ON THE OPTIMAL TRANSMISSION STRATEGY FOR THE MIMO MAC WITH MMSE RECEIVER

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ABSTRACT

In this work, we study the multiuser mulitple-input multiple-output (MIMO) multiple access channel (MAC) under the assumption that the base station performs linear multiuser minimum mean-square error (MMSE) detection. We derive the average sum MSE and minimize it under a sum power constraint with respect to the transmit covariance matrices of the users. Furthermore, we characterize the optimum power allocation among the users in regards of the single-user region. For low SNR values, the optimum strategy is to only have the best user transmitting at a time. The single-user region decreases with the number of receive antennas at the base station and with the number of users in the system. In addition, we derive the individual MSE using single user MMSE detectors and study the fulfillment of MSE requirements. We illustrate all theoretical results by numerical simulations.

1. INTRODUCTION

The increasing need for fast and reliable wireless communication links has opened the discussion about systems with multiple antennas both located at the transmitter and the receiver, so called multiple-input multiple-output (MIMO) systems [1]. Systems with multiple antennas at one side of the link are well known [2] for increasing the capacity and performance. Recently, it was discovered that MIMO systems have the ability to reach higher transmission rates than one-sided array links [3],[4].

In this work, we study the uplink transmission of K users, each equipped with n_T transmit antennas, to the base station which has n_R receive antennas. In [5], the authors maximize the ergodic sum capacity of the MIMO MAC for fixed individual power constraints for the transmit covariance matrices. For fixed power constraints $\tilde{P}_1, ..., \tilde{P}_K$, the sum capacity is maximized with $Tr(\mathbf{Q}_k) \leq \tilde{P}_k \quad 1 \leq k \leq K$. It is shown that the optimal transmit covariance matrices are characterized by an iterative water-filling solution which treats the other users like noise with $Tr(\hat{\mathbf{Q}}_k) = \tilde{P}_k$, $1 \leq k \leq K$.

We consider the case in which the base station uses the linear MMSE detector in order to detect the signals from the K users. The performance criterium for this receiver is the MSE. In [6], the linear MMSE multiuser receiver for synchronous CDMA systems is analyzed. We apply and extend the results of [6] for the MIMO transmission model and compute the MSE as a function of the transmit covariance matrices and power allocation of the users. In addition to this, we derive the optimization problem which minimizes the sum MSE and the optimization problem which balances the MSE requirements of the users. Using optimization theory we provide an algorithm for the case in which only one user is transmitting. The single-user region is the SNR range in which only the best user transmits simultaneously. The analysis of the structure of the optimum transmission strategy which minimizes the sum MSE is difficult. In order to analyze the properties of the optimum transmission strategy, we study the behaviour at low SNR values. We consider the point at which the second user is allowed to transmit. These effects at low SNR values give insight into the general structure of the optimal transmit covariance matrices.

We illustrate the impact of the number of users and the number of receive antennas on the single-user range. We show that the single-user range decreases as the number of receive antennas at the base station increases. In addition to this, the number of users lowers the single-user range. Finally, we illustrate all our theoretical results by numerical simulations.

2. SIGNAL MODEL, SUM AND, INDIVIDUAL MSE

In this section, we present the signal model and review the linear MMSE receiver at the base station. Furthermore, we derive the sum MSE and the individual MSEs.

2.1. Signal model

In figure (1), we show the signal model for the MIMO MAC with the multiuser MMSE receiver. K mobiles with n_T antennas each transmit to the base station with n_R receive antennas. The transmit signal from user i at each point k is given by $\boldsymbol{x}_i(k)$. The transmit covariance matrix of the i-th user is given by $\mathbf{Q}_i$.

The received signal at the base station is given by

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{x}_k + \mathbf{n}. \tag{1}$$

We assume a flat fading MIMO channel $\mathbf{H}_k$ for all users in (1). Addionally, we have the additive white Gaussian noise vector $\mathbf{n}$ with noise variance σ_n^2.

Equation (1) can be rewritten in compact form as

$$\mathbf{y} = \hat{\mathbf{H}}\hat{\mathbf{x}} + \mathbf{n} \tag{2}$$

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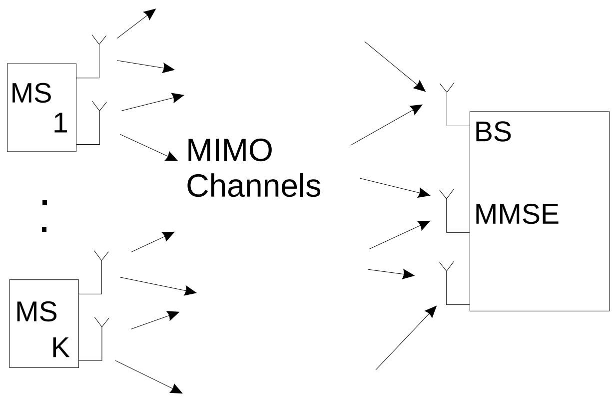

Fig. 1. MIMO MAC with MMSE multiuser receiver

with $\hat{\mathbf{H}} = [\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_K]$ and $\hat{\mathbf{x}} = [\mathbf{x}_1^T, ..., \mathbf{x}_K^T]^T$. We collect the transmit covariance matrices in

$$\hat{\mathbf{Q}} = \begin{pmatrix} \mathbf{Q}_1 & 0 & 0 & ... & 0 \\ 0 & \mathbf{Q}_2 & 0 & ... & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{Q}_K \end{pmatrix}. \quad (3)$$

2.2. Normalized sum MSE of the linear MMSE receiver

We follow the definition and derivation of the normalized MSE in [6] for the synchronous CDMA system. The linear MMSE multiuser receiver computes the data estimate

$$\tilde{\mathbf{x}} = \hat{\mathbf{Q}}\hat{\mathbf{H}}^H \left(\sigma_n^2 \mathbf{I} + \hat{\mathbf{H}}\hat{\mathbf{Q}}\hat{\mathbf{H}}^H\right)^{-1} \mathbf{y}. \quad (4)$$

The covariance matrix of the estimation error ϵ is given as

$$\mathbf{K}_\epsilon = \hat{\mathbf{Q}} - \hat{\mathbf{Q}}\hat{\mathbf{H}}^H \left(\hat{\mathbf{H}}\hat{\mathbf{Q}}\hat{\mathbf{H}}^H + \sigma_n^2 \mathbf{I}\right)^{-1} \hat{\mathbf{H}}\hat{\mathbf{Q}}. \quad (5)$$

We define the matrix $\mathbf{A}$ for convenience

$$\mathbf{A} = \sigma_n^2 \mathbf{I} + \sum_{k=1}^{K} \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H \quad (6)$$

From (5), it follows the normalized MSE as

$$\begin{aligned} MSE &= tr(\hat{\mathbf{Q}}^{-1/2}\mathbf{K}_\epsilon \hat{\mathbf{Q}}^{-1/2}) = Kn_T - \sum_{i=1}^{n_R} \frac{\mu_i}{\sigma_n^2 + \mu_i} \\ &= Kn_T - n_R + \sigma_n^2 \sum_{i=1}^{n_R} \frac{1}{\sigma_n^2 + \mu_i} \\ &= Kn_T - n_R + \sigma_n^2 tr\left(\mathbf{A}^{-1}\right) \quad (7) \end{aligned}$$

with μ_i as the eigenvalues of $\hat{\mathbf{H}}\hat{\mathbf{Q}}\hat{\mathbf{H}}^H$. The MSE is minimized by minimizing the sum in the RHS of (7). It is worth mentioning that the term $\sum_{i=1}^{n_R} \frac{1}{\sigma_n^2 + \mu_i}$ is a Schur-convex function with respect to the μ_i [7]. Therefore, the term is minimized if all eigenvalues μ_i are equal. We cannot directly control the μ_i. We can influence the eigenvalues μ_i indirectly by choicing $\mathbf{Q}_1, ..., \mathbf{Q}_K$. This leads us to the first problem statement:

Problem 1: In order to minimize the sum MSE in (7) find the optimal transmit covariance matrices $\mathbf{Q}_1^*, ..., \mathbf{Q}_K^*$ which minimize the RHS of (7) with (6)

$$\mathbf{Q}_k^* = \arg \min_{\substack{\mathbf{Q}_i \geq 0 \\ \sum_{k=1}^K tr(\mathbf{Q}_k) \leq P}} tr\left(\mathbf{A}^{-1}\right). \quad (8)$$

2.3. Normalized individual MSEs

In this scenario, we assume that each user has an indiviual requirement regarding his MSE. Because each mobile uses different services it requires different MSE. Let the MSE requirements be given by $\gamma_1, ..., \gamma_K$. The individual MMSE estimation for user k is given by

$$\hat{x}_k = \mathbf{Q}_k \mathbf{H}_k^H \left(\sum_{l=1}^{K} \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H + \sigma_n^2 \mathbf{I}\right)^{-1} \mathbf{y}. \quad (9)$$

The normalized MSE of the k-th user is given by

$$\begin{aligned} MSE_k = & \quad (10) \\ n_T - tr & \left(\mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H \left(\sum_{l=1}^K \mathbf{H}_l \mathbf{Q}_l \mathbf{H}_l^H + \sigma_n^2 \mathbf{I}\right)^{-1}\right) \end{aligned}$$

The MSE requirements are feasible for given channel realizations and SNR if

$$MSE_k \leq \gamma_k \quad \forall k \in [1...K].$$

Hence, the MSE balancing problem is given by

Problem 2: For fixed channel realizations $\mathbf{H}_1, ..., \mathbf{H}_K$ and fixed SNR, solve the following optimization problem

$$r = \min_{\sum_{i=1}^K tr(\mathbf{Q}_i) \leq P} \left(\max_{k \in [1,...,K]} \frac{MSE_k}{\gamma_k} \right). \quad (11)$$

In order to solve Problem 1, we present the following theorem.

Theorem 1: For the optimal covariance matrices $\mathbf{Q}_1^*, ..., \mathbf{Q}_K^*$ with $\sum_{l=1}^K tr(\mathbf{Q}_l^*) \leq P$ and $\max_{k \in [1...K]} \frac{MSE_k(\mathbf{Q}^*)}{\gamma_k} = r$ we have

1. $\sum_{k=1}^K tr(\mathbf{Q}_k^*) = P$ and
2. $\frac{MSE_1(\mathbf{Q}^*)}{\gamma_k} = ... = \frac{MSE_K(\mathbf{Q}^*)}{\gamma_K}$.

Remark: The MSE for the k-th user can be written with (6) as

$$MSE_k = n_T - tr\left(\mathbf{A}^{-1/2} \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H \mathbf{A}^{-1/2}\right). \quad (12)$$

Following the arguments in [8], we can balance the terms

$$u_k = \frac{n_R tr\left(\mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H\right)}{tr\left(\mathbf{A}\right)} \quad (13)$$

using the so called 'trace-balancing' algorithm.

3. SINGLE-USER RANGE FOR NORMALIZED SUM MSE

3.1. Characterization of optimality conditions

At low SNR values, the optimal transmission strategy is to have one single user performing beamforming, i.e. the single user uses only its largest eigenvalue. Let us order the users with respect to their largest eigenvalues, i.e. their L_2-norm:

$$||\mathbf{H}_1||_2 \geq ||\mathbf{H}_2||_2 \geq ... \geq ||\mathbf{H}_K||_2$$

Next, we provide an algorithm which decides whether it is optimal to have the user with the best channel transmitting only. Let us prove the following theorem which characterizes the single-user region of the MIMO MAC with linear multiuser MMSE receiver

Theorem 2: If user i is not active in the optimum solution then

$$\left(\mathbf{H}_i^H \mathbf{A}^{-2} \mathbf{H}_i\right)^T \leq \mu \mathbf{I} \tag{14}$$

and if user j is active in the optimum solution then

$$\left(\mathbf{H}_j^H \mathbf{A}^{-2} \mathbf{H}_j\right)^T + \mathbf{\Psi}_j = \mu \mathbf{I} \tag{15}$$

with $tr(\mathbf{\Psi}_j \mathbf{Q}_j) = 0$.

Proof: We sketch the proof by using the Karush-Kuhn-Tucker conditions for the optimization problem in (8). The Lagrange function for this optimization problem is given by

$$\begin{aligned} L(\mathbf{Q}_1, ..., \mathbf{Q}_K, \mu, \mathbf{\Psi}_1, ..., \mathbf{\Psi}_K) = \quad (16) \\ tr\left(\left(\sigma_n^2 \mathbf{I} + \textstyle\sum_{k=1}^K \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^H\right)^{-1}\right) - \\ \textstyle\sum_{k=1}^K tr(\mathbf{\Psi}_k \mathbf{Q}_k) + \mu\left(\textstyle\sum_{k=1}^K tr(\mathbf{Q}_k - P\right). \end{aligned}$$

The first derivative of (17) is given by

$$\frac{\delta L}{\delta \mathbf{Q}_i} = \left(-\mathbf{H}_i^H \mathbf{A}^{-1} \mathbf{A}^{-1} \mathbf{H}_i\right)^T - \mathbf{\Psi}_i + \mu \mathbf{I}. \tag{17}$$

The Karush-Kuhn-Tucker conditions which are necessary and sufficient for the optimality of $\mathbf{Q}_1^*, ..., \mathbf{Q}_K^*$ are given by

$$\mathbf{Q}_i^* \geq 0 \ \forall i \in [1...K] \tag{18}$$

$$\sum_{k=1}^K tr(\mathbf{Q}_k^*) \leq P \tag{19}$$

$$\mu\left(\sum_{k=1}^K tr(\mathbf{Q}_k^*) - P\right) = 0 \tag{20}$$

$$tr(\mathbf{\Psi}_k \mathbf{Q}_k^*) = 0 \ \forall k \in [1...K] \tag{21}$$

$$\frac{\delta L}{\delta \mathbf{Q}_i} = 0. \tag{22}$$

We directly obtain from (21) the following necessary and sufficient condition for the optimum covariance matrices $\mathbf{Q}_i$ and their corresponding Lagrange multiplier:

$$tr(\mathbf{Q}_i^*) > 0 \Longrightarrow \mathbf{\Psi}_i = 0 \tag{23}$$

$$tr(\mathbf{Q}_j^*) = 0 \Longrightarrow \mathbf{\Psi}_j > 0 \tag{24}$$

(23) means that if user i is active, i.e. its transmit covariance matrix is positive definite, its Langrange multiplier $\mathbf{\Psi}_i$ is equal to the zero matrix. (24) means that if the user j is not active, i.e. its transmit covariance matrix is equal to zero, its Langrange multiplier $\mathbf{\Psi}_j$ is positive definite. From that follows the conditions in (14) and (15).

3.2. Development of algorithm

The Theorem 1 leads us to the following algorithm:

1. Search the user with the best channel, i.e. with the maximum L_2-norm:

$$j = \arg \max_{k \in [1...K]} ||\mathbf{H}_k||_2.$$

2. The eigenvalues of the channel $\mathbf{H}_j \mathbf{H}_j^H$ are named λ_n^H. Compute the optimum transmission strategy for user j with the formula for $n = 1...n_T$

$$p_n^j = \left(\frac{\sigma_n^2}{\nu}(\lambda_n^H)^{-1/2} - \sigma_n^2(\lambda_n^H)^{-1}\right)^+. \tag{25}$$

   from [9].

3. Compute the Langrangian multiplier μ from (15). Note that the eigenvectors of $\mathbf{\Psi}_j$, $\mathbf{H}_j^H \mathbf{A}^{-2} \mathbf{H}_j$ and $\mathbf{Q}_j$ are identical. Hence, we write for (15) diagonal matrices

$$\mathbf{D}_\Psi + \mathbf{D}_{\mathbf{H}_j^H \mathbf{A}^{-2} \mathbf{H}_j} = \mu \mathbf{I}.$$

   The diagonal entries in (26) of $\mathbf{H}_j^H \mathbf{A}^{-2} \mathbf{H}_j$ which correspond to the eigenvalues of $\mathbf{Q}_j$ in which power is allocated are given by $\frac{\lambda_k^H}{(\sigma_n^2 + p_k \lambda_k^H)^2}$ and are all equal. Therefore, we set

$$\mu = \frac{\lambda_k^H}{(\sigma_n^2 + p_k \lambda_k^H)^2} \tag{26}$$

   for all $k \in \{\kappa : p_\kappa > 0\}$.

4. Next, we compute the Lagrangian multiplier $\mathbf{\Psi}_j$ with μ from (26). For all $k \in \{\kappa : p_\kappa > 0\}$ we set $D_\Psi^k = 0$. For all $k \in \{\kappa : p_\kappa = 0\}$ we choose

$$D_\Psi^k = \mu - \frac{\lambda_k^H}{(\sigma_n^2 + p_k \lambda_k^H)^2}. \tag{27}$$

5. Finally, we test for the second largest user l with

$$l = \arg \max_{l \in [1...K]/j} ||\mathbf{H}_l||_2$$

   if the condition (14) is fulfilled with the Lagrangian mutliplier μ from (26):

$$\text{Single-user region} \quad \text{if} \quad \mathbf{H}_l^H \mathbf{A}^{-2} \mathbf{H}_l \leq \mu \mathbf{I}. \tag{28}$$

Remark: The algorithm is deterministic because the computation in each step is unique. The optimal single-user power allocation in (25) is unique. The determination of the Lagrangian multiplier μ in (26) is unique, too. The Lagrangian multiplier $\mathbf{\Psi}_j$ results directly in (27). Therefore, for given channel realization and SNR, the algorithm can directly decide whether we are in the single-user region or not.

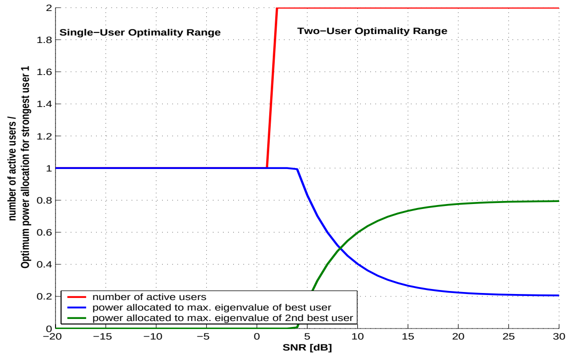

Fig. 2. Optimum number of users and optimum power allocation for strongest user: $K = 2$, $n_T = 2$, $n_R = 2$, one channel realization

4. NUMERICAL SIMULATIONS

In figure (2), we show a simulation result for a two user MIMO MAC with two transmit antennas each and a base station with four receive antennas. In figure (2), for the strongest user we show the optimum power allocation depending on the SNR and the optimum number of users. For low SNR values only the strongest user is transmitting. The single-user region reaches up to 1 dB. At this point the second strongest user is allowed to transmit.

In figure (3), we show the average number of active users for two users each with two transmit antennas and a base station with various number of receive antennas. We average the number of active users over 1000 channel realizations. In figure (3), we observe that the more receive antennas we have the lower is the single-user range.

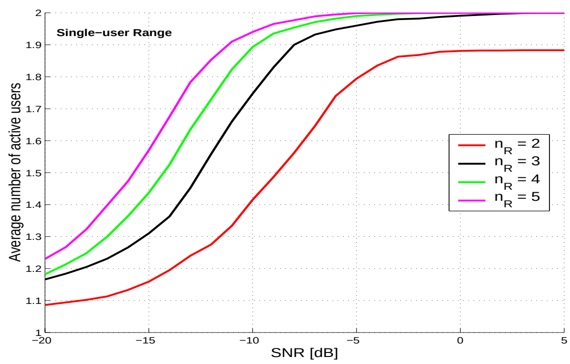

Fig. 3. Average number of active users: $K = 2$, $n_T = 2$, $n_R = \{2, 3, 4, 5\}$

Furthermore, for two receive antennas and SNR approaching infininty, the average number of active users is less than two. I.e. less than two users are supported on average.

In figure (4), we show the impact of the number of users on the average number of active users as a function of the SNR. From figure (4), we observe that the single-user range decreases as the number of user increases.

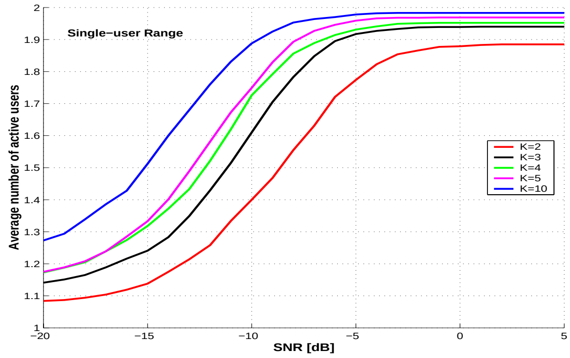

Fig. 4. Average number of active users: $K = \{2, 3, 4, 5, 10\}$, $n_T = 2$, $n_R = 2$

5. CONCLUSIONS

In this work, we study the multiuser MIMO MAC with a linear MMSE multiuser detector at the base station. We compute the average sum MSE depending on the transmit covariance matrices of the users. We formulate the optimization problem of minimizing the sum MSE. Furthermore, we analyze the single-user range, i.e. the SNR range in which only the strongest user transmits. We propose an algorithm which easily checks whether the single-user strategy achieves the minimum MSE or not. We illustrate all theoretical results by numerical simulations.

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