sum_{m=1}^{M} \mathbf{q}(\theta_m, \phi_m) z_{ml} s_k + \mathbf{n}_{lk} \ , \qquad (8)$$

where $\mathbf{A}_l(\theta_m, \phi_m)$ is the lth row of the matrix $\mathbf{A}(\theta_m, \phi_m)$, the scalar $z_{ml} = \mathbf{A}_l(\theta_m, \phi_m)\boldsymbol{\zeta}_m$ denotes the response of the lth type sensor for DOA: θ_m, ϕ_m, and $\mathbf{n}_{lk}$ stands for the corresponding noise vector. This observation implies that each type of sensor array measurements provides a different linear combination of the vectors $\mathbf{q}(\theta_1, \phi_1), \cdots, \mathbf{q}(\theta_M, \phi_M)$, as it would be the case for uncorrelated signals. The information acquired by the L different sensor types helps to obtain a measurement space in which the signals are not fully correlated. We utilize this concept in order to span the signal subspace, which is a necessary requirement of the eigenstructure-based algorithms for source localization.

Eq. (8) can be rewritten in matrix form as

$$\mathbf{y}_{lk} = \mathbf{Q}(\boldsymbol{\theta}, \boldsymbol{\phi})\mathbf{z}_l s_k + \mathbf{n}_{lk} \ , k = 1, \cdots, K, \ \ l = 1, \cdots, L \ , \ (9)$$

where $\mathbf{Q}(\boldsymbol{\theta}, \boldsymbol{\phi}) \triangleq [\mathbf{q}(\theta_1, \phi_1), \ \cdots, \ \mathbf{q}(\theta_M, \phi_M)]$ and $\mathbf{z}_l \triangleq [z_{1l}, \cdots, z_{Ml}]^T$. Therefore, the covariance matrix of each sensor type is given by

$$\mathbf{R}_{\mathbf{y}\,l} = E\left[\mathbf{y}_{lk}\mathbf{y}_{lk}^H\right] = \sigma_s^2 \mathbf{Q}\mathbf{z}_l \mathbf{z}_l^H \mathbf{Q}^H + \mathbf{R}_{\mathbf{n}\,l} \ , \ \ l = 1, \cdots, L \ , \qquad (10)$$

in which $\sigma_s^2 = E\,|s_k|^2$ denotes the signal power and $\mathbf{R}_{\mathbf{n}\,l}$ denotes the corresponding noise covariance matrix. In this problem, the M signals are fully correlated. Accordingly, the signal covariance matrix of each sensor array type, $\sigma_s^2 \mathbf{Q}\mathbf{z}_l \mathbf{z}_l^H \mathbf{Q}^H$ is of rank one. In the proposed VSS method, the covariance matrices $\{\mathbf{R}_{\mathbf{y}\,l}\}_{l=1}^L$, are smoothed for the L elements of the vector sensor. Consequently, the signal subspace is extended by averaging the L sensor type covariance matrices, i.e.

$$\mathbf{R} = \frac{1}{L}\sum_{l=1}^{L} \mathbf{R}_{\mathbf{y}\,l} = \sigma_s^2 \mathbf{Q}\mathbf{R}_{\mathbf{z}}\mathbf{Q}^H + \frac{1}{L}\sum_{l=1}^{L} \mathbf{R}_{\mathbf{n}\,l} \ , \qquad (11)$$

in which $\mathbf{R}_{\mathbf{z}}$ is defined as $\mathbf{R}_{\mathbf{z}} \triangleq \frac{1}{L}\sum_{l=1}^{L} \mathbf{z}_l \mathbf{z}_l^H$. The rank of the smoothed signal covariance matrix is limited by $\min(rank(\mathbf{R}_{\mathbf{z}}), M)$. Algorithms such as MUSIC, ESPRIT, etc, can use the corresponding sample covariance matrix, $\hat{\mathbf{R}} = \frac{1}{KL}\sum_{l=1}^{L}\sum_{k=1}^{K} \mathbf{y}_{lk}\mathbf{y}_{lk}^H$, with steering function, $\mathbf{q}(\theta, \phi)$ for source localization.

For determination of the M-dimensional signal subspace, it is required that $M \leq \min(L, N)$. This requirement can be alleviated if one can use other methods for signal decorrelation. For example, by applying the Forward-Backward averaging, the maximum number of the fully correlated signals, which can be localized, is doubled.

The vector sensor array contains NL sensors and therefore NL receivers are required for data collection. However, the VSS computes the smoothed covariance matrix by averaging the $N \times N$ matrices $\{\mathbf{R}_{\mathbf{y}\,l}\}_{l=1}^L$. In stationary scenarios, these matrices can be calculated in different periods. This implies that by an appropriate switching scheme, one can use N receivers in order to collect the required data.

4. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed techniques for different vector sensor types and scenarios. Simulations are conducted with the MUSIC using the following preprocessing techniques: 1) No preprocessing, 2) Forward-Backward averaging (FB), 3) VSS and 4) VSS combined with Forward-Backward averaging (VSS-FB). The CRB for source localization using a vector sensor array was derived in [2] and calculated for the scenarios presented below.

For the simulations, a 12 elements linear array of vector sensors along the y-axis with half-wavelength inter-element spacing was chosen. Three types of vector sensors were considered:

- A - Vertical polarized sensor (see sensor no. 3 in Fig. 1) - scalar sensor case. The VSS preprocessing cannot be applied in this case.
- B - Dual polarized vector sensor (see sensors no. 1 and 3 in Fig. 1 with $\delta = 0°$) - the vector sensors consist of vertical and horizontal electric dipoles.
- C - Quadrature polarized vector sensor (see sensors no. 1, 2, 4 and 5 in Fig. 1 with $\delta = 45°$) - the vector sensors consist of two electric dipoles and two magnetic dipoles. The value of w, in Eq. (1) was chosen to be 1. In the simulations we assumed that the sources were located in the azimuth plane, $\theta = 90°$.

In the first scenario, two equal power, fully correlated sources with DOA's $4°, 0°$ and elliptical polarizations, $\mathbf{p}_1 = (0.707e^{-j60°}, 0.707)$, $\mathbf{p}_2 = (0.707e^{j80°}, 0.707)$ were considered. The phase difference between the two incident signals at the origin was $110°$ such that $\boldsymbol{\zeta}_1 = (0.707e^{j50°}, 0.707e^{j110°})$, $\boldsymbol{\zeta}_2 = (0.707e^{j80°}, 0.707)$. The number of samples taken from the array was 100.

Fig. 2 shows the root-mean-square error (RSME) of the first source DOA estimation versus signal-to-noise ratio (SNR). The VSS-MUSIC and VSS-FB-MUSIC achieve the CRB, although at higher SNR's. It can be seen that the RMSE of the FB-MUSIC decreases as the SNR increases, but it is not an efficient estimator even asymptotically. MUSIC with no preprocessing fails as expected in case of fully correlated sources.

Fig. 3 presents the spatial spectrum of the MUSIC algorithm for the three types of vector sensors, mentioned above, and different methods of preprocessing. Eight equal power, fully correlated sources with DOA's $-70°$, $-50°$, $-30°$, $-10°$, $0°$, $20°$, $40°$, $60°$ and randomly chosen polarizations were considered. The SNR's of all the signals were 15 dB and 100 samples were collected from the array. The MUSIC and FB-MUSIC algorithms were applied to an array with vector sensors of type A, while the VSS-MUSIC and VSS-FB-MUSIC algorithms were applied to array of vector sensors of types B and C. This figure shows that only the VSS-FB-MUSIC with type C vector sensor array is able to resolve the 8 fully correlated signals, while all other methods fail.

5. CONCLUSIONS

The problem of fully correlated source localization using vector sensor array was addressed in this paper. A novel preprocessing method based on vector sensor smoothing (VSS) for "decorrelating" the signals was presented. This method enables the use of eigenstructure-based techniques, such as MUSIC, for fully correlated signals. In contrast to other preprocessing methods, such as spatial smoothing, forward-backward averaging and redundancy averaging, the VSS method is not limited to any specific array structure. By combining the VSS and forward-backward averaging, one is able to resolve up to $\min(N-1, 2L)$ coherent sources where L is the number of dipoles in each of the N vector sensors in the array. Simulations were carried out to evaluate the performance of the proposed method combined with MUSIC. The method was compared to forward-backward averaging and CRB, and it was shown to be asymptotically efficient.

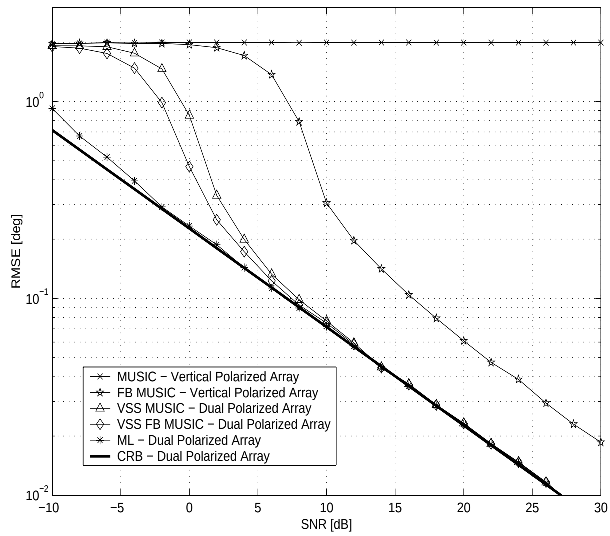

Figure 2: DOA estimation RMSE versus SNR.

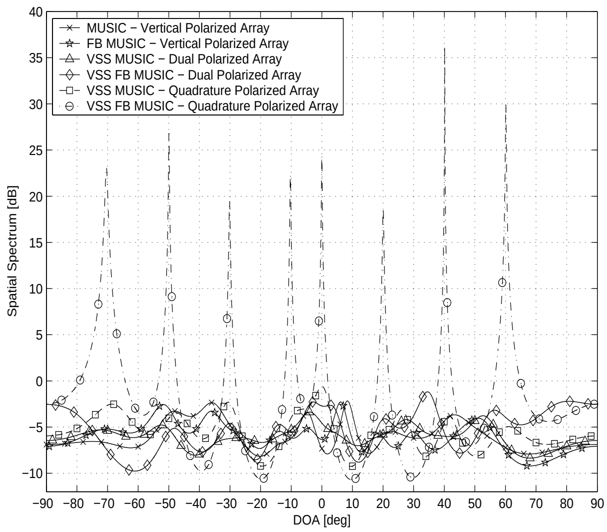

Figure 3: Spatial spectrum of MUSIC with different preprocessing techniques and vector sensor structures with 8 fully correlated sources at $-70°$, $-50°$, $-30°$, $-10°$, $0°$, $20°$, $40°$, $60°$.

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