ROBUST POWER ADJUSTMENT FOR TRANSMIT BEAMFORMING IN CELLULAR COMMUNICATION SYSTEMS

Mehrzad Biguesh[†] *Shahram Shahbazpanahi*[*] *Alex B. Gershman*[*]

[†]Department of Communication Systems,
Mercator University, Duisburg, Germany

[*]Department of ECE, McMaster University
Hamilton, Ontario, Canada

ABSTRACT

A new robust power adjustment method is proposed for transmit beamforming in cellular communication systems that use antenna arrays at base stations (BS's). Our method provides an improved robustness against imperfect knowledge of the wireless channel by means of maintaining the required quality of service (QoS) for the worst-case channel uncertainty.

1. INTRODUCTION

Downlink beamforming and power adjustment techniques have been a recent focus of intensive studies in application to cellular communication systems [1]-[6]. The user signal-to-interference-plus-noise ratio (SINR) criterion has been used in these papers to optimize the transmit beamformer weights and adjust the transmitted powers to ensure that the QoS requirements are satisfied for all users. A serious shortcoming of these methods is that they assume the exact knowledge of the user downlink channel correlation (DCC) matrices. In practical situations the channel may be uncertain and these matrices may be subject to substantial errors. As DCC matrices are estimated at BS arrays by means of uplink channel measurements or through a feedback from the users, such errors may be caused by channel variability, user mobility, finite data length effects, etc. In the presence of DCC matrix errors, the QoS constraints can be violated. Hence, the existing transmit beamforming methods can break down in this case.

In this paper, we propose a new closed-form solution for the power adjustment problem in transmit beamforming that has an improved robustness against DCC matrix estimation errors.

Supported in part by the Natural Sciences and Engineering Research Council (NSERC) of Canada, Premier Research Excellence Award Program of the Ministry of Energy, Science, and Technology (MEST) of Ontario, and Wolfgang Paul Award Program of the Alexander von Humboldt Foundation.

2. BACKGROUND

Consider a cellular wireless communication system with K users and M BS's equipped with antenna arrays. Let $\mathbf{w}_k$, P_k, and $c(k)$ be the transmit beamformer weight vector, the transmitted power, and the cell site index for the kth user, respectively, where $\|\mathbf{w}_k\|^2 = 1$.

Assuming that the weight vectors $\mathbf{w}_k$ $(k = 1, \dots, K)$ are known (computed in advance) for each user, the goal of power adjustment is to find all $P_k > 0$ such that the total transmitted power

$$P = \sum_{m=1}^{M} P_m \tag{1}$$

is minimized while the required QoS is guaranteed for each user [4]. The QoS for the kth user is defined by means of its receive SINR [3], [4]

$$\text{SINR}_k = \frac{P_k \mathbf{w}_k^H \mathbf{R}_{k,c(k)} \mathbf{w}_k}{\sigma_k^2 + \sum_{l=1;\, l\neq k}^{K} P_l \mathbf{w}_l^H \mathbf{R}_{k,c(l)} \mathbf{w}_l} \tag{2}$$

where σ_k^2 is the noise power of the kth user, $\mathbf{R}_{k,c(l)}$ is the DCC matrix between the kth user and the BS with the index $c(l)$, and $(\cdot)^H$ stands for the Hermitian transpose.

The QoS requirements are satisfied for all users if

$$\text{SINR}_k \geq \gamma_k\,, \qquad k = 1, \dots, K \tag{3}$$

where γ_k $(k = 1, \dots, K)$ are positive QoS constants.

Using (2) and the fact that the transmitted power is minimized when the inequalities in (3) become equalities, the optimal transmitted powers can be computed as [5]

$$\mathbf{p}_{\rm t} = \mathbf{\Psi}^{-1} \mathbf{p}_{\rm n} \tag{4}$$

where

$$\mathbf{p}_{\rm t} = [P_1, \cdots, P_K]^T \tag{5}$$

$$\mathbf{p}_{\rm n} = [\sigma_1^2, \cdots, \sigma_K^2]^T \tag{6}$$

are the $K \times 1$ vectors of the transmitted and noise powers, respectively, $(\cdot)^T$ stands for the transpose, and

$$[\boldsymbol{\Psi}]_{i,j} = \begin{cases} \mathbf{w}_i^H \mathbf{R}_{i,c(i)} \mathbf{w}_i / \gamma_i & \text{for} \quad i = j \\ -\mathbf{w}_j^H \mathbf{R}_{i,c(j)} \mathbf{w}_j & \text{for} \quad i \neq j \end{cases} \tag{7}$$

Note that all transmitted powers must be positive and, therefore, the positiveness of P_k has to be checked for all $k = 1, \ldots, K$. If $P_k \leq 0$ for some values of k then the underlying problem is infeasible and the parameters γ_k can be decreased to enable problem feasibility.

In practice, the DCC matrices $\mathbf{R}_{i,c(j)}$ may be known imprecisely and, as a result, the QoS constraints (3) may become violated for some of the users. Therefore, the robustness of the transmit beamforming and downlink power adjustment algorithms is of primary importance.

3. ROBUST DOWNLINK POWER ADJUSTMENT

In the presence of DCC matrix errors, we can write

$$\mathbf{R}_{k,c(m)} = \tilde{\mathbf{R}}_{k,c(m)} + \mathbf{E}_{k,c(m)}\,, \quad m = 1, \ldots, K \tag{8}$$

where $\tilde{\mathbf{R}}_{k,c(m)}$ is the *presumed* DCC matrix, $\mathbf{R}_{k,c(m)}$ is its *actual* value, and $\mathbf{E}_{k,c(m)}$ stands for an unknown DCC matrix error. We assume that the Frobenius norm of each error matrix $\mathbf{E}_{k,c(m)}$ is bounded by some known constant:

$$\|\mathbf{E}_{k,c(m)}\| \leq \varepsilon \|\tilde{\mathbf{R}}_{k,c(m)}\| \triangleq \varepsilon_{k,c(m)}\,, \quad m = 1, \ldots, K \tag{9}$$

Let us modify the QoS conditions (3) to incorporate robustness against DCC matrix errors. Instead of (3) (which is formulated for the ideal error-free DCC matrix case), we require the QoS conditions to be satisfied for all possible mismatched DCC matrices. That is, for the kth user we require that

$$\frac{P_k \mathbf{w}_k^H (\tilde{\mathbf{R}}_{k,c(k)} + \mathbf{E}_{k,c(k)}) \mathbf{w}_k}{\sigma_k^2 + \sum_{l=1;\, l \neq k}^{K} P_l \mathbf{w}_l^H (\tilde{\mathbf{R}}_{k,c(l)} + \mathbf{E}_{k,c(l)}) \mathbf{w}_l} \geq \gamma_k \tag{10}$$
$$\text{for all} \ \ \|\mathbf{E}_{k,c(m)}\| \leq \varepsilon_{k,c(m)}\,, \quad m = 1, \ldots, K$$

Note that (10) is equivalent to the *worst-case QoS constraint* which should be satisfied for the worst-case SINR of the kth user. This constraint can be rewritten as

$$\min_{\{\mathbf{E}_{k,c(m)}\}_{m=1}^K} \frac{P_k \mathbf{w}_k^H (\tilde{\mathbf{R}}_{k,c(k)} + \mathbf{E}_{k,c(k)}) \mathbf{w}_k}{\sigma_k^2 + \sum_{l=1;\, l \neq k}^{K} P_l \mathbf{w}_l^H (\tilde{\mathbf{R}}_{k,c(l)} + \mathbf{E}_{k,c(l)}) \mathbf{w}_l} \geq \gamma_k \tag{11}$$

where the norms of all $\mathbf{E}_{k,c(m)}$ $(m = 1, \ldots, K)$ in (11) are bounded according to (9).

Unfortunately, the complexity of (11) does not allow us to obtain any closed-form solution. Therefore, let us strengthen the QoS constraints (11) by replacing the worst-case user SINR by its *lower bound* in each of them. The left-hand side of (11) can be lower-bounded by

$$\frac{P_k \min_{\mathbf{E}_{k,c(k)}} \mathbf{w}_k^H (\tilde{\mathbf{R}}_{k,c(k)} + \mathbf{E}_{k,c(k)}) \mathbf{w}_k}{\sigma_k^2 + \sum_{l=1;\, l \neq k}^{K} P_l \max_{\mathbf{E}_{k,c(l)}} \mathbf{w}_l^H (\tilde{\mathbf{R}}_{k,c(l)} + \mathbf{E}_{k,c(l)}) \mathbf{w}_l} \tag{12}$$

We will make use of the following lemma.

Lemma 1: For any vector $\mathbf{w}$*, matrix* $\mathbf{R}$*, and positive scalar* δ*,*

$$\min_{\|\mathbf{E}\| \leq \delta} \mathbf{w}^H (\mathbf{R} + \mathbf{E}) \mathbf{w} \ = \ \mathbf{w}^H (\mathbf{R} - \delta \mathbf{I}) \mathbf{w} \tag{13}$$
$$\max_{\|\mathbf{E}\| \leq \delta} \mathbf{w}^H (\mathbf{R} + \mathbf{E}) \mathbf{w} \ = \ \mathbf{w}^H (\mathbf{R} + \delta \mathbf{I}) \mathbf{w} \tag{14}$$

where $\mathbf{I}$ *is the identity matrix.*

Proof: Let us consider the following problems

$$\min_{\mathbf{E}} \ \mathbf{w}^H (\mathbf{R} + \mathbf{E}) \mathbf{w} \quad \text{subject to} \quad \|\mathbf{E}\| \leq \delta \tag{15}$$
$$\max_{\mathbf{E}} \ \mathbf{w}^H (\mathbf{R} + \mathbf{E}) \mathbf{w} \quad \text{subject to} \quad \|\mathbf{E}\| \leq \delta \tag{16}$$

From the linearity of the objective function $\mathbf{w}^H (\mathbf{R} + \mathbf{E}) \mathbf{w}$ with respect to $\mathbf{E}$, it follows that the inequality constraint $\|\mathbf{E}\| \leq \delta$ in (15) and (16) can be replaced by the equality constraint $\|\mathbf{E}\| = \delta$. Therefore, the solutions to (15) and (16) can be obtained using Lagrange multiplier method, by means of minimizing/maximizing the function

$$L(\mathbf{E}, \lambda) = \mathbf{w}^H (\mathbf{R} + \mathbf{E}) \mathbf{w} - \lambda (\|\mathbf{E}\|^2 - \delta^2) \tag{17}$$

Equating the gradient $\partial L(\mathbf{E}, \lambda) / \partial \mathbf{E}$ to zero and noting that $\|\mathbf{E}\| = \delta$, we obtain that

$$\mathbf{E} = \mp \delta \, \frac{\mathbf{w}\mathbf{w}^H}{\|\mathbf{w}\|^2} \tag{18}$$

Inserting the latter equation into the objective function yields

$$\begin{aligned} \mathbf{w}^H (\mathbf{R} + \mathbf{E}) \mathbf{w} \ &= \ \mathbf{w}^H (\mathbf{R} \mp \delta \, \frac{\mathbf{w}\mathbf{w}^H}{\|\mathbf{w}\|^2}) \mathbf{w} \\ &= \ \mathbf{w}^H (\mathbf{R} \mp \delta \mathbf{I}) \mathbf{w} \end{aligned} \tag{19}$$

Since δ is positive,

$$\mathbf{w}^H (\mathbf{R} - \delta \mathbf{I}) \mathbf{w} < \mathbf{w}^H (\mathbf{R} + \delta \mathbf{I}) \mathbf{w} \tag{20}$$

and this proves equations (13) and (14). □

Replacing the worst-case user SINR in (11) by its lower bound (12), using Lemma 1 and the equalities $\|\mathbf{w}_k\| = 1$ $(k = 1, \ldots, K)$, and taking into account that the total transmitted power is minimized when the inequality constraints

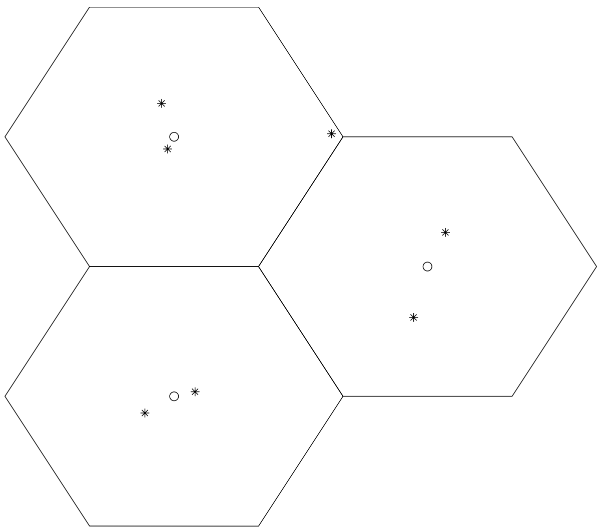

Fig. 1. Simulated scenario. Positions of BS's and users are indicated by $\circ$ and $*$, respectively.

become equalities, we obtain the following robust QoS constraint for the kth user

$$\frac{P_k(\mathbf{w}_k^H \tilde{\mathbf{R}}_{k,c(k)} \mathbf{w}_k - \varepsilon_{k,c(k)})}{\sigma_k^2 + \sum\limits_{l=1;\, l\neq k}^{K} P_l(\mathbf{w}_l^H \tilde{\mathbf{R}}_{k,c(l)} \mathbf{w}_l + \varepsilon_{k,c(l)})} = \gamma_k \qquad (21)$$

The solution to K linear equations in (21) is given by

$$\mathbf{p}_{\mathrm{t,rob}} = \mathbf{\Phi}^{-1}\mathbf{p}_{\mathrm{n}} \qquad (22)$$

where

$$[\mathbf{\Phi}]_{i,j} = \begin{cases} (\mathbf{w}_i^H \tilde{\mathbf{R}}_{i,c(i)} \mathbf{w}_i - \varepsilon_{i,c(i)})/\gamma_i & \text{for} \quad i = j \\ -\mathbf{w}_j^H \tilde{\mathbf{R}}_{i,c(j)} \mathbf{w}_j - \varepsilon_{i,c(j)} & \text{for} \quad i \neq j \end{cases} \qquad (23)$$

Equation (22) is the core of the proposed robust power adjustment algorithm.

4. SIMULATIONS

In our numerical simulations, we consider a cellular time division duplex (TDD) system with three cells and 7 cochannel users. The geometry of the simulated scenario is shown in Fig. 1. The signal attenuation is assumed to be proportional to r^{-4} where r is the BS-user distance. Each BS is assumed to have a transmit uniform circular array of 9 omnidirectional sensors spaced half a wavelength apart. The users are assumed to be incoherently locally scattered sources [7]-[9] with uniform angular distribution, characterized by the central angle and angular spread. To model DCC matrix errors, the presumed and the true angular spreads are assumed

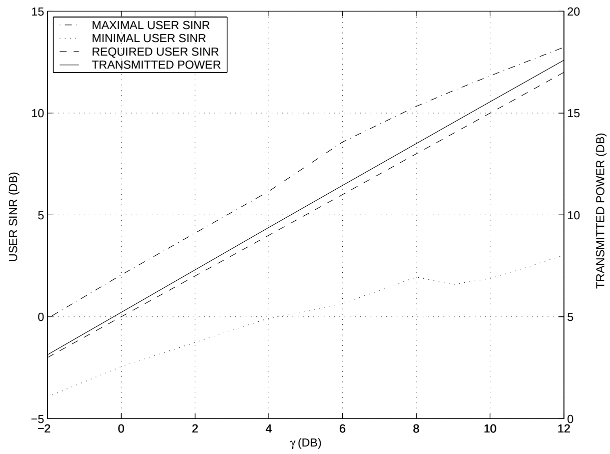

Fig. 2. User SINR's and the transmitted power for the algorithm (4) versus γ.

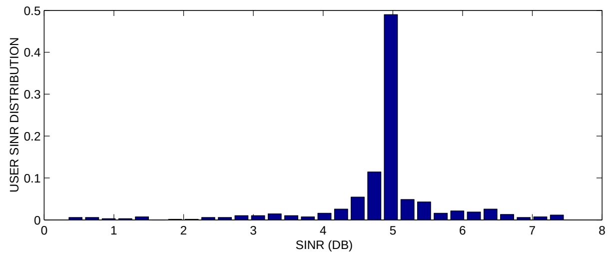

Fig. 3. Histogram of the received user SINR's for the algorithm (4). The required user SINR is equal to 5 dB.

to be the same for each user, whereas the presumed user central angles are randomly selected from the interval of $[-3°, 3°]$ around the corresponding true central angles. We have assumed that the required receive SINR is identical for each user so that $\gamma_i = \gamma$ $(i = 1, \ldots, K)$. Also, we assume that $\sigma_i^2 = \sigma^2$ $(i = 1, \ldots, K)$. 100 independent simulation runs are used to compare the performance of transmit beamforming with the conventional and proposed power adjustment methods (4) and (22), respectively. The algorithm B of [3] is used to compute the transmit weight vectors $\mathbf{w}_k$, $k = 1, \ldots, K$.

Fig. 2 shows the minimal and maximal user SINR's as well as the required user SINR (the latter curve corresponds to the ideal case when there are no DCC matrix errors) of transmit beamforming with the conventional power adjustment algorithm (4) versus γ. In the same figure, the total transmitted power is displayed. This figure shows that, because of DCC matrix errors, the minimal user SINR is much lower than required by the QoS constraints (i.e., these constraints are completely violated). From Fig. 2 we also observe that there is no way to satisfy these constraints by

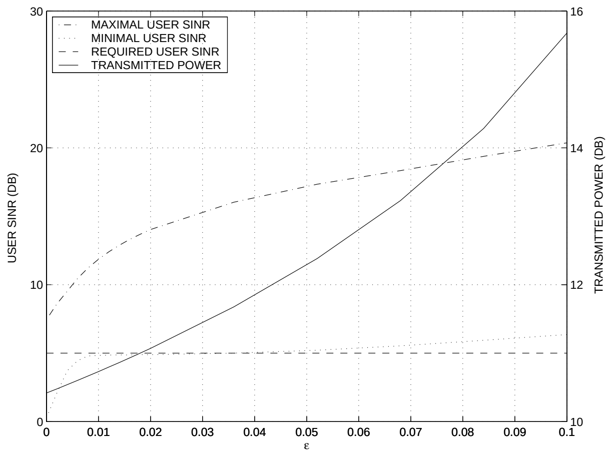

Fig. 4. User SINR's and the transmitted power for the algorithm (22) versus ε.

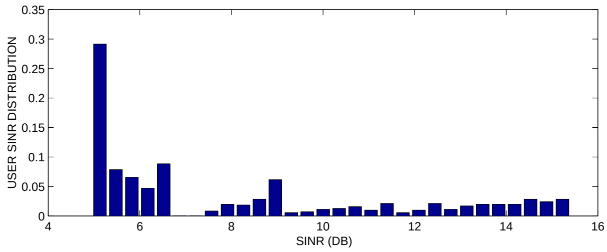

Fig. 5. Histogram of the received user SINR's for the algorithm (22). The required user SINR is equal to 5 dB.

boosting the parameter γ as this causes an unacceptable increase of the total transmitted power.

Fig. 3 displays the histogram of the user SINR's achieved by the conventional algorithm (4) in the case when the required user SINR is 5 dB. Obviously, the QoS constraints are completely violated in this figure as the SINR's of a part of users are below 5 dB.

Fig. 4 shows the minimal and maximal user SINR's of the robust power control algorithm (22) versus ε for the case when the required user SINR is equal to 5 dB. Furthermore, the total transmitted power is displayed in the same plot. This figure demonstrates that $\varepsilon = 0.03$ is enough to guarantee that the QoS constraints are satisfied because the minimal user SINR is above the required value of 5 dB. This necessitates only a moderate (1 dB) increase of the total transmitted power (as compared with more than 17 dB power increase required by the conventional method (4) in Fig. 2 to satisfy the QoS constraints). The histogram of the user SINR's for the robust method (22) is shown in Fig. 5 where $\varepsilon = 0.03$ is chosen. The latter figure demonstrates that the SINR's of all users are higher than 5 dB and, therefore, the QoS constraints are satisfied.

5. CONCLUSIONS

A new robust downlink power adjustment method has been proposed. This method can be used for transmit beamforming in cellular wireless communication systems with antenna arrays at BS's. Simulations have validated an improved robustness of our algorithm as compared to the conventional approach.

6. REFERENCES

[1] D. Gerlach and A. Paulraj, "Base station transmitting antenna arrays for multipath environments," *Signal Processing*, vol. 54, pp. 59-73, Oct. 1996.

[2] C. Farsakh and J. A. Nossek, "Spatial covariance based downlink beamforming in an SDMA mobile radio system," *IEEE Trans. Communications*, vol. 46, , pp. 1497-1506, Nov. 1998.

[3] F. Rashid-Farokhi *et al.*, "Transmit beamforming and power control for cellular wireless systems," *IEEE Journ. Selected Areas in Communications*, vol. 16, pp. 1437-1450, Oct. 1998.

[4] M. Bengtsson, "Robust and constrained downlink beamforming," in *Proc. EUSIPCO'00*, Tampere, Finland, Apr. 2000, pp. 1433-1436.

[5] H. Boche and M. Schubert, "A new approach to power adjustment for spatial covariance based downlink beamforming," in *Proc. ICASSP'01*, Salt Lake City, UT, pp. 2957-2960, Apr. 2001.

[6] H. Boche and M. Schubert, "SIR balancing for multiuser downlink beamforming – a convergence analysis," in *Proc. ICC'02*, New York, NY, pp. 841-845, Apr. 2002.

[7] S. Valaee, B. Champagne, and P. Kabal, "Parametric localization of distributed sources," *IEEE Trans. Signal Processing*, vol. 43, pp. 2144-2153, Sept. 1995.

[8] K. I. Pedersen *et al.*, "A stochastic model of the temporal and azimuthal dispersion seen at the base station in outdoor propagation environments," *IEEE Trans. Veh. Technology*, vol. 49, pp. 437-447, March 2000.

[9] O. Besson, F. Vincent, P. Stoica, and A. B. Gershman, "Approximate maximum likelihood estimators for array processing in multiplicative noise environments," *IEEE Trans. Signal Processing*, vol. 48, pp. 2506-2518, Sept. 2000.