LINEAR PRECODING OVER TIME-VARYING CHANNELS IN TDD SYSTEMS

Frank Dietrich, Raphael Hunger, Michael Joham, and Wolfgang Utschick

Institute for Circuit Theory and Signal Processing, Munich University of Technology,
80290 Munich, Germany, E-Mail: Dietrich@nws.ei.tum.de

ABSTRACT

Linear transmit filters depend on current channel state information, which is available from the uplink channel estimation in time division duplex systems. For multiple antenna elements at the transmitter, we illustrate the influence of out-dated channel estimates on link level performance for symmetric and asymmetric traffic, comparing the transmit matched, zero-forcing, and Wiener filter. A Wiener predictor is proposed to improve channel knowledge of the transmitter. We observe an inherent robustness of the transmit matched filter and explain the occurrence of a minimum in the bit error ratio at a finite SNR for the transmit Wiener filter.

1. INTRODUCTION

Under the assumption that the transmitter knows the channel state information, the signal at the receiver can be equalized by applying a filter prior to transmission which leads to simplified receivers and is therefore especially attractive for the downlink. Since the two links in a *time division duplex* (TDD) system share the same frequency band, this assumption is fulfilled as long as the calibration works correctly and the coherence time of the channel is large enough. Unfortunately, the uplink estimate of the channel is not available in the following downlink slot, as we have to consider a delay of the necessary processing. Moreover, the number of uplink slots can be expected to be smaller than the number of downlink slots, because multimedia applications lead to asymmetric traffic. Therefore, the transmit filters at the *base station* (BS) can only be constructed by means of an out-dated channel estimate which leads to deteriorated performance.

In [1], Kowalewski et al. considered a TD-CDMA system [2] with user velocities up to 30 km/h and compared the transmit zero-forcing filter (TxZF) with its equivalent at the receiver in terms of raw *bit error ratio* (BER) and concluded that the main reason for the degradation of the transmit filter is due to the inaccurate channel estimates, but did not illustrate this assertion.

We do not only concentrate on the TxZF, but also examine the influence of varying channels on the *transmit matched filter* (TxMF) or *prerake* [3] and the *transmit Wiener filter* (TxWF) [4]. Moreover, we include a linear predictor at the BS to improve the quality of the channel estimates.

After explaining the system model in Section 2, we briefly review the construction of the transmit filters in Section 3. In Section 4, we discuss the utilized channel estimation and derive the predictor. The simulation results are presented in Section 5.

2. SYSTEM MODEL

In the TDD mode of UMTS the signal is divided into frames, which consist of 15 slots [2]. Each slot contains a midamble and a direct sequence spread data signal. From various ways of assigning slots to the up- (“↑”) or downlink (“↓”), we pick the following 2 proposed frame structures for medium and high data rates [2]. They have multiple switching points and are, therefore, less sensitive to out-dated channel information than their counterparts with a single switching point between up- and downlink mode.

↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓

Symmetric multiple switching point downlink frame.

↓	↓	↓	↑	↓	↓	↓	↑	↓	↓	↓	↑	↓	↓	↓

Asymmetric multiple switching point downlink frame.

We consider a single user MISO system with spreading factor one (proposed in [2]) and a transmitted QPSK symbol sequence $\boldsymbol{s} \in \mathbb{C}^W$ with correlation matrix $\boldsymbol{R_s} = \mathrm{E}\left[\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}\right] \in \mathbb{C}^{W \times W}$, which is precoded with $\boldsymbol{P} \in \mathbb{C}^{MW \times W}$ and transmitted over M antenna elements. The received signal in the downlink is given by (Fig. 1)

$$\hat{\boldsymbol{s}} = \boldsymbol{H}_n \boldsymbol{P} \boldsymbol{s} + \boldsymbol{\eta}. \tag{1}$$

The discrete time frequency selective channel of length Q is constant during slot n and described by the block Toeplitz matrix

$$\boldsymbol{H}_n = \sum_{q=0}^{Q-1} \boldsymbol{S}^{\mathrm{T}}_{(q,W,Q-1)} \otimes \boldsymbol{h}^{\mathrm{T}}_{n,q} \in \mathbb{C}^{(W+Q-1)\times MW} \tag{2}$$

with the selection matrix

$$\boldsymbol{S}_{(q,M,N)} = \left[\boldsymbol{0}_{M\times q}, \boldsymbol{1}_M, \boldsymbol{0}_{M\times(N-q)}\right] \in \{0,1\}^{M\times(M+N)}$$

and the vector channel coefficient $\boldsymbol{h}_{n,q} \in \mathbb{C}^M$ of tap q. Here, $\boldsymbol{\eta} \in \mathbb{C}^{W+Q-1}$ is the additive zero mean complex Gaussian noise with correlation matrix $\boldsymbol{R}_{\boldsymbol{\eta}} = \mathrm{E}\left[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}\right]$. Moreover, signal and noise are assumed to be uncorrelated, i. e. $\mathrm{E}\left[\boldsymbol{s}\boldsymbol{\eta}^{\mathrm{H}}\right] = \mathbf{0}_{W\times(W+Q-1)}$. Throughout the paper, $\hat{A}$ denotes an estimate of A, '$\otimes$' the Kronecker product, $\mathbf{0}_{M\times N}$ the $M \times N$ zero matrix, $\mathbf{1}_M$ the $M \times M$ identity matrix, and $\boldsymbol{e}_i$ the i-th column of $\mathbf{1}_{15p}$.

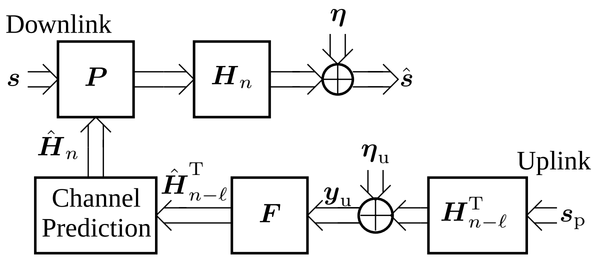

Fig. 1. Downlink transmit processing based on channel estimation and prediction from the uplink.

3. LINEAR TRANSMIT FILTERS

The most intuitive transmit filter is the TxZF $\boldsymbol{P}_{\mathrm{ZF}}$ (e. g. [1]), which removes the interference and uses the whole available transmit power E_{tr}, can be denoted [4]:

$$[\boldsymbol{P}_{\mathrm{ZF}}, \beta_{\mathrm{ZF}}] = \arg\min_{\boldsymbol{P},\beta} \beta^{-2} \quad (3)$$
$$\text{s. t.: } \boldsymbol{H}_n\boldsymbol{P} = \beta\boldsymbol{\Psi} \text{ and } \mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{s}\|_2^2\right] = E_{\mathrm{tr}}.$$

We decided to employ block filters which do not process the whole slot at once, but split the slot into smaller windows with W symbols to reduce the necessary complexity. Therefore, we have to suppress the interference generated for the following window, leading to $\boldsymbol{\Psi} = [\mathbf{1}_W, \mathbf{0}_{W\times Q-1}]^{\mathrm{T}}$. The solution of above optimization reads as

$$\boldsymbol{P}_{\mathrm{ZF}} = \beta_{\mathrm{ZF}}\boldsymbol{H}_n^{\mathrm{H}}\left(\boldsymbol{H}_n\boldsymbol{H}_n^{\mathrm{H}}\right)^{-1}\boldsymbol{\Psi}, \quad (4)$$

where β_{ZF} is chosen to fulfill the transmit power constraint.

The TxMF $\boldsymbol{P}_{\mathrm{MF}}$ maximizes the received desired signal portion and can be expressed as [3]

$$\boldsymbol{P}_{\mathrm{MF}} = \beta_{\mathrm{MF}}\boldsymbol{H}_n^{\mathrm{H}}\boldsymbol{\Psi}. \quad (5)$$

Again, β_{MF} is necessary to fulfill $\mathrm{E}[\|\boldsymbol{P}_{\mathrm{MF}}\boldsymbol{s}\|_2^2] = E_{\mathrm{tr}}$.

The *modified mean square error* is minimized by the TxWF $\boldsymbol{P}_{\mathrm{WF}}$:

$$[\boldsymbol{P}_{\mathrm{WF}}, \beta_{\mathrm{WF}}] = \arg\min_{\boldsymbol{P},\beta} \mathrm{E}\left[\|\boldsymbol{\Psi}\boldsymbol{s} - \beta^{-1}\hat{\boldsymbol{s}}\|_2^2\right] \quad (6)$$
$$\text{s. t.: } \mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{s}\|_2^2\right] = E_{\mathrm{tr}}$$

and can be written as [4]

$$\boldsymbol{P}_{\mathrm{WF}} = \beta_{\mathrm{WF}}\left(\boldsymbol{H}_n^{\mathrm{H}}\boldsymbol{H}_n + \xi\mathbf{1}_{MW}\right)^{-1}\boldsymbol{H}_n^{\mathrm{H}}\boldsymbol{\Psi}, \quad (7)$$

where $\xi = \mathrm{tr}(\boldsymbol{R}_{\boldsymbol{\eta}})/E_{\mathrm{tr}}$ and β_{WF} is used to set the transmit power to E_{tr}.

4. CHANNEL ESTIMATION AND PREDICTION

The transmit filters in Section 3 require knowledge about the instantaneous channel coefficients $\boldsymbol{h}_{n,q}$ at the transmitter. Ideally, the channels in up- and downlink in the TDD mode are reciprocal. Thus, the channel coefficients can be estimated using the N_{p} pilot symbols $\boldsymbol{s}_{\mathrm{p}}$ of the $(n-\ell)$-th uplink slot. The received pilot sequence in the uplink is (Fig. 1)

$$\boldsymbol{y}_{\mathrm{u}} = \boldsymbol{H}^{\mathrm{T}}_{n-\ell}\boldsymbol{s}_{\mathrm{p}} + \boldsymbol{\eta}_{\mathrm{u}} = \boldsymbol{S}_{\mathrm{p}}\boldsymbol{h}_{n-\ell} + \boldsymbol{\eta}_{\mathrm{u}} \in \mathbb{C}^{M(N_{\mathrm{p}}-Q+1)}$$

with the spatio-temporal channel vector

$$\boldsymbol{h}_{n-\ell} = \left[\boldsymbol{h}^{\mathrm{T}}_{n-\ell,0}, \cdots, \boldsymbol{h}^{\mathrm{T}}_{n-\ell,Q-1}\right]^{\mathrm{T}}$$

and the matrix of pilot symbols

$$\boldsymbol{S}_{\mathrm{p}} = \begin{bmatrix} s_{\mathrm{p}}[Q] & \cdots & s_{\mathrm{p}}[0] \\ \vdots & \ddots & \vdots \\ s_{\mathrm{p}}[N_{\mathrm{p}}] & \cdots & s_{\mathrm{p}}[N_{\mathrm{p}}-Q] \end{bmatrix} \otimes \mathbf{1}_M.$$

The *maximum likelihood channel estimator* for white Gaussian noise $\boldsymbol{\eta}_{\mathrm{u}}$ with $\boldsymbol{R}_{\boldsymbol{\eta}_{\mathrm{u}}} = \sigma_{\mathrm{u}}^2\mathbf{1}$ is given by [5]

$$\hat{\boldsymbol{h}}_{n-\ell} = \boldsymbol{F}\boldsymbol{y}_{\mathrm{u}}, \quad \boldsymbol{F} = (\boldsymbol{S}_{\mathrm{p}}^{\mathrm{H}}\boldsymbol{S}_{\mathrm{p}})^{-1}\boldsymbol{S}_{\mathrm{p}}^{\mathrm{H}}. \quad (8)$$

Unfortunately, only an out-dated channel estimate from an uplink slot is available for downlink processing due to the frame structure. Additionally, we assume a processing delay, such that the channel estimate is only available for filter design one slot later. For a symmetric frame (cf. Section 2) this results in a difference ℓ of 2-4 slots between the current downlink slot and an uplink slot, from which a channel estimate is available. This is particularly severe in the asymmetric format with a delay ℓ of 2-8 slots, as we have only few uplink slots.

To reduce the degradation due to out-dated channel estimates, we propose to use a *Wiener predictor* at the transmitter (Fig. 1). The predictor $\boldsymbol{w}$ uses the channel estimates from the previous p frames to predict the channel in slot n, which is the k-th slot within a frame,

$$\hat{\boldsymbol{h}}_n[q] = \left[\hat{\boldsymbol{h}}_{n-15p,q}, \cdots, \hat{\boldsymbol{h}}_{n-1,q}\right]\boldsymbol{T}_k\boldsymbol{w} = \hat{\boldsymbol{\mathcal{H}}}\boldsymbol{T}_k\boldsymbol{w}, \quad (9)$$

where $\boldsymbol{T}_k = \mathrm{diag}(\boldsymbol{B}\boldsymbol{C}^{k-1}\boldsymbol{c}) \in \{0,1\}^{15p\times 15p}$ selects the uplink slots among the previous $15p$ slots. The permutation matrix $\boldsymbol{C} = [\boldsymbol{e}_2, \cdots, \boldsymbol{e}_{15p}, \boldsymbol{e}_1] \in \{0,1\}^{15p\times 15p}$ cyclically shifts the elements of $\boldsymbol{c} = [1, \cdots, 1]^{\mathrm{T}} \otimes \boldsymbol{c}_0 \in \{0,1\}^{15p}$, where the i-th element of $\boldsymbol{c}_0 \in \{0,1\}^{15}$ is one if the i-th slot in the frame is used for the uplink. $\boldsymbol{B} = [\mathbf{0}, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_{15p}] \in$

$\{0,1\}^{15p \times 15p}$ describes the processing delay of one slot. A Wiener filter $\boldsymbol{w}$ minimizes the mean square error

$$\mathrm{MSE} = \mathrm{E}[\|\hat{\boldsymbol{h}}_{n,q} - \boldsymbol{h}_{n,q}\|_2^2],$$

which results in the Wiener-Hopf equation

$$\boldsymbol{T}_k \boldsymbol{R}_{\hat{\boldsymbol{\mathcal{H}}}} \boldsymbol{T}_k \boldsymbol{w} = \boldsymbol{T}_k \boldsymbol{r}_q. \tag{10}$$

The solution can be written as

$$\boldsymbol{w} = \left(\boldsymbol{T}_k \boldsymbol{R}_{\hat{\boldsymbol{\mathcal{H}}}} \boldsymbol{T}_k\right)^{\dagger} \boldsymbol{T}_k \boldsymbol{r}_q \tag{11}$$

using the Moore-Penrose inverse $(\bullet)^{\dagger}$ [5], the auto-correlation matrix $\boldsymbol{R}_{\hat{\boldsymbol{\mathcal{H}}}} = \mathrm{E}[\hat{\boldsymbol{\mathcal{H}}}^{\mathrm{H}} \hat{\boldsymbol{\mathcal{H}}}]$, and the cross-correlation vector $\boldsymbol{r}_q = \mathrm{E}[\hat{\boldsymbol{\mathcal{H}}}^{\mathrm{H}} \boldsymbol{h}_{n,q}]$.

5. NUMERICAL RESULTS

In the downlink 128 symbols are transmitted per slot over $M = 2$ antenna elements using a window of size $W = 8$ and with a carrier frequency of $2\,\mathrm{GHz}$. Channel estimates are obtained from $N_{\mathrm{p}} = 256$ pilot symbols in an uplink slot, which is received at a SNR of $3\,\mathrm{dB}$. The channel coefficients are i.i.d. complex Gaussian distributed in space and delay domain ($Q = 4$) assuming a Jakes power spectrum [6] describing temporal correlations with maximum Doppler frequency $f_{\mathrm{dmax}} \in \{10\,\mathrm{Hz}, 100\,\mathrm{Hz}\}$. Moreover, we assume $\boldsymbol{R}_{\boldsymbol{\eta}} = \sigma_{\mathrm{n}}^2 \mathbf{1}_{W+Q-1}$ and $\boldsymbol{R}_{\boldsymbol{s}} = \mathbf{1}_W$. In all results below, the SNR is defined as the ratio of transmit power and noise variance at the receiver.

MSE performance: As all channel coefficients are i.i.d. distributed we consider the mean square error

$$\mathrm{E}[\|\boldsymbol{\epsilon}_{n,q}\|_2^2] = \mathrm{E}[\|\boldsymbol{h}_{n,q} - \hat{\boldsymbol{h}}_{n,q}\|_2^2] \tag{12}$$

of the transmitter's knowledge of the channel vector at delay q to quantify the influence of the time varying channel. Four system configurations are evaluated: Prediction with $p \in \{1,2\}$ based on out-dated channel estimates (Eqn. 9) and the use of out-dated exact ($\hat{\boldsymbol{h}}_{n,q} = \boldsymbol{h}_{n-\ell,q}$) or estimated channel coefficients ($\hat{\boldsymbol{h}}_{n,q} = \hat{\boldsymbol{h}}_{n-\ell,q}$). The MSE for the predicted and the out-dated exact knowledge is given by ($c_q[\ell] = \mathrm{E}[\boldsymbol{h}_{n,q}^{\mathrm{H}} \boldsymbol{h}_{n-\ell,q}]$)

$$\mathrm{E}\left[\|\boldsymbol{\epsilon}_{n,q}\|_2^2\right] = \begin{cases} c_q[0] - \boldsymbol{r}_q^{\mathrm{H}} \boldsymbol{T}_k \left(\boldsymbol{T}_k \boldsymbol{R}_{\hat{\boldsymbol{\mathcal{H}}}} \boldsymbol{T}_k\right)^{\dagger} \boldsymbol{T}_k \boldsymbol{r}_q \\ 2(c_q[0] - \mathrm{Re}(c_q[\ell])) \end{cases}.$$

Figures 2 and 3 compare the MSE (cf. Eqn. 12) of these configurations normalized by M for the *asymmetric* frame. The MSE of uplink slots is set to zero. For $f_{\mathrm{dmax}} = 10\,\mathrm{Hz}$ the channel estimation variance has a notable influence on the MSE, which is reduced significantly by prediction with $p = 2$ (cf. Eqn. 11). At higher speed ($f_{\mathrm{dmax}} = 100\,\mathrm{Hz}$) channel knowledge is degraded by an order of magnitude even after prediction (cf. Fig. 3). Channel estimation variance is negligible in this case.

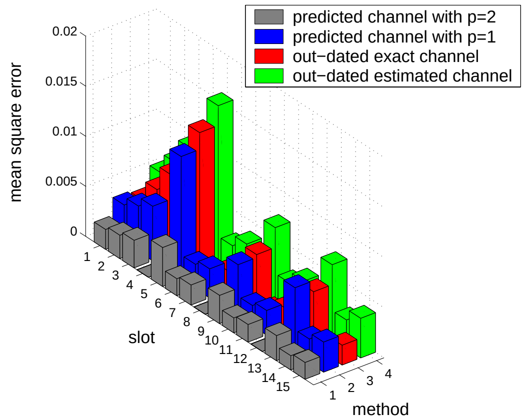

Fig. 2. MSE of 1 channel coefficient for the *asymmetric* frame structure and $f_{\mathrm{dmax}} = 10\,\mathrm{Hz}$ ($5.4\,\mathrm{km/h}$).

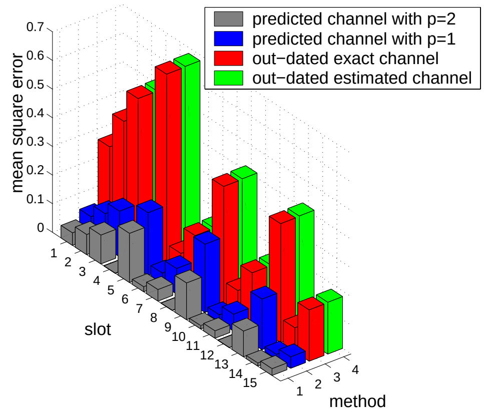

Fig. 3. MSE of 1 channel coefficient for the *asymmetric* frame structure and $f_{\mathrm{dmax}} = 100\,\mathrm{Hz}$ ($54\,\mathrm{km/h}$).

BER performance: The BER of the matched filter (cf. Eqn. 5) is not increased by the out-dated channel estimates for $f_{\mathrm{dmax}} = 10\,\mathrm{Hz}$, i.e. it is identical with the ideal case of instantaneous channel knowledge in Fig. 4, which saturates at a rather high BER level. This is due to its inherent robustness, which results from its simplicity. There is no difference in BER between out-dated estimated and exact knowledge as well as for prediction order $p = 1$ and $p = 2$.

As expected, the TxWF and TxZF saturate at a much lower level, but are more sensitive to out-dated channel knowledge even at low mobile speed (Figures 5 and 6). The TxZF approach is clearly outperformed by the TxWF due to the limited number of degrees of freedom available. For $f_{\mathrm{dmax}} = 100\,\mathrm{Hz}$ prediction is necessary for all transmit filters to achieve an uncoded BER $= 10^{-1}$, which is a typical point of operation for speech services. As in the MSE results above, there is no loss due to estimation errors in the out-dated coefficients at $f_{\mathrm{dmax}} = 100\,\mathrm{Hz}$.

At first glance, it is surprising to find a minimum in the BER of the TxWF, i.e. the TxWF's BER increases for high SNR (Fig. 6 and 7). It finally converges to the TxZF performance as $\xi \to 0$. The TxZF inverts the channel in case of perfect channel knowledge, but causes severe interference when only out-dated channel coefficients are available. In the minimum the scaled unity matrix in the TxWF ensures that the filter stays closer to the TxMF, i.e. it finds the best trade-off between interference suppression and serving the available paths with signal power.

For asymmetric frames, the TxWF performance (Fig. 7) shows that the use of transmit filters is questionable at speeds larger than 54 km/h, as the BER already approaches that of a TxMF with prediction ($p = 2$), which saturates at about $\text{BER} = 1.5 \cdot 10^{-1}$ in this case (not shown here).

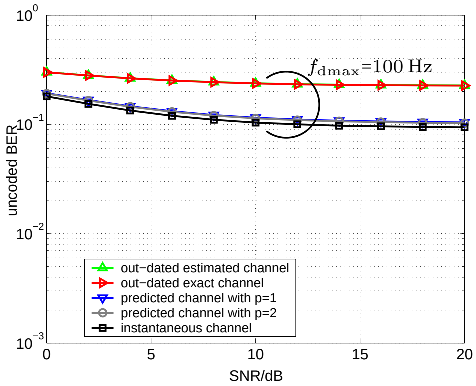

Fig. 4. BER for transmit matched filter (Prerake), *symmetric* downlink frame structure, and $f_{\text{dmax}} = 100\,\text{Hz}$.

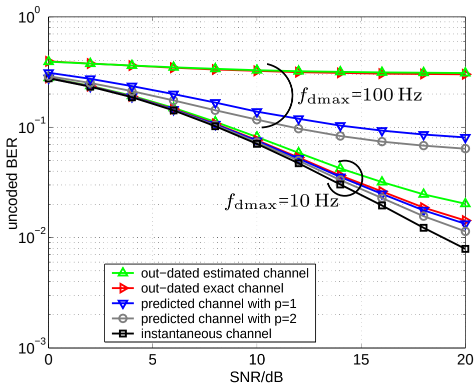

Fig. 5. BER for transmit zero-forcing filter, *symmetric* downlink frame structure, and $f_{\text{dmax}} \in \{10\,\text{Hz}, 100\,\text{Hz}\}$.

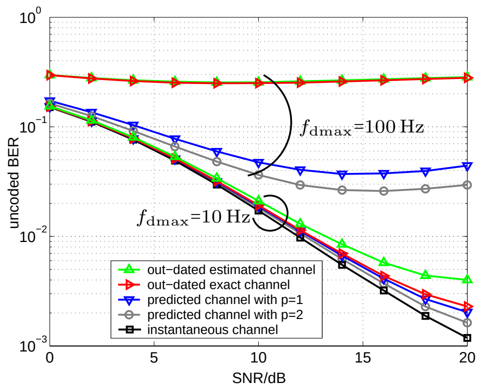

Fig. 6. BER for transmit Wiener filter, *symmetric* downlink frame structure, and $f_{\text{dmax}} \in \{10\,\text{Hz}, 100\,\text{Hz}\}$.

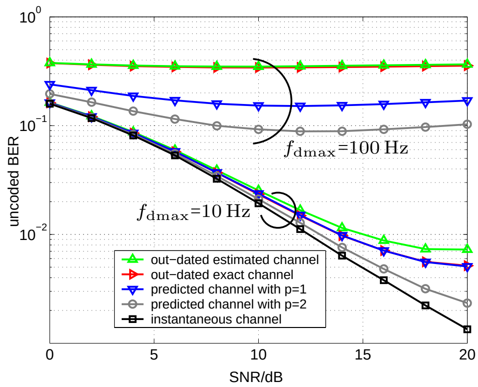

Fig. 7. BER for transmit Wiener filter with *asymmetric* downlink frame structure and $f_{\text{dmax}} \in \{10\,\text{Hz}, 100\,\text{Hz}\}$.

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