

# DECODING OF FULL RATE SPACE-TIME BLOCK CODE WITHOUT CHANNEL STATE INFORMATION IN FREQUENCY-SELECTIVE CHANNELS<sup>1</sup>

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**Abstract**—Space-time coding and modulation exploit the presence of multiple transmit antennas to improve performance on multipath radio channels. Thus far, most work on space-time coding has assumed that perfect channel estimates are available at the receiver. In certain situations, however, it may be difficult or costly to estimate the channel accurately. In this paper, we propose a new decoding algorithm for full rate space-time block coding in frequency selective channels, which accomplishes the decoding directly from the over-sampled system outputs without the knowledge of the channel. Combined with the unique feature of space-time block coding, the subspace of the transmitted signal is exploited to carry out the decoding. Monte Carlo simulations give the performance comparison of this algorithm against that with available channel information.

## 1. INTRODUCTION

Transmit diversity has gained much attention as a promising technique for improving performance on multipath radio channels [1]. Space-time block coding (STBC) discovered by Alamouti is a remarkable transmit diversity scheme to resolve the issue of decoding complexity [2]. An attractive property of space-time block codes is that maximum-likelihood decoding can be performed using only linear processing.

STBCs were originally designed for known flat fading channels. The maximum-likelihood decoding of STBCs proposed in [2] can not be applied directly to frequency-selective channels. [3] and [4] deal with applications of STBCs to frequency-selective channels assuming that perfect channel state information (CSI) is estimated at the receiver. Channel estimation requires training symbols. Increasing the number of transmit antennas increases the required training interval and reduces the available time in which data may be transmitted before the fading coefficients change.

[5]-[7] accomplish decoding or channel estimation for STBC systems in frequency-selective channel without CSI. A deterministic channel estimator was derived in [5] when the channel transfer functions are coprime (no common zeros) and the transmitted signals have constant-modulus (CM). Relying on symbol blocking and space-time redundancy, the system in [6] was shown capable of providing guaranteed symbol recovery regardless of the underlying channels. [7] enables blind subspace-based channel estimation for space time orthogonal frequency division multiplexing (ST-OFDM). However, all of them need redundant precoder and the redundancy reduces the transmit rate of symbols.

In this paper, we propose a novel decoding algorithm for full rate STBC in frequency-selective channels. It combines the blind subspace-based equalization with the unique feature of STBCs to accomplish the decoding directly from the over-sampled system

outputs without the knowledge of the channel. Because the STBC system has multiple inputs, the equalization results are still the mixture of these multiple input signals. However, thanks to the combination, we can easily work out the estimation of the transmitted symbols from the equalization results. In contrast to some decoding methods that use the information from previous data, the new approach provides a closed form solution of decoding based on a given set of outputs. This solution makes it more suitable for dynamic systems such as mobile communication channels.

The paper is organized in five sections following this introduction. Section 2 presents the system model. Section 3 includes the equalization of the system based on the subspace method, and Section 4 presents the decoding with the results of equalization, and Section 5 gives Monte Carlo simulation results. Section 6 concludes this paper.

## 2. SYSTEM AND CHANNEL MODEL

We will consider a system with  $M$  transmit antennas and one receive antenna. The extension of the result to the system with any number of receive antennas is straightforward. Let  $h_j(t)$  be the channel impulse response between transmit antenna  $j$  and the receive antenna. Without loss of generality, an important assumption we make here is that the channel response is invariant within a data burst. In many cases, the length of a data burst is smaller than the coherence time, so the assumption is also reasonable.

Let  $d_j(n)$  be the transmitted symbols from antenna  $j$ . The "noiseless" received signal due to signals transmitted from antenna  $j$  can be written as

$$r_j(t) = \sum_n d_j(n) h_j(t - nT) \quad (1)$$

The received signal is over-sampled by a factor of  $Q$ , then the sampling times are

$$t = kT + \frac{qT}{Q} \quad k = 0, 1, 2, \dots \quad q = 0, 1, 2, \dots, Q-1 \quad (2)$$

Then we have

$$r_j(kT + \frac{qT}{Q}) = \sum_n d_j(n) h_j((k-n)T + \frac{qT}{Q}) \quad (3)$$

Also, a reasonable assumption is that the pulse response will have a finite duration and hence it will have a finite number of taps. Let  $L+1$  be the number of taps. Let  $k-n=m$ . Hence we will have

$$r_j(kT + \frac{qT}{Q}) = \sum_{m=0}^L d_j(k-m) \tilde{h}_j(m, q) \quad (4)$$

where

$$\tilde{h}_j(m, q) = h_j[mT + \frac{qT}{Q}]$$

is the over-sampled discrete time channel response for transmit

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antenna  $j$ . For simplicity of notation, let

$$\tilde{\mathbf{h}}_{j,m} = [\tilde{h}_j(m,0), \tilde{h}_j(m,1), \dots, \tilde{h}_j(m, Q-1)]^T \quad (5)$$

$$\mathbf{r}_j(k) = [r_j(kT), r_j(kT + \frac{T}{Q}), \dots, r_j(kT + \frac{(Q-1)T}{Q})]^T \quad (6)$$

Therefore, we can write the noiseless discrete time model for the received signal due to transmit antenna  $j$  as

$$\mathbf{r}_j(k) = \sum_{m=0}^L \tilde{\mathbf{h}}_{j,m} d_j(k-m) \quad (7)$$

In the above discussion, we assume that the orders of all transmit antennas are  $L+1$ , but in fact they are different. We can let the maximum of the orders be  $L+1$ . For the channels with order less than  $L+1$ , the zero tail of the channel response does not affect the correct channel expression.

The total received signal can be written as:

$$\begin{aligned} \mathbf{r}(k) &= \sum_{j=0}^M \mathbf{r}_j(k) \\ &= \sum_{j=0}^M \sum_{m=0}^L \tilde{\mathbf{h}}_{j,m} d_j(k-m) \end{aligned} \quad (8)$$

### 3. BLIND EQUALIZATION

To eliminate intersymbol interference in frequency-selective channels, we equalize the received signal based on the subspace method without CSI at the receiver.

We rewrite the received signal due to transmit antenna  $j$  from symbol period  $L+1$  in the form of a Hankel matrix as

$$\begin{aligned} \mathbf{X}_j(K) &= \begin{bmatrix} \mathbf{r}_j(L+1) & \mathbf{r}_j(L+2) & \dots & \mathbf{r}_j(N-K+1) \\ \mathbf{r}_j(L+2) & \mathbf{r}_j(L+3) & \dots & \mathbf{r}_j(N-K+2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{r}_j(L+K) & \mathbf{r}_j(L+K+1) & \dots & \mathbf{r}_j(N) \end{bmatrix} \\ &= \begin{bmatrix} \tilde{\mathbf{h}}_{j,L} & \tilde{\mathbf{h}}_{j,L-1} & \dots & \tilde{\mathbf{h}}_{j,0} & \mathbf{0} & \dots & \dots & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{h}}_{j,L} & \tilde{\mathbf{h}}_{j,L-1} & \dots & \tilde{\mathbf{h}}_{j,0} & & & \mathbf{0} \\ \vdots & & & & & & & \vdots \\ \mathbf{0} & \dots & \dots & \dots & \mathbf{0} & \tilde{\mathbf{h}}_{j,L} & \tilde{\mathbf{h}}_{j,L-1} & \dots \tilde{\mathbf{h}}_{j,0} \end{bmatrix} \\ &\quad \bullet \begin{bmatrix} d_j(1) & d_j(2) & \dots & d_j(N-r+1) \\ d_j(2) & d_j(3) & \dots & d_j(N-r+2) \\ \vdots & \vdots & \ddots & \vdots \\ d_j(r) & d_j(r+1) & \dots & d_j(N) \end{bmatrix} \\ &= \mathbf{H}_j \mathbf{D}_j(r) \end{aligned} \quad (9)$$

where  $r = K+L$ , and  $K=1,2,\dots$  is a smooth coefficient.  $\mathbf{H}_j$  is a  $KQ \times r$  Toeplitz matrix.  $\mathbf{D}_j(r)$  is a  $r \times (N-r+1)$  Hankel matrix.  $\mathbf{X}_j(K)$  is  $KQ \times (N-r+1)$ , the sampled received signal matrix.

Correspondingly, when there are  $M$  transmit antennas, the overall received signal constructed in the form of a Hankel matrix can be represented as:

$$\mathbf{X}(K) = \sum_{j=1}^M \mathbf{H}_j \mathbf{D}_j(r) = [\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_M] \begin{bmatrix} \mathbf{D}_1(r) \\ \mathbf{D}_2(r) \\ \vdots \\ \mathbf{D}_M(r) \end{bmatrix}$$

$$= \mathbf{H} \mathbf{D}(r) \quad (10)$$

where  $\mathbf{H}$  is a  $KQ \times Mr$  matrix.  $\mathbf{D}(r)$  is an  $Mr \times (N-r+1)$  matrix.  $\mathbf{X}(K)$  is the  $KQ \times (N-r+1)$  sampled received signal matrix. With the subspace of received signal matrix  $\mathbf{X}(K)$ , we can realize the blind equalization for the multiple-input multiple-output (MIMO) system.

Let the space spanned by the rows of  $\mathbf{D}(r)$  be the "signal subspace", and its orthogonal complement space is called the "noise subspace", represented as  $\mathbf{V}_o(r)$ , so we have  $\mathbf{V}_o(r) \cdot \mathbf{D}(r)^H = \mathbf{0}$ . If the columns of  $\mathbf{H}$  is full-rank,  $\mathbf{X}(K)$  has the same signal subspace as  $\mathbf{D}(r)$ , that is:

$$\Re\{\mathbf{X}(K)\} = \Re\{\mathbf{D}(r)\} \quad (11)$$

In the transmit diversity systems, the propagation environment results in significant decorrelation among all the transmit antenna channels. From theory analysis [8], we can easily derive that in frequency-selective channels, when the over-sampled factor  $Q \geq M+1$ , increasing the smooth coefficient  $K$  can make  $\mathbf{H}$  to be full column rank for any  $L$ .

A subspace decomposition can be performed on  $\mathbf{X}(K)$  using a singular value decomposition (SVD) [9]:

$$\mathbf{X}(K) = [\mathbf{U}_s(r) \quad \mathbf{U}_o(r)] \begin{bmatrix} \Sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s(r) \\ \mathbf{V}_o(r) \end{bmatrix} \quad (12)$$

$\mathbf{V}_o(r)$  is an  $(N-2r+1) \times (N-r+1)$  matrix.

Because of  $\mathbf{V}_o(r) \cdot \mathbf{D}(r)^H = \mathbf{0}$ , we have

$$\mathbf{V}_o(r) [\mathbf{D}_1(r)^H, \mathbf{D}_2(r)^H, \dots, \mathbf{D}_M(r)^H] = \mathbf{0} \quad (13)$$

From (13), it follows that  $\mathbf{V}_o(r) \cdot \mathbf{D}_j(r)^H = \mathbf{0}$ . And  $\mathbf{D}_j(r)$  is a Hankel matrix, from the special structure of Hankel matrix we can derive that  $\mathbf{V}_o(r)$  is orthogonal to any  $N-r+1$  consecutive elements of  $\mathbf{d}_j$ , where  $\mathbf{d}_j = [d_j(1), d_j(2), \dots, d_j(N)]^T$ , an  $N \times 1$  vector. The relation between  $\mathbf{V}_o(r)$  and  $\mathbf{d}_j$  can be represented as:

$$\underbrace{[\mathbf{0} \dots \mathbf{0}]_{i \text{ blocks}}} \mathbf{V}_o(r) \underbrace{[\mathbf{0} \dots \mathbf{0}]_{r-i-1 \text{ blocks}}} \mathbf{d}_j = \mathbf{0} \quad (i=0, \dots, r-1; j=1, \dots, M)$$

Then, we obtain

$$\mathbf{V} \mathbf{d}_j = \mathbf{0} \quad (14)$$

where

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_o(r) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{V}_o(r) & \dots & \vdots \\ \vdots & \dots & \dots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{V}_o(r) \end{bmatrix}$$

$\mathbf{0}$  is an  $(N-2r+1) \times 1$  vector,  $\mathbf{V}$  is an  $r(N-2r+1) \times N$  matrix.

From (14), we can derive that  $\mathbf{V}$  is orthogonal to the  $M$  vectors,  $\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_M$ . So, (14) can be represented as

$$\mathbf{V} \mathbf{D} = \mathbf{0} \quad (15)$$

where  $\mathbf{D} = [\mathbf{d}_1 \mathbf{d}_2 \dots \mathbf{d}_M]$  is an  $N \times M$  matrix, the columns of which are the transmitted symbol sequences of transmit antennas.

Note that (15) reveals a result of the blind equalization for the MIMO system: the solutions for the transmitted sequences  $\mathbf{D}$  are included in the orthogonal complement space of  $\mathbf{V}$ . From (15), we can obtain infinite number of possible solutions for  $\mathbf{D}$  and

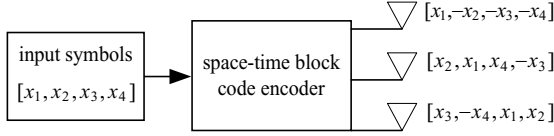


Fig. 1 Space-time block encoder

every time the solution is different. Let  $\mathbf{Y}$  be a possible solution for  $\mathbf{D}$ , and the relation between  $\mathbf{Y}$  and  $\mathbf{D}$  is given by:

$$\mathbf{Y} = [\mathbf{d}_1 \mathbf{d}_2 \cdots \mathbf{d}_M] \mathbf{W} = \mathbf{D} \mathbf{W} \quad (16)$$

where  $\mathbf{W}$  is an unknown  $M \times M$  matrix. So, from (15) we can not acquire the solution for transmitted symbols.

ST block encoder transmits the same symbols among all of the transmit antennas, so there are some relations among the symbol sequences of transmitted antennas. Making use of these relations, without any training symbol we can work out the input of ST block encoder from the noise subspace  $\mathbf{V}$  of  $\mathbf{D}$ .

#### 4. DECODING WITH THE NOISE SUBSPACE

The STBC in this paper is of the maximal rate. Fig. 1 depicts the ST block encoder considered in this paper, where the transceiver is equipped with 3 transmit antennas. The input symbols to the ST block encoder are divided into groups of four symbols. For a group, the ST block encoder takes as input four consecutive symbols  $\{x_1, x_2, x_3, x_4\}$  to output the following  $4 \times 3$  code matrix

$$\mathbf{C} = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \end{bmatrix} \quad (17)$$

where the  $j$ th column of  $\mathbf{C}$  is the transmitted symbol sequence of transmit antenna  $j$  in the four consecutive symbol periods. Based on this STBC, the code matrix  $\mathbf{C}$  and input vector  $\mathbf{c} = \{x_1, x_2, x_3, x_4\}^T$  have following relation (see Appendix for the proof):

**Theorem 1:** Denote an  $m \times 4$  matrix  $\mathbf{U} = [\mathbf{u}_1 \mathbf{u}_2 \mathbf{u}_3 \mathbf{u}_4]$ , where  $\mathbf{u}_j$  ( $j=1,2,3,4$ ) is the  $j$ th column of  $\mathbf{U}$ , and with the columns of  $\mathbf{U}$ , construct a  $3m \times 4$  matrix  $\bar{\mathbf{U}}$

$$\bar{\mathbf{U}} = \begin{bmatrix} \mathbf{u}_1 & -\mathbf{u}_2 & -\mathbf{u}_3 & -\mathbf{u}_4 \\ \mathbf{u}_2 & \mathbf{u}_1 & -\mathbf{u}_4 & \mathbf{u}_3 \\ \mathbf{u}_3 & \mathbf{u}_4 & \mathbf{u}_1 & -\mathbf{u}_2 \end{bmatrix}$$

Then,  $\bar{\mathbf{U}}\mathbf{c} = \text{vec}(\mathbf{U}\mathbf{C})$ , where  $\text{vec}(\mathbf{A})$  represents the vectorization of matrix  $\mathbf{A}$ , that is, a vector is constructed by one column of  $\mathbf{A}$  following the previous one.

Thus, the product of a matrix  $\mathbf{U}$  and the code matrix  $\mathbf{C}$  is transformed to the product of  $\bar{\mathbf{U}}$ , which is coming from  $\mathbf{U}$ , and the input symbol vector  $\mathbf{c}$ . With this feature of the STBC, we can work out  $\mathbf{D}$  from  $\mathbf{V}\mathbf{D} = \mathbf{0}$  in (15).

Let  $\mathbf{D}$  be the output of  $B$  groups of ST block coding. If the  $i$ th group input is  $\mathbf{c}_i = [x_{4i-3}, x_{4i-2}, x_{4i-1}, x_{4i}]$  and the output matrix is  $\mathbf{D}_i$ , then  $\mathbf{D}$  is the output for the input  $\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_N]$ , ( $N = 4B$ ), and

$$\mathbf{D} = [\mathbf{D}_1^T \ \mathbf{D}_2^T \ \cdots \ \mathbf{D}_B^T]^T$$

Separate the  $r[N - (M+1)r + 1] \times N$  matrix  $\mathbf{V}$  to  $B$  segments

and each segment has 4 columns, that is:

$$\mathbf{V} = [\mathbf{Z}_1 \ \mathbf{Z}_2 \ \cdots \ \mathbf{Z}_B] \quad (18)$$

where  $\mathbf{Z}_i$  is an  $r[N - (M+1)r + 1] \times 4$  matrix. Then,  $\mathbf{V}\mathbf{D} = \mathbf{0}$  in (15) can be represented as

$$\mathbf{V}\mathbf{D} = \mathbf{Z}_1\mathbf{D}_1 + \mathbf{Z}_2\mathbf{D}_2 + \cdots + \mathbf{Z}_B\mathbf{D}_B = \mathbf{0} \quad (19)$$

Because  $\mathbf{D}_i$  is the ST block encoder output of one group, it has the same structure as  $\mathbf{C}$ . Based on theorem 1, the product of  $\mathbf{Z}_i$  with the matrix  $\mathbf{D}_i$  can be changed to the product of a matrix  $\bar{\mathbf{Z}}_i$  with the  $i$ th group input symbols  $[x_{4i-3}, x_{4i-2}, x_{4i-1}, x_{4i}]^T$ . Let  $\mathbf{Z}_i = [\mathbf{z}_{i,1} \ \mathbf{z}_{i,2} \ \mathbf{z}_{i,3} \ \mathbf{z}_{i,4}]$ , where  $\mathbf{z}_{i,j}$  is the  $j$ th column of  $\mathbf{Z}_i$ . If  $\mathbf{z}_{i,j}$  is considered as  $\mathbf{u}_j$  of  $\bar{\mathbf{U}}$  in theorem 1, we can construct a  $3r[N - (M+1)r + 1] \times 4$  matrix  $\bar{\mathbf{Z}}_i$ :

$$\bar{\mathbf{Z}}_i = \begin{bmatrix} \mathbf{z}_{i,1} & -\mathbf{z}_{i,2} & -\mathbf{z}_{i,3} & -\mathbf{z}_{i,4} \\ \mathbf{z}_{i,2} & \mathbf{z}_{i,1} & -\mathbf{z}_{i,4} & \mathbf{z}_{i,3} \\ \mathbf{z}_{i,3} & \mathbf{z}_{i,4} & \mathbf{z}_{i,1} & -\mathbf{z}_{i,2} \end{bmatrix} \quad (20)$$

Based on theorem 1, we have

$$\bar{\mathbf{Z}}_i [x_{4i-3}, x_{4i-2}, x_{4i-1}, x_{4i}]^T = \text{vec}(\mathbf{Z}_i \mathbf{D}_i) \quad (21)$$

From (15), we obtain

$$\text{vec}(\mathbf{V}\mathbf{D}) = \text{vec}(\sum_{i=1}^B \mathbf{Z}_i \mathbf{D}_i) = \sum_{i=1}^B \text{vec}(\mathbf{Z}_i \mathbf{D}_i) = \mathbf{0}. \quad (22)$$

From (21), we can derive that

$$\sum_{i=1}^B \text{vec}(\mathbf{Z}_i \mathbf{D}_i) = \sum_{i=1}^B \bar{\mathbf{Z}}_i [x_{4i-3}, x_{4i-2}, x_{4i-1}, x_{4i}]^T \quad (23)$$

Let

$$\bar{\mathbf{Z}} = [\bar{\mathbf{Z}}_1 \ \bar{\mathbf{Z}}_2 \ \cdots \ \bar{\mathbf{Z}}_B] \quad (3r[N - (M+1)r + 1] \times N) \quad (24)$$

It follows from (22) and (23) that

$$\bar{\mathbf{Z}}\mathbf{x} = \mathbf{0} \quad (25)$$

where  $\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_N]^T$  is an  $N \times 1$  vector, which is the input of ST block encoder and just what we want to work out. With the SVD, from  $\bar{\mathbf{Z}}\mathbf{x} = \mathbf{0}$ , we can get the estimation  $\hat{\mathbf{x}}$  of  $\mathbf{x}$ . However, if  $\hat{\mathbf{x}} = \gamma \mathbf{x}$ ,  $\hat{\mathbf{x}}$  also satisfies  $\bar{\mathbf{Z}}\mathbf{x} = \mathbf{0}$ . So, there is a complex coefficient  $\gamma$  between the estimation  $\hat{\mathbf{x}}$  and the actual input  $\mathbf{x}$ . With the finite alphabet of the communication signals, the complex coefficient can be removed and does not affect the decision of symbols. So, in frequency selective channel, we accomplish the decoder of full rate STBC without CSI.

#### 5. SIMULATION RESULTS

In this section, we show the performance results of Monte Carlo simulation. The base station is equipped with three transmit antennas and the mobile station with one antenna. The channel is assumed to be frequency-selective fading. It is assumed that the channel fading coefficient has a Gaussian distribution with zero-mean and a variance 1. The delays are distributed uniformly within  $[0, T]$ . The number of multipaths from each antenna is distributed uniformly within  $[4, 8]$ . The over-sampled factor  $Q$  is 4. The noise is zero-mean white Gaussian noise with variable variance. The DBPSK modulation is used.

We assume that the coherence time is 80 symbol periods. Channel responses are invariant within the coherence time. Because of the assumption of coherence time,  $N$  must be less than 80. Consider a vehicle transmitting at a symbol rate of 30 kHz and a frequency of 1.9GHz. If the vehicle moves at 60 mi/h,

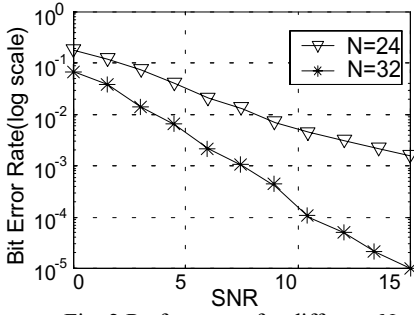


Fig. 2 Performance for different  $N$

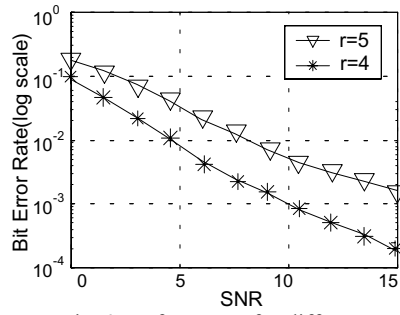


Fig. 3 Performance for different  $r$

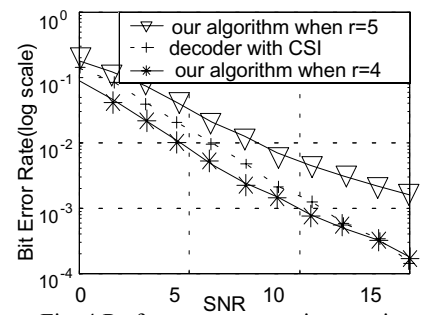


Fig. 4 Performance comparison against the decoder with known CSI

the coherence time is on the order of 50-100 symbols. So, this assumption of coherence time is reasonable.

**Performance of our decoder:** Considering the presence of noise in (8), from the theory of subspace, we know that the more the columns of  $\mathbf{D}$  or the less the rows of  $\mathbf{D}$ , the more accurate the estimated subspace of  $\mathbf{D}$ . If let  $r$  be constant and  $N$  increased, the number of columns of  $\mathbf{D}$  increases while the number of the rows is invariant. So, the performance will be better. The simulation results are shown in Fig.2, where  $r = 5$ .

On the other hand, if let  $N$  be constant and  $r$  increased, the column number of  $\mathbf{D}$  decreases while the row number increases. The performance will be worse. The simulation results are shown in Fig.3, where  $N=24$ .

**Performance comparison against the decoder with known CSI:** We compare the performance of our algorithm against that of the decoder with known CSI. Both of them have the same channel model and ST block encoder. In order to be in the similar conditions, we choose the following algorithm, which has the same received signal model  $\mathbf{X}(K)$  as we have constructed in Section 3.

$$\mathbf{D}(r) = \mathbf{H}^+ \mathbf{X}(K)$$

$(\bullet)^+$  stands for the Moore-Penrose pseudo-inverse. According to the rule of equal gain combining, we combine the same symbols in  $\mathbf{D}(r)$  and according to the rule of ST block coding, the input of ST block encoder is worked out.

The performance results are shown in Fig. 4, where  $N = 24$ , from which we can see that the performance of the decoder with known CSI is between those of our algorithm for  $r = 4$  and  $r = 5$ , while our algorithm does not use CSI. But the computation load of our algorithm is a little larger than that of the decoder with known CSI. Because the coherence time is 80 symbol periods, increasing  $N$  can make the subspace more accurate and the performance will be better.

## 6. CONCLUSION

We have developed a novel space-time decoder for STBC in unknown frequency-selective channels. Relying on the subtle combination of blind subspace-based equalization with the special space-time structure of STBC, the decoding is accomplished for the maximal rate STBC, which does not need channel state information and the redundancy of transmitted symbols in time domain. Simulation results show that the performance of our algorithm approaches that of the decoder with known channel state information.

## APPENDIX

### (PROOF OF THEOREM 1)

$$\mathbf{UC} = [\mathbf{u}_1 \ \mathbf{u}_2 \ \mathbf{u}_3 \ \mathbf{u}_4] \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \end{bmatrix} = [\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3]$$

where  $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$  are  $m \times 1$  vectors,

$$\mathbf{a}_1 = \mathbf{u}_1 x_1 - \mathbf{u}_2 x_2 - \mathbf{u}_3 x_3 - \mathbf{u}_4 x_4,$$

$$\mathbf{a}_2 = \mathbf{u}_1 x_2 + \mathbf{u}_2 x_1 + \mathbf{u}_3 x_4 - \mathbf{u}_4 x_3,$$

$$\mathbf{a}_3 = \mathbf{u}_1 x_3 - \mathbf{u}_2 x_4 + \mathbf{u}_3 x_1 + \mathbf{u}_4 x_2,$$

$$\bar{\mathbf{U}}\mathbf{c} = \begin{bmatrix} \mathbf{u}_1 x_1 - \mathbf{u}_2 x_2 - \mathbf{u}_3 x_3 - \mathbf{u}_4 x_4 \\ \mathbf{u}_2 x_1 + \mathbf{u}_1 x_2 - \mathbf{u}_4 x_3 + \mathbf{u}_3 x_4 \\ \mathbf{u}_3 x_1 + \mathbf{u}_4 x_2 + \mathbf{u}_1 x_3 - \mathbf{u}_2 x_4 \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{bmatrix} = \text{vec}(\mathbf{UC})$$

So, we have  $\mathbf{U}\mathbf{c} = \text{vec}(\mathbf{UC})$   $\square$

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