

# CONSTRAINED LS ALGORITHM FOR CHANNEL VECTOR ESTIMATION IN 2-D RAKE RECEIVER

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## ABSTRACT

In conventional 2-D RAKE receiver, channel vector is usually estimated based on LS criterion only using pilot. When the power of the pilot is low or the variation of the channel is fast, the error of the channel estimation becomes large. We propose an improved algorithm in this paper. The steering vector of the desired signal is estimated firstly using subspace decomposition method and a constrained condition is then configured employing this estimated steering vector. Channel vector is estimated based on constrained LS criterion applying traffic signals. The pilot is only used to get the initial value of the estimation. The estimation accuracy of channel vector can progressively increase through iteration. Simulation results demonstrate that the improved algorithm upgrades the estimation accuracy effectively.

## 1. INTRODUCTION

It is well known that the use of antenna arrays can increase the link capacity of mobile radio systems considerably. Various algorithms of space-time signal processing can be used to enhance the system performance. In CDMA system, 2-D RAKE receiver is in common use. The essence of 2-D RAKE receiver is to calculate a vector weighting the signals received by the array to make the system performance as optimum as possible. The calculation of this vector is determined by channel vector usually [1]. Thus, the estimation of channel vector takes an important role in 2-D RAKE receiver.

In the conventional 2-D RAKE receiver, channel vector estimation is usually performed based on least squares (LS) criteria when continuous pilot is available [2]. The accuracy of estimation is determined by two factors, one is the time window, and the other is the pilot power. The wider the time window or the higher the pilot power,

the more accurate the estimation is. However, the bottlenecks that time-varying channel restricts the window length and the high pilot power introduces high capacity loss restrain the high accuracy achievable in the conventional receiver.

To solve this problem, we propose an improved channel vector estimation algorithm in this paper. In mobile communications, the variation of DOA (direction of arrival) is slower than that of the amplitude and phase [3]. The new algorithm exploits this character to estimate the steering vector in a relatively wider time window and uses the estimated result to set a constraint condition. Then, we use traffic signals to estimate the channel vector based on constrained LS criteria. The solution can be obtained using the Iterative LS with Projection (ILSP) algorithm [4]. The pilot is only employed to provide the initial estimate for the ILSP algorithm. The iteration in the estimate procedure can render high accuracy performance even if the pilot power is very low. Thus, two bottlenecks that the conventional LS algorithm suffers are mitigated.

## 2. SYSTEM MODEL

The up-link of mobile communication system is considered in this paper. The base station is equipped with  $M$ -element  $d$ -spaced uniform linear array. Assume that there are  $K$  users in the section and the  $k$ th user has  $L_k$  paths. Also assume that the plane wave from the  $l$ th path of the  $k$ th user impinges upon the array at an angle  $\theta_{kl}$  and its corresponding delay is  $\tau_{kl}$  ( $l=1,2,\dots,L_k$ ). In this situation, the base band equivalent signals received by the array can be shown as follows:

$$\mathbf{x}(t) = \sum_{k=1}^K [A_k^t \sum_{l=1}^{L_k} \rho_{kl} \mathbf{a}(\theta_{kl}) \sum_{n=-\infty}^{\infty} b_k^t(n) c_k^t(t - nT_b - \tau_{kl}) + A_k^p \sum_{l=1}^{L_k} \rho_{kl} \mathbf{a}(\theta_{kl}) \sum_{n=-\infty}^{\infty} b_k^p(n) c_k^p(t - nT_b - \tau_{kl})] + \mathbf{n}(t) \quad (1)$$

Where  $b_k^t(n)$  and  $b_k^p(n)$  are the  $n$ th symbol of  $k$ th user for traffic channel and pilot channel respectively. In this paper, we assume the symbol is BPSK.  $c_k^t(t)$  and  $c_k^p(t)$  is the spreading waveform of  $k$ th user for traffic channel and pilot channel respectively.  $A_k^t$  is the amplitude of traffic signal and  $A_k^p$  is pilot amplitude. Without loss of generality, we assume  $A_k^t=1$  and  $A_k^p = \beta$ . Then  $\beta^2$  is the pilot-to-traffic power ratio.  $T_b$  is the symbol interval and  $\mathbf{n}(t)$  is  $M$  by 1 additive gaussian noise vector.  $\rho_{kl}$  is the channel complex fading (including amplitude and phase) of  $l$ th path for  $k$ th user and  $\mathbf{a}(\theta_{kl})$  is its corresponding steering vector. We define the channel vector:

$$\mathbf{h}_{kl} = \rho_{kl} \mathbf{a}(\theta_{kl}) \quad (2)$$

An algorithm of 2-D RAKE receiver is presented in [2], Assuming the first user is the desired user, the  $m$ th path despreading signals of its pilot channel and traffic channel can be written respectively as follows [2]:

$$\mathbf{r}_{1m}^p(n) = \beta \mathbf{h}_{1m} b_1^p(n) + \mathbf{i}_{1m}^p(n) \quad (3)$$

$$\mathbf{r}_{1m}^t(n) = \mathbf{h}_{1m} b_1^t(n) + \mathbf{i}_{1m}^t(n) \quad (4)$$

If the MRC (maximal ratio combining) algorithm [5] is used, the output of 2-D RAKE receiver is

$$\hat{b}_1^t(n) = \text{sgn} \left\{ \text{real} \left( \sum_{m=1}^{L_1} \hat{\mathbf{h}}_{1m}^H \mathbf{r}_{1m}^t(n) \right) \right\} \quad (5)$$

Where  $\hat{\mathbf{h}}_{1m}$  is the estimate of  $\mathbf{h}_{1m}$ ,  $(\cdot)^H$  denotes conjugate transpose. From Eq.(5), we can know that the estimation of channel vector is important to 2-D RAKE receiver. In conventional 2-D RAKE receiver, pilot is used to estimate the channel vector based on LS criteria, which can be described as:

$$\hat{\mathbf{h}}_{1m} = \arg \min_{\mathbf{h}_{1m}} \sum_{n=1}^J \|\mathbf{r}_{1m}^p(n) - \beta \mathbf{h}_{1m} b_1^p(n)\|^2 \quad (6)$$

The solution is:

$$\hat{\mathbf{h}}_{1m} = \frac{1}{J\beta} \sum_{n=1}^J \mathbf{r}_{1m}^p(n) [b_1^p(n)]^* \quad (7)$$

It is well known that the estimation accuracy of  $\mathbf{h}_{1m}$  is limited by  $J$  and  $\beta$ . The selection of  $J$  is limited by the condition that channel vector must be almost invariable during  $J$ -symbol interval. According to Eq.(2), the channel

vector is determined by the steering vector and the fading. We know that the variation of fading is faster than that of steering vector in the practical situation. As a result,  $J$  is mainly determined by the speed of fading variation. This limits the accuracy of channel vector estimation. The next section we propose an improved algorithm.

### 3. IMPROVED ALGORITHM OF CHANNEL VECTOR ESTIMATION

Assuming that  $\theta_{1m}$  is almost invariable during the observation period, we can derive the following equation from Eq. (2), (3), and (4) :

$$\mathbf{a}_{1m}(\theta_{1m}) \mathbf{a}_{1m}^H(\theta_{1m}) = \frac{\mathbf{R}_{1m}^t - \mathbf{R}_{1m}^p}{(1 - \beta^2)E(|\rho_{1m}|^2)} \quad (8)$$

where  $\mathbf{R}_{1m}^t$  and  $\mathbf{R}_{1m}^p$  are the covariance matrices of  $\mathbf{r}_{1m}^t$  and  $\mathbf{r}_{1m}^p$  respectively. They can be estimated by time averaging as follows respectively

$$\hat{\mathbf{R}}_{1m}^t = \frac{1}{N} \sum_{n=1}^N [\mathbf{r}_{1m}^t(n) (\mathbf{r}_{1m}^t(n))^H] \quad (9)$$

$$\hat{\mathbf{R}}_{1m}^p = \frac{1}{N} \sum_{n=1}^N [\mathbf{r}_{1m}^p(n) (\mathbf{r}_{1m}^p(n))^H] \quad (10)$$

Where  $N$  is determined by the speed of DOA variation. Because the variation of the DOA is slow,  $N$  can be selected bigger than  $J$  used in conventional algorithm which is determined by the speed of fading variation. Assuming  $\mathbf{e}_{1m}$  is the eigenvector corresponding to the largest eigenvalue of  $\mathbf{R}_{1m}^t - \mathbf{R}_{1m}^p$ , and according to Eq.(8), we know that  $\mathbf{a}_{1m} \in \text{span}(\mathbf{e}_{1m})$ . Eq.(2) shows that  $\mathbf{h}_{1m}$  is in the same space as  $\mathbf{a}_{1m}$ . Thus  $\mathbf{h}_{1m} \in \text{span}(\mathbf{e}_{1m})$ . We now get the following equation[6].

$$P_{\mathbf{e}_{1m}}^\perp \mathbf{h}_{1m} = 0 \quad (11)$$

Where  $P_{\mathbf{e}_{1m}}^\perp$  is the projector on the orthogonal complement of the column space of  $\mathbf{e}_{1m}$ . Because  $\mathbf{R}_{1m}^t$  and  $\mathbf{R}_{1m}^p$  are estimated in a wider time window, the steering vector  $\mathbf{e}_{1m}$  can be estimated more accurate. Furthermore, in order to overcome the limit of pilot power, we take Eq.(11) as a

constraint condition and apply the traffic signals to estimate the channel vector based on constrained LS criteria. In this circumstance, pilot is only used to get the initial estimate.

$$\hat{\mathbf{h}}_{1m} = \arg \min_{\hat{\mathbf{h}}_{1m}} \sum_{n=1}^J \|\mathbf{r}_{1m}^t(n) - \hat{\mathbf{h}}_{1m} \hat{b}_1^t(n)\|^2$$

subject to  $P_{\mathbf{e}_{1m}}^\perp \hat{\mathbf{h}}_{1m} = 0$  (12)

The initial condition is

$$\hat{\mathbf{h}}_{1m} = \arg \min_{\hat{\mathbf{h}}_{1m}} \sum_{n=1}^J \|\mathbf{r}_{1m}^p(n) - \beta \hat{\mathbf{h}}_{1m} b_1^p(n)\|^2$$

subject to  $P_{\mathbf{e}_{1m}}^\perp \hat{\mathbf{h}}_{1m} = 0$  (13)

where  $\hat{b}_1^t(n)$  is the estimate of  $b_1^t(n)$ . The ILSP algorithm is employed to solve this problem. Its details are illustrated as follows:

Step1: let  $k=0$ , for  $m=1, \dots, L_1$ .

$$^{(0)}\hat{\mathbf{h}}_{1m} = \mathbf{e}_{1m} \mathbf{e}_{1m}^H \sum_{n=1}^J \mathbf{r}_{1m}^p(n) b_1^p(n) / J\beta$$
 (14)

Step2: set  $k=k+1$ ; for  $n=1, \dots, J$ ;  $m=1, \dots, L_1$ .

$$^{(k)}\hat{b}_1^t(n) = \text{sgn} \left( \text{real} \left( \sum_{m=1}^{L_1} {}^{(k-1)}\hat{\mathbf{h}}_{1m}^H \mathbf{r}_{1m}^t(n) \right) \right)$$
 (15)

$$^{(k)}\hat{\mathbf{h}}_{1m} = \mathbf{e}_{1m} \mathbf{e}_{1m}^H \sum_{n=1}^J \mathbf{r}_{1m}^t(n) {}^{(k)}\hat{b}_1^t(n) / J$$
 (16)

Step3: repeat step 2 until  $k>1$  and

$$^{(k)}b_1^t(n) = {}^{(k-1)}b_1^t(n) \quad (n=1, 2, \dots, J).$$
 (17)

Comparing Eq.(14) and Eq.(7), we find that the estimated result by pilot must be projected on to the  $\mathbf{e}_{1m}$ . As mentioned above,  $\mathbf{e}_{1m}$  is estimated in a wider time window. Thus, its estimation accuracy is higher. The projection procedure can eliminate the estimation error of channel vector. This algorithm also shows that the pilot is only used to get the initial estimate of the channel vector. The estimated traffic signals are feedback to re-estimate the channel. The estimation accuracy can progressively improved through iteration. Thus, this algorithm can mitigate the two bottlenecks mentioned in the abstract.

#### 4. SIMULATION RESULTS

The system parameters are as follows: the carrier frequency is 2GHz, the chip rate is 3.84Mbit/s and the spread factor is 64. The 6-element half-wavelength spaced uniform linear array is equipped in base station. The BPSK signals of desired user attain the array through Rayleigh fading channel. Assuming there are 4 paths impinge the array with injection angle  $\{40^\circ, 10^\circ, 60^\circ, 80^\circ\}$ , average power  $\{1, 0.72, 0.25, 0.03\}$  and delay chips  $\{0, 2, 3, 5\}$ , respectively. The vehicular velocity is 100km/h and the maximal Doppler frequency shift is 185Hz. During simulations, the DOA varies  $0.001^\circ$  per symbol. The time window length is 32-symbol interval in the fading estimation and 1000-symbol interval in the steering vector estimation.

The normalized mean square error (NMSE) of the channel vector estimation and symbol error rate(SER) using different algorithm are given in Fig.1 and Fig.2 respectively, here the pilot-to-traffic power ratio (PTR) is 0.25. According to the figures, the performances of improved algorithm are better than the conventional LS algorithm under various signal to interference plus noise ratio (SINR) conditions. The reason is that the improved algorithm employs the pilot and traffic information together, and exploits the character that DOA varies more slowly than the fading at the same time.

Fig.3 compares the NMSE performances of two algorithms in different PTR when the SINR is  $-3$  dB. The figure shows that the estimation error increases with the drop of the pilot power in conventional LS algorithm. In the improved algorithm, however, the pilot is only used to get the initial estimate, it is the iterative procedure that progressively increases the accuracy of channel vector estimation, thus the estimation error is nearly constant.

Fig.4 illustrates the average iteration number for different SINR when the PTR is 0.25. It can be observed from the figure that the iteration number decreases when the SINR increases. It is because the initial estimate becomes more accuracy when the SINR increases. This will accelerate the algorithm's convergence speed. Fig.4 also shows that the iteration number is small even when the SINR is  $-6$ dB. This demonstrates that this algorithm has good convergence property.

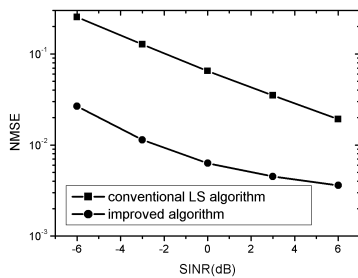


Fig.1 NMSE for different SINR with PTR=0.25

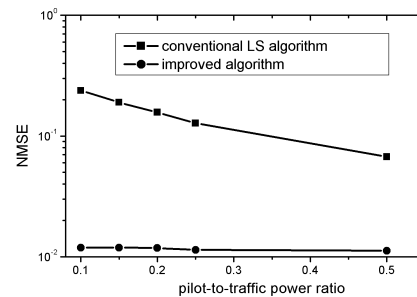


Fig.3 NMSE for different PTR with SINR=-3dB

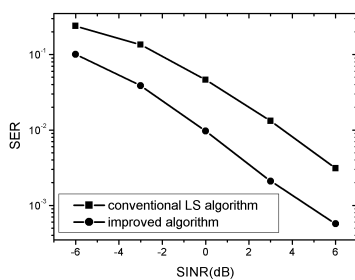


Fig.2 SER for different SINR with PTR=0.25

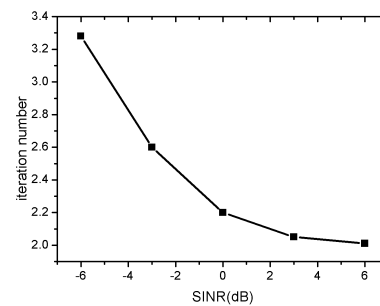


Fig.4 iteration number for different SINR with PTR=0.25

## 5. CONCLUSIONS

This paper proposes an improved algorithm to estimate the steering vector in a wider time window based on the fact that DOA varies more slowly than fading. The traffic signals, as well as the pilot, are then used to estimate the channel vector based on the constrained LS criteria. The initial value of the improved algorithm is already more accurate than that of the conventional algorithm due to projection procedure. The accuracy of channel vector estimation can further increase through iteration. The proposed algorithm efficiently solves the pilot power and time window problem that the conventional LS algorithm suffers.

## 6. REFERENCES

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