DECODING ALGORITHMS FOR RECONFIGURABLE SPACE-TIME TURBO CODES

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ABSTRACT*

In this paper the flexibility of Space-Time Turbo Codes to work with different antenna configurations and constellations is examined. Original contributions are presented for the enhancement of current decoding algorithms and their extension to four transmit antennas. Besides, a metric is proposed for the evaluation of Quality of Service (QoS) without explicit PER measurement.

1. INTRODUCTION

The motivation of this paper is the search for space-time coding schemes that can be used in reconfigurable transceivers for the provision of spectrally efficient radio links [2]

Space-Time Turbo Codes (STTC) can be regarded as an instance of the concatenation of a Turbo Code (TC) and Space Division Multiplexing (SDM) scheme or as a Space-Time Bit-Interleaved Modulation [11] where, in both cases, the number of transmit antennas is matched to the TC code rate. From this point of view, STTC can accommodate different constellations, transmit/receive antenna configurations and coding rates in order to fulfill the target PER with minimum cost/complexity. However, the matching between the code rate and the number of antennas in STTC also allows to use specific decoding algorithms that offer different complexity vs performance trade-offs.

In this paper we address the problem of STTC decoding for different numbers of transmit antennas (2,3,4) and different constellations (BPSK, QPSK). We show that the same algorithms can be applied in all cases with minor changes in the decoder structure, and we point out that performance of previously proposed algorithms can be improved with a negligible complexity increase. Finally, a metric is proposed to evaluate the QoS without requiring explicit measurement of the Packet Error Rate (PER). The advantage of the new metric is that it synthesizes in one single parameter the dependence of the PER on the Eb/No, antenna configuration and delay spread.

2. SYSTEM MODEL

Several configurations for STTC have been proposed in the literature. In this paper we adopt the same approach as [9] and [10]. The proposed STTC is based on the parallel concatenation of two or three identical binary systematic convolutional codes (PCCC) with generator polynomials 7/5 (see Fig.1). The coded bits corresponding to each branch of the PCCC will be denoted as c_i. As the paper does not aim to analyze the performance of a specific code but to show the flexibility of STTC schemes, this simple code was selected. Note that the same convolutional code (CC) has been used in all configurations, minimizing hardware complexity. The interleavers were designed as S-random interleavers and full diversity conditions [7] were satisfied for all antenna configurations. A frame length of 2·54 bytes was selected.

The propagation channel is modeled as a MIMO (Multiple Input Multiple Output) Rayleigh fading channel. Two possible scenarios have been considered in the simulations. First, some results are shown for the frequency flat fading channel (section 3) and the frequency selective channel (sections 4-6). In the first case, the turbo code outputs were BPSK modulated and were sent to the antennas. In the second case, the symbols applied to each antenna are OFDM modulated using the same parameters as HIPERLAN/2 (BPSK or QPSK, 48 data carriers).

The use of OFDM decouples the frequency selective channel into a set of frequency flat fading channels, so the signal model is essentially the same in all cases: at the subcarrier basis the received signal $\boldsymbol{r}$ is written as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \qquad [\mathbf{H}]_{ij} = h_{ij} \quad i = 1,\ldots,M \quad j = 1,\ldots,N$$

where $\mathbf{H}$ stands for the MIMO channel matrix of size M×N and $\mathbf{n}$ for the additive white Gaussian noise $N(\mathbf{0},\sigma^2\mathbf{I})$.

3. DECODING ALGORITHMS

The iterative algorithm used for turbo decoding relies on the assumption that transmitted symbols can be observed at the receiver independently. However, this assumption fails in STTC's because all signals are superimposed at the receiver. In this section we review the decoding algorithms for STTC that have been proposed in the literature (called hereafter *IR, JR*) and we show how a minor modification that takes into account the MIMO channel effect can improve significantly the performance of both of them with nearly no additional complexity (the new algorithms are called here *Enhanced IR* and *Enhanced JR).*

The four alternatives are based on the well-known BCJR algorithm, so we assume that the reader is familiar with the notation in [1] and we only list the most important equations for the decoding of the first CC (output bits c_1, c_2) in the 2 or 3 transmit antenna configuration and BPSK modulation. For a more complete description of the four algorithms refer to [3].

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Two problems arise in the application of turbo decoding to STTC. First, to compute trellis transition metrics, the receiver needs to evaluate $P(\mathbf{r}|\ c_1,c_2)$ but it can only have access to $P(\mathbf{r}|c_1,c_2,c_3)$. Second, the iterative decoder relies on the assumption that the two constituent codes exchange uncorrelated decisions (the extrinsic ratios), but this is difficult to satisfy since both decoders base their decisions on the same observation $\mathbf{r}$. The algorithms described next approach these problems differently.

In the paper LLR^{ap_i} and LLR^{ext} denote the a priori information coming from the i-th decoder and the extrinsic information delivered by the current decoder. Besides, the following conventions are used to simplify notation:

$$LLR_i(a)=\log\frac{P(c_i=a)}{P(c_i=0)} \qquad LLR_{ij}(a,b)=\log\frac{P(c_i=a,c_j=b)}{P(c_i=0,c_j=0)}$$

$$P_{c_i}(\mathbf{r}\,|\,a)=P(\mathbf{r}\,|\,c_i=a) \quad P_{c_ic_j}(\mathbf{r}\,|\,a,b)=P(\mathbf{r}\,|\,c_i=a,c_j=b)$$

$$P_{c_ic_jc_k}(\mathbf{r}\,|\,a,b,c)=P(\mathbf{r}\,|\,c_i=a,c_j=b,c_k=c)$$
$$P_{c_ic_jc_kc_l}(\mathbf{r}\,|\,a,b,c,d)=P(\mathbf{r}\,|\,c_i=a,c_j=b,c_k=c,c_l=d)$$

3.1. Independent ratios (IR)

This algorithm [9] evaluates one soft value for each input bit c_i, given by:

$$\Delta(c_i=a)=\log\frac{P_{c_i}(\mathbf{r}\,|\,a)}{P_{c_i}(\mathbf{r}\,|\,0)}=\log\frac{\sum_{\mathbf{x}:c_i=a}P(\mathbf{r}\,|\,\mathbf{x})}{\sum_{\mathbf{x}:c_i=0}P(\mathbf{r}\,|\,\mathbf{x})}=$$
$$=\log\frac{\sum_{\mathbf{x}:c_i=a}\prod_{j=1}^{M}\exp\left(-\frac{1}{\sigma^2}\left\|r_j-\sum_{z=1}^{N}h_{zj}x_z\right\|^2\right)}{\sum_{\mathbf{x}:c_i=0}\prod_{j=1}^{M}\exp\left(-\frac{1}{\sigma^2}\left\|r_j-\sum_{z=1}^{N}h_{zj}x_z\right\|^2\right)}$$

Afterwards, the IR algorithm applies the traditional decoder used for a TC, using Δ as the trellis transition metrics:

$$\log P_{c_1c_2}(\mathbf{r}\,|\,a,b)\cong\Delta(c_1=a)+\Delta(c_2=b) \tag{1}$$

This approach is widely used in SDM and STTC because of its flexibility and in traditional turbo codes (e.g [8]). However the approximation in equation (1) is only true when the channel matrix is diagonal, a situation that never appears in practice.

3.2. Enhanced Independent Ratios (EIR)

A minor change in equation (1) allows to improve significantly the performance of the decoder. The independence assumption required to apply (1) can be avoided computing one soft value for each trellis transition (including one information bit and one redundancy bit):

$$\Delta(c_1=a,c_2=b)=\log\frac{P_{c_1c_2}(\mathbf{r}\,|\,a,b)}{P_{c_1c_2}(\mathbf{r}\,|\,0,0)}=\log\frac{\sum_{\mathbf{x}:c_1=a,c_2=b}P(\mathbf{r}\,|\,\mathbf{x})}{\sum_{\mathbf{x}:c_1=0.c_2=0}P(\mathbf{r}\,|\,\mathbf{x})}$$

Thus, the transition metrics can be computed without the need of the approximation in IR algorithm: $P_{c_1c_2}(\mathbf{r}|a,b)=\Delta(c_1=a,\ c_2=b)$

Simulations will show that this small change improves performance significantly.

3.3. Joint Ratios (JR)

The algorithm in [10] is based on the definition of joint ratios for the systematic and the redundancy bit of each constituent code:

$$LLR_{1,2}(a,b)=\log\left(\frac{\sum_{(s',s)(c_1,c_i)=(a,b)}\alpha(s')\cdot\gamma(s',s)\cdot\beta(s)}{\sum_{(s',s)(c_1,c_i)=(0,0)}\alpha(s')\cdot\gamma(s',s)\cdot\beta(s)}\right) \tag{2}$$

This simplifies the exchange of extrinsic information between decoders so the transition metrics can be evaluated without approximations as follows:

$$\log\gamma(s',s)=\Gamma\Big(LLR_{1,3}^{ap}(a,1)+\log P_{c_1c_2c_3}(\mathbf{r}\,|\,a,b,1),\; LLR_{1,3}^{ap}(a,0)+\log P_{c_1c_2c_3}(\mathbf{r}\,|\,a,b,0)\Big)$$

where Γ(x,y)=log(e^x+e^y). However, this algorithm does not compute a truly extrinsic ratio. The proposed metrics:

$$LLR_{1,2}^{ext}(a,b)=LLR_{1,2}(a,b)-LLR_1^{ap}(a) \tag{3}$$

are correlated with the channel observations, and this results in a performance degradation of iterative decoding.

3.4. Enhanced Joint Ratios (EJR)

A minor change in equation (3) allows to improve significantly the performance of the JR in [10]. Equation (2) can be rephrased as:

$$LLR_{1,2}(a,b)=\log\left(\frac{\gamma(s',s)_{(a,b)}}{\gamma(s',s)_{(0,0)}}\right)+\log\left(\frac{\sum_{(s',s)(c_1,c_2)=(a,b)}\alpha(s')\cdot\beta(s)}{\sum_{(s',s)(c_1,c_2)=(0,0)}\alpha(s')\cdot\beta(s)}\right)$$

Thus, it can be seen in this equation that the channel observation can be easily subtracted from the LLR to lower the correlation of the extrinsic information:

$$LLR_{1,2}^{ext}(a,b)=LLR_{1,2}(a,b)-LLR_1^{ap}(a)-\log\left(\frac{\gamma(s',s)_{(a,b)}}{\gamma(s',s)_{(0,0)}}\right)$$

In [3] another alternative is presented for the definition of LLR^{ext} that yields similar performance.

3.5. Performance comparison

A comparison on the performance of the previously introduced decoding algorithms is presented in Figure 2. Simulations have been carried out in the Frequency Flat Fading channel, but the extension to the OFDM modulation leads to similar conclusions. For this comparison the 3x2 configuration was simulated in a 1% Doppler spread channel running three decoding iterations.

The IR algorithm is shown to offer the poorest performance due to its strong assumptions, but it also has lowest complexity. The minor change introduced to derive its enhanced version (EIR) leads to a significant BER reduction while its increase in complexity is very small. Nevertheless, the best results are obtained by joint-ratios based decoding algorithms, which model more accurately the MIMO scenario. In this case the improved computation of the extrinsic ratios in EJR also provides a performance improvement over JR, even though the difference is not as large as for IR and EIR.

4. TRANSMITTER CONSTELLATION

The STTC's in the literature work mostly with BPSK, since its analysis is simplest. In this section, a possible extension to QPSK is presented.

The same binary CC presented in Figure 1 was implemented for flexibility reasons, even though best performance could be achieved if a double binary code were used. The QPSK mapping was implemented by grouping pairs of bits at the CC output and using a symbol interleaver. To keep complexity low, the EIR algorithm was applied to a MAP decoder for the non-binary equivalent trellis working on trellis steps of two bits. This only introduced a small increase in complexity compared to BPSK since, although four LLR's were required, the length of the trellis was halved.

Figure 3 compares the performance for BPSK and QPSK simulation in a 3x3 configuration for a W-LAN transmission system with an exponential power delay profile with delay spread 50ns (transmission rate 12 and 24Mbps respectively). As expected, QPSK incurs in some losses with respect to BPSK that depend on the antenna configuration and, according to the figure, they range from 0.9 to 1.6 dB.

5. NUMBER OF TRANSMIT ANTENNAS

The STTC's analyzed in the literature only consider two or three transmit antennas. However, the configuration of four transmit antennas has a great interest. In this paper we propose the extension of the decoding algorithms in section 3 to four antennas based on the results described in [6] for a TC with three CC. More specifically, we show here the equations for the JR algorithm with four antennas based on option ES of [6]. For a more detailed discussion the reader is referred to [4]. It can be shown that the extension to four antennas boils down to the inclusion of the term $-\frac{1}{2}(LLR_1^{ap_3}(a)+LLR_1^{ap_4}(a))$ in the computation of state accumulated metrics (*alphas, betas*) and LLR's, and to the extension of the computation of metric transitions *gammas* to include four terms instead of two:

$$LLR_{1,2}(a,b) = -\frac{1}{2}\left(LLR_1^{ap_3}(a)+LLR_1^{ap_4}(a)\right)+$$
$$\log\left(\frac{\gamma(s',s)_{(a,b)}}{\gamma(s',s)_{(0,0)}}\right)+\log\left(\frac{\sum_{(s',s):(c_1,c_2)=(a,b)}\alpha_{k-1}(s')\cdot\beta_k(s)}{\sum_{(s',s):(c_1,c_2)=(0,0)}\alpha_{k-1}(s')\cdot\beta_k(s)}\right)$$
$$\log\gamma(s',s)=\Gamma\Big(LLR_{1,3}^{ap}(a,1)+LLR_{1,4}^{ap}(a,1)+\log P_{c_1c_2c_3c_4}(r\mid a,b,1,1),$$
$$LLR_{1,3}^{ap}(a,1)+LLR_{1,4}^{ap}(a,0)+\log P_{c_1c_2c_3c_4}(r\mid a,b,1,0),$$
$$LLR_{1,3}^{ap}(a,0)+LLR_{1,4}^{ap}(a,1)+\log P_{c_1c_2c_3c_4}(r\mid a,b,0,1)$$
$$LLR_{1,3}^{ap}(a,0)+LLR_{1,4}^{ap}(a,0)+\log P_{c_1c_2c_3c_4}(r\mid a,b,0,0)\Big)$$
$$LLR_{1,2}^{ext}(a,b)=LLR_{1,2}(a,b)-\frac{1}{2}\left(LLR_1^{ap_3}(a)+LLR_1^{ap_4}(a)\right)$$

Therefore, even though equations may seem complex at first sight, the complexity is doubled only in the computation of γ. The same decoder used for two or three antenna transmission can work with four antennas if minor changes are introduced in its equations.

Figure 3 compares the performance using three and four transmit antennas. The configuration with four antennas outperforms that one with three at high Eb/No, thanks to the increased slope of the curve, as was expected. For this particular scenario the cross-over between the two plots appears at PER values close to the typical target value PER=10^{-2}, so in order to benefit from the gain provided by the four antennas other code/interleaver configurations should be tested.

6. MONITORISATION OF PER FOR LINK ADAPTATION

Reconfigurable systems adaptively change the modulation and coding scheme according to channel conditions to satisfy a certain target QoS, that is usually measured in terms of the PER, transmission delay, etc. Hence, the performance of reconfigurable systems depends on their ability to track of these parameters.

While direct measurement of the PER is too time consuming, the alternative of inferring its value through the measurement of the SIR, delay spread, etc is not desirable in the multiple antenna case because the number of parameters to take into account is very large.

In this paper we propose a single metric that summarizes many of the parameters that define the PER for a given scenario. This metric is based on the measurement of the mean and variance of the STTC decoder soft outputs of error-free frames.

It is well known that the statistics of the LLR's of a TC in a binary transmission system on a AWGN channel follow a Gaussian distribution [5]. The statistics of the STTC decoder output are more involved due to the space and frequency diversity provided by the MIMO Rayleigh channel. Figure 4 depicts the probability density function (p.d.f.) of the LLR's at the STTC output for several combinations of transmission scheme and channel delay spread that provide the same PER=10^{-2}, illustrating how different the p.d.f. can be. In spite of that, it can be analytically shown that for the SIMO (Single Input Multiple Output) case there is a one to one relationship between the mean of the LLR's (μ_{LLR}), their variance (σ^2_{LLR}) and the uncoded BER through the following parameter when the M is large.

$$\chi = \frac{\mu_{LLR}^3}{\sigma_{LLR}^2}$$

Furthermore, simulation results for the STTC with N={2,3}, M={2,3,4}, different delay spreads and decoding algorithms show that the dependency of the PER and coded BER on these parameters is mostly summarized in the metric χ as well. Figure 5 shows that all configurations are aligned or correspond to parallel curves in the low BER region of the plot BER=f(χ), in spite of the fact that their respective plots BER=f(Eb/No) are very different for all of them.

Therefore, the parameter χ summarizes most of the information on the transmission/reception scheme and scenario, so it can be used monitor the PER in a simple manner.

Detailed analysis of the LLR statistics of the STTC is now under investigation by the authors.

7. CONCLUSIONS

In this paper STTC's have been analyzed from the reconfigurability point of view. Contribution on decoding algorithms include the improvement of existing algorithms and the extension of STTC to four transmit antennas. Thus, the proposed algorithms can be applied to 2,3,4 transmit antennas (and any number of receive antennas) with nearly no changes in the decoding algorithms. Moreover, a new metric is provided for QoS evaluation purposes that can be used to reduce considerably the number of parameters that are required for link adaptation.

8. REFERENCES

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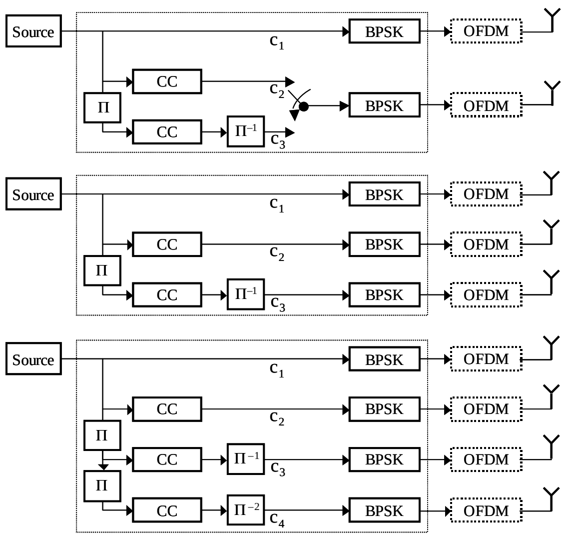

Figure 1. STTC for transmission with 2, 3, 4 antennas

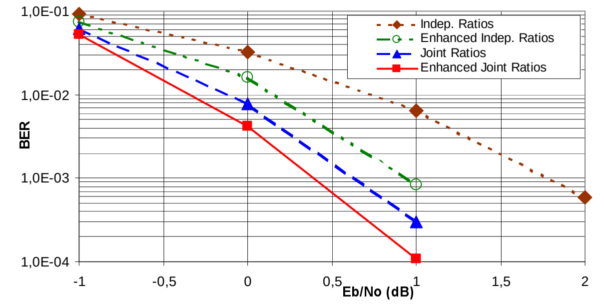

Figure 2.BER of IR, EIR, JR, EJR algorithms for 3 decoding iterations

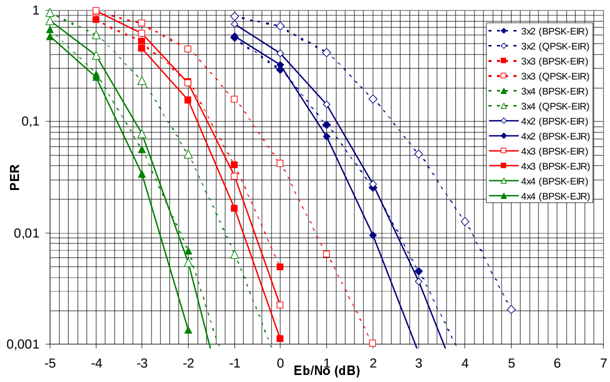

Figure 3. PER for several antenna/constellation configurations and 6 decoding iterations

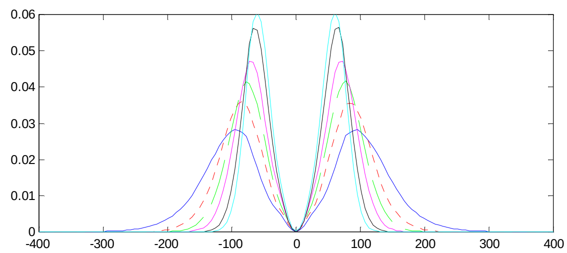

Figure 4. Probability Density Function of LLR's

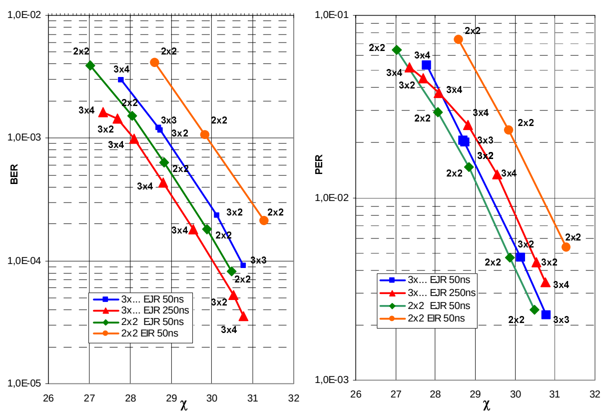

Figure 5. PER performance in terms of parameter χ for different antenna configurations, channel delay spreads and decoding algorithms (BPSK constellation, 6 decoding iterations)