FAST ANTENNA SUBSET SELECTION IN WIRELESS MIMO SYSTEMS

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ABSTRACT

Multiple antenna wireless communication systems have recently attracted significant attention due to their higher capacity as compared to the systems that employ a single antenna. For systems with a large number of antennas, there is a strong motivation to develop techniques with reduced hardware and computational costs. An efficient approach to achieve this goal is the optimal antenna subset selection. In this paper, we propose a fast antenna selection algorithm for wireless multiple-input multiple-output (MIMO) systems. Our algorithm achieves almost the same outage capacity as the optimal selection technique while having a lower computational complexity than the existing nearly optimal antenna selection methods.

1. INTRODUCTION

The capacity of wireless communication systems operating in fading environments can be increased substantially by using multiple antennas at the transmitter and receiver. In [1]-[3], it has been shown that the capacity of MIMO systems increases almost linearly with the minimum of the numbers of the transmit and receive antennas. In practice, however, the main limitation of increasing the number of transmit and receive antennas is typically not the number of sensors, but the cost of the corresponding RF channels for these antennas and a high amount of computations required for signal encoding and decoding. This limitation may be more severe when there are some power constraints.

A promising way of capturing a large portion of the channel capacity in MIMO systems at reduced hardware costs and computational complexity is to select optimally a small number of “best” antennas from the larger set of antennas available.

Antenna subset selection problems have been intensively studied in the literature. To select the receive antennas in the optimal way, the channel capacity has to be computed for all possible combinations of them and, as a result, the computational cost of such a procedure may be prohibitively high.

Gore and Paulraj studied antenna selection for the Alamouti’s space-time codes [4]. They showed that for the transmit/receive antenna selection, choosing two columns/rows of the channel matrix with the largest 2-norms results in the highest SNR at the decoder.

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A promising approach for the fast antenna subset selection was proposed by Gorokhov [5]. This algorithm finds a nearly optimal selection of receive/transmit antennas based on the capacity maximization. The algorithm begins with the full set of antennas available and then removes one antenna per step. In each step, the antenna with the lowest contribution to the system capacity is removed. The reduction in capacity due to removing of each single antenna is evaluated using a proper updating formula. This process is repeated until the required number of antennas remains.

In this paper, we propose a novel fast nearly optimal selection algorithm whose computational complexity is substantially lower than that of the algorithm of [5]. In contrast to the technique of [5], our algorithm starts with the empty set of selected antennas and then adds one antenna per step to this set. In each step, the antenna with the highest contribution to the system capacity is added to the set of selected antennas.

Similar to the algorithm of [5], our approach is applicable both to the receive and transmit antenna selection. However, for the sake of simplicity, only the receive antenna selection case will be considered below.

2. MIMO SIGNAL MODEL

Let us denote the numbers of the available receive and transmit antennas by N_r and N_s, respectively, and number of the selected receive antennas by L_r. The input-output relationship for a MIMO system is given by [6]

$$\mathbf{r}(t) = \sqrt{\frac{\rho}{N_s}}\,\mathbf{H}\mathbf{s}(t) + \mathbf{w}(t) \tag{1}$$

where $\mathbf{s}(t)$, $\mathbf{r}(t)$, and $\mathbf{w}(t)$ are the transmitted signal, the received signal, and the zero-mean additive noise vectors, respectively, $\mathbf{H}$ is the $N_r \times N_s$ channel matrix, ρ is the average SNR, and $(\cdot)^T$ stands for the transpose. Without loss of generality, it is assumed that $E\{\mathbf{w}(t)\mathbf{w}^H(t)\} = \mathbf{I}_{N_r}$ where $\mathbf{I}_{N_r}$ is the $N_r \times N_r$ identity matrix and $(\cdot)^H$ stands for the Hermitian transpose.

We make use of the standard assumption that the channel matrix $\mathbf{H}$ is known at the receiver [3]. For any $\mathbf{H}$, the capacity of the MIMO channel in (1) is given by [1]

$$C(\mathbf{H}) = \log_2 \det\left(\mathbf{I}_{N_s} + \frac{\rho}{N_s}\mathbf{H}^H\mathbf{H}\right) \tag{2}$$

3. PROPOSED ALGORITHM

In order to make the receive antenna selection, instead of computing the capacity for all possible combinations of selected L_r

Table 1. The proposed fast antenna selection algorithm.

FastAntSel$(N_r, L_r, N_s, \mathbf{h}_1, \ldots, \mathbf{h}_{N_r}, \rho)$	
$\mathcal{I} := \{1, 2, \ldots, N_r\}$	
$\mathbf{B} := \mathbf{I}_{N_s}$	
for $j := 1$ to N_r	
$\quad \alpha_j := \mathbf{h}_j^H \mathbf{h}_j$	$O(N_s N_r)$
end	
for $n := 1$ to L_r	
$\quad J := \underset{j \in \mathcal{I}}{\operatorname{argmax}} \ \alpha_j$	$O(N_r L_r)$
$\quad \mathcal{I} := \mathcal{I} - \{J\}$	
$\quad$ if $n < L_r$	
$\qquad \mathbf{a} := \frac{1}{\sqrt{N_s/\rho + \alpha_J}} \mathbf{B} \, \mathbf{h}_J$	$O(N_s^2 \ L_r)$
$\qquad \mathbf{B} := \mathbf{B} \ - \mathbf{a} \, \mathbf{a}^H$	$O(N_s^2 L_r)$
$\qquad$ for all $j \in \mathcal{I}$	
$\qquad\quad \alpha_j := \alpha_j - \lvert \mathbf{a}^H \ \mathbf{h}_j \rvert^2$	$O(N_s N_r L_r)$
$\qquad$ end	
$\quad$ end	
end	
return $\{1, 2, \ldots, N_r\} - \mathcal{I}$	

Table 2. An alternative form of the proposed fast antenna selection algorithm.

FastAntSel$(N_r, L_r, N_s, \mathbf{h}_1, \ldots, \mathbf{h}_{N_r}, \rho)$	
$\mathcal{I} := \{1, 2, \ldots, N_r\}$	
for $j := 1$ to N_r	
$\quad \alpha_j := \mathbf{h}_j^H \mathbf{h}_j$	$O(N_s N_r)$
end	
for $n := 1$ to L_r	
$\quad J := \underset{j \in \mathcal{I}}{\operatorname{argmax}} \ \alpha_j$	$O(N_r L_r)$
$\quad \mathcal{I} := \mathcal{I} - \{J\}$	
$\quad$ if $n < L_r$	
$\qquad \mathbf{a}_n := \frac{1}{\sqrt{\frac{N_s}{\rho} + \alpha_J}} \left(\mathbf{h}_J - \sum_{i=1}^{n-1} (\mathbf{a}_i^H \mathbf{h}_J) \mathbf{a}_i \right)$	$O(N_s L_r^2)$
$\qquad$ for all $j \in \mathcal{I}$	
$\qquad\quad \alpha_j := \alpha_j - \lvert \mathbf{a}_n^H \ \mathbf{h}_j \rvert^2$	$O(N_s N_r L_r)$
$\qquad$ end	
$\quad$ end	
end	
return $\{1, 2, \ldots, N_r\} - \mathcal{I}$	

antennas (as in the optimal selection procedure), the proposed algorithm starts with the empty set of selected antennas and then adds one antenna per step to this set. In each step, our objective is to select one more antenna which leads to the highest increase of the capacity (2). In the n-th step of the algorithm, the $n \times N_s$ channel matrix corresponding to the selected n receive antennas is denoted by $\mathbf{H}_n$. The matrix $\mathbf{H}_n$ contains n rows of $\mathbf{H}$ (in the same order as they appear in $\mathbf{H}$) which correspond to the n antennas selected. We will denote the j-th row of $\mathbf{H}$ by $\mathbf{f}_j$ and its Hermitian transpose by $\mathbf{h}_j$.

Let us assume that in the $(n+1)$-st step, the receive antenna corresponding to the J-th row of $\mathbf{H}$ is selected. By this selection, $\mathbf{f}_J$ is inserted in a proper position in $\mathbf{H}_n$ to obtain the $(n+1) \times N_s$ channel matrix $\mathbf{H}_{n+1}$. Then, using (2), we have

$$C(\mathbf{H}_{n+1}) = \log_2 \det \left(\mathbf{I}_{N_s} + \frac{\rho}{N_s} \mathbf{H}_{n+1}^H \mathbf{H}_{n+1} \right). \tag{3}$$

Noting that

$$\mathbf{H}_{n+1}^H \mathbf{H}_{n+1} = \mathbf{H}_n^H \mathbf{H}_n + \mathbf{h}_J \mathbf{h}_J^H \tag{4}$$

making use of the Sherman-Morrison formula for determinants, and using the notations

$$\mathbf{B}_n \triangleq \left(\mathbf{I}_{N_s} + \frac{\rho}{N_s} \mathbf{H}_n^H \mathbf{H}_n \right)^{-1}, \quad \alpha_{j,n} \triangleq \mathbf{h}_j^H \mathbf{B}_n \mathbf{h}_j \tag{5}$$

we obtain that (3) can be rewritten as

$$C(\mathbf{H}_{n+1}) = C(\mathbf{H}_n) + \log_2 \left(1 + \frac{\rho}{N_s} \alpha_{J,n} \right) \tag{6}$$

where, for any j, the value of $\alpha_{j,n}$ represents the contribution of the j-th receive antenna to the expression under the $\log$ function in (3) if this antenna is selected in the $(n+1)$-st step of the algorithm. Finding J that maximizes $C(\mathbf{H}_{n+1})$ in (6) is equivalent to obtaining

$$J = \underset{j}{\operatorname{argmax}} \ \alpha_{j,n}. \tag{7}$$

The matrix $\mathbf{B}_n$ can be updated using the matrix inversion lemma,

$$\mathbf{B}_{n+1} = \mathbf{B}_n - \mathbf{a}\mathbf{a}^H \tag{8}$$

where $\mathbf{a} \triangleq \frac{1}{\sqrt{N_s/\rho + \alpha_{J,n}}} \mathbf{B}_n \mathbf{h}_J$. To reduce the number of computations, let us use the following updating formula

$$\begin{aligned} \alpha_{j,n+1} &= \mathbf{h}_j^H \mathbf{B}_{n+1} \mathbf{h}_j \\ &= \mathbf{h}_j^H \left(\mathbf{B}_n - \mathbf{a}\mathbf{a}^H \right) \mathbf{h}_j \\ &= \alpha_{j,n} - \lvert \mathbf{a}^H \mathbf{h}_j \rvert^2. \end{aligned} \tag{9}$$

Equation (9) demonstrates that $\alpha_{j,n}$ cannot increase as n increases.

The proposed algorithm is summarized in Table 1 with the right column showing the complexity corresponding to each part of the algorithm. In our technique, $\mathbf{B}_n$ and $\alpha_{j,n}$ are computed in-place, and, therefore, their subscript n is dropped. Furthermore, we assume that $N_r \gg L_r$ because the complexity of antenna subset selection algorithms becomes an important issue only when the number of computations is high (i.e., when N_r is large), while the selected antenna subset L_r is usually much smaller than N_r.

With these assumptions, the total order of complexity is given by $O(\max\{N_s, N_r\} N_s L_r)$. An alternative formulation of our algorithm is shown in Table 2, where storing and updating of $\mathbf{B}_n$ is avoided, and an equivalent of the updating formula for $\mathbf{B}_n$ is directly used to compute $\mathbf{a}$.

Let us compare the computational complexity of our algorithm to the fast algorithm of [5] which is summarized in Table 3. In the case $N_r \gg L_r$, the complexity of this algorithm is not less than $O\left(N_s^2 N_r^2\right)$. Therefore, our algorithm has a significantly lower computational complexity than the algorithm of [5]. This complexity reduction is due to the fact that our algorithm starts from the empty set of selected antennas rather than from the full set (as the algorithm of [5] does). Moreover, in our method, the quantity $\alpha_{j,n}$ is updated rather than recomputed. At last, our algorithm is devoid of the matrix inverse operation. This certainly simplifies its implementation as compared to the algorithm of [5].

Table 3. The antenna selection algorithm of [5].

FastAntSel($N_r, L_r, N_s, \mathbf{h}_1, \ldots, \mathbf{h}_{N_r}, \rho$)	
$\mathcal{I} := \{1, 2, \ldots, N_r\}$	
$\mathbf{H} := [\ \mathbf{h}_1 \quad \mathbf{h}_2 \quad \ldots \quad \mathbf{h}_{N_r}\ ]^H$	
$\mathbf{B} := \left(\mathbf{I}_{N_s} + \frac{\rho}{N_s}\mathbf{H}^H\,\mathbf{H}\right)^{-1}$	$O(N_s^3 + N_s^2 N_r)$
for $n := 1$ to $N_r - L_r$	
for all $j \in \mathcal{I}$	
$\alpha_j := \mathbf{h}_j^H\,\mathbf{B}\,\mathbf{h}_j$	$O(N_s^2 N_r^2)$
end	
$J := \underset{j \in \mathcal{I}}{\operatorname{argmin}}\ \alpha_j$	$O(N_r^2)$
$\mathcal{I} := \mathcal{I} - \{J\}$	
if $n < N_r - L_r$	
$\mathbf{a} := \mathbf{B}\,\mathbf{h}_J$	$O(N_s^2 N_r)$
$\mathbf{B} := \mathbf{B} + \frac{1}{N_s/\rho - \alpha_J}\,\mathbf{a}\,\mathbf{a}^H$	$O(N_s^2 N_r)$
end	
end	
return $\mathcal{I}$	

Table 4. A QR decomposition-based interpretation of the proposed algorithm.

FastAntSel(N_r, L_r, $\mathbf{g}_1, \ldots, \mathbf{g}_{N_r}$)	
$\mathcal{I} := \{1, 2, \ldots, N_r\}$	
for $j := 1$ to N_r	
$\beta_j := \mathbf{g}_j^H\,\mathbf{g}_j$	$O(N_r^2)$
end	
for $n := 1$ to L_r	
$J_n := \underset{j \in \mathcal{I}}{\operatorname{argmax}}\ \beta_j$	$O(N_r L_r)$
$\mathcal{I} := \mathcal{I} - \{J_n\}$	
if $n < L_r$	
$\hat{\mathbf{g}}_{J_n} := \frac{1}{\sqrt{\beta_{J_n}}}\left(\mathbf{g}_{J_n} - \sum_{i=1}^{n-1}(\hat{\mathbf{g}}_{J_i}^H\mathbf{g}_{J_n})\hat{\mathbf{g}}_{J_i}\right)$	$O(N_r L_r^2)$
for all $j \in \mathcal{I}$	
$\beta_j := \beta_j - \lvert\hat{\mathbf{g}}_{J_n}^H\ \mathbf{g}_j\rvert^2$	$O(N_r^2 L_r)$
end	
end	
end	
return $\{1, 2, \ldots, N_r\} - \mathcal{I}$	

4. QR DECOMPOSITION-BASED INTERPRETATION

In this section, a QR decomposition-based interpretation of the proposed algorithm is developed. This interpretation explicitly shows why the proposed algorithm tends to avoid selection of the rows of $\mathbf{H}$ that result in a rank-deficient channel matrix.

The alternative form of the proposed algorithm in Table 2 can be seen to have a high level of similarity to the Gram-Schmidt orthogonalization procedure. The only difference between the algorithm of Table 2 and the Gram-Schmidt orthogonalization of $\mathbf{h}_J$'s is the presence of the term N_s/ρ in line 10 of Table 2. For large values of the SNR, the term N_s/ρ can be ignored and our algorithm becomes identical to the Gram-Schmidt procedure.

Let us show that for *any* value of the SNR, Table 2 corresponds to the Gram-Schmidt orthogonalization of the rows of another matrix $\mathbf{G}$ (defined below) which are related to the rows of $\mathbf{H}$.

If all the available receive antennas are used, the channel capacity can be alternatively expressed as

$$C(\mathbf{H}) = N_r \log\frac{\rho}{N_s} + \log_2\det\left(\frac{N_s}{\rho}\,\mathbf{I}_{N_r} + \mathbf{H}\mathbf{H}^H\right). \qquad (10)$$

Since the matrix $N_s/\rho\,\mathbf{I}_{N_r} + \mathbf{H}\mathbf{H}^H$ in (10) is Hermitian and non-singular, there exists a $N_r \times N_r$ full rank matrix $\mathbf{G}$ such that

$$\frac{N_s}{\rho}\,\mathbf{I}_{N_r} + \mathbf{H}\mathbf{H}^H = \mathbf{G}\mathbf{G}^H. \qquad (11)$$

Note that the matrix $\mathbf{G}$ has the same number of rows as $\mathbf{H}$ but not necessarily the same number of columns. Removing some rows from $\mathbf{H}$ results in removal of the corresponding rows from $\mathbf{G}$. If we denote the j-th column of $\mathbf{G}^H$ by $\mathbf{g}_j$ then it is clear from (11) that

$$\mathbf{g}_i^H\,\mathbf{g}_j = \begin{cases} \mathbf{h}_i^H\mathbf{h}_i + \frac{N_s}{\rho} & \text{if } i = j \\ \mathbf{h}_i^H\mathbf{h}_j & \text{if } i \neq j\,. \end{cases} \qquad (12)$$

Let us use the QR decomposition $\mathbf{G}^H = \mathbf{Q}\mathbf{R}$. Here, $\mathbf{Q}$ is an $N_r \times N_r$ orthogonal matrix, and $\mathbf{R}$ is a $N_r \times N_r$ upper-triangular matrix. Then, from (11) we obtain

$$\frac{N_s}{\rho}\,\mathbf{I}_{N_r} + \mathbf{H}\mathbf{H}^H = \mathbf{R}^H\mathbf{R}\,. \qquad (13)$$

Inserting (13) into (10) and taking into account that the determinant of a triangular matrix is the product of its diagonal entries, we have

$$C(\mathbf{H}) = N_r \log\frac{\rho}{N_s} + \log_2\left(\prod_{j=1}^{N_r} R_{jj}\right)^2 \qquad (14)$$

where R_{jj} is the j-th diagonal element of $\mathbf{R}$. From the theory of the QR decomposition, it is well-known that R_{jj}'s are the norms of the residuals of $\mathbf{g}_j$'s after the Gram-Schmidt orthogonalization. Note that any permutation of the rows of $\mathbf{H}$ corresponds to the corresponding permutation of the rows of $\mathbf{G}$, which, in turn, corresponds to changing the order of orthogonalization of $\mathbf{g}_j$'s. However, such a permutation does not affect $C(\mathbf{H})$. Therefore, even though the individual values R_{jj} depend on the order of rows in $\mathbf{H}$, the value $\prod_{j=1}^{N_r} R_{jj}$ does not.

Now, we are interested in finding a subset of L_r rows of $\mathbf{H}$, or equivalently, a subset of L_r rows of $\mathbf{G}$, such that $\prod_{j=1}^{L_r} R_{jj}$ is maximized. One possible approach would be to permute the rows of $\mathbf{G}$ in such a way that the selection of the first row would correspond to the largest R_{11}, and then to select the second row such that R_{22} is maximized, and so on. An algorithm that is based on this concept is shown in Table 4. In this algorithm, J_n corresponds to the index of the row of $\mathbf{G}$ selected in the n-th step and $\hat{\mathbf{g}}_{J_n}$ corresponds to the normalized version of $\mathbf{g}_{J_n}$.

Comparing the algorithms of Tables 2 and 4 and making use of (12), we see that these two techniques lead to equivalent results. In particular, using (12) it is easy to show by induction that in the n-th step, the parameters α_j and β_j in line 4 of these algorithms are related as $\beta_j = \alpha_j + N_s/\rho$, while in line 12, $\hat{\mathbf{g}}_{J_n}^H\mathbf{g}_j$ in Table 4 results in the same value as $\mathbf{a}_n^H\mathbf{h}_j$ in Table 2.

In summary, the proposed algorithms of Table 2 and Table 1 can be interpreted as QR decomposition-based techniques applied

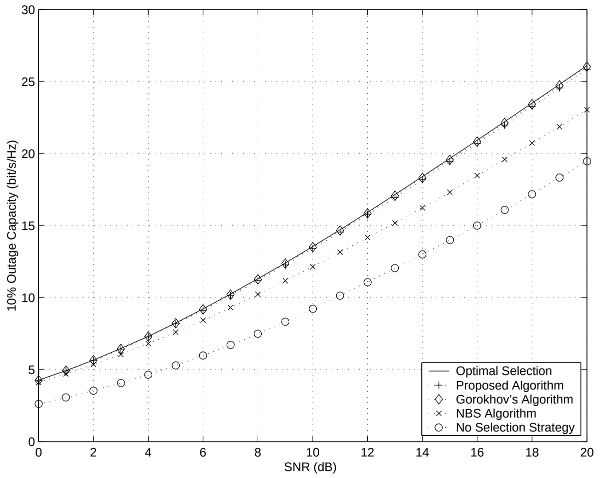

Fig. 1. 10% outage capacity versus the SNR. First example.

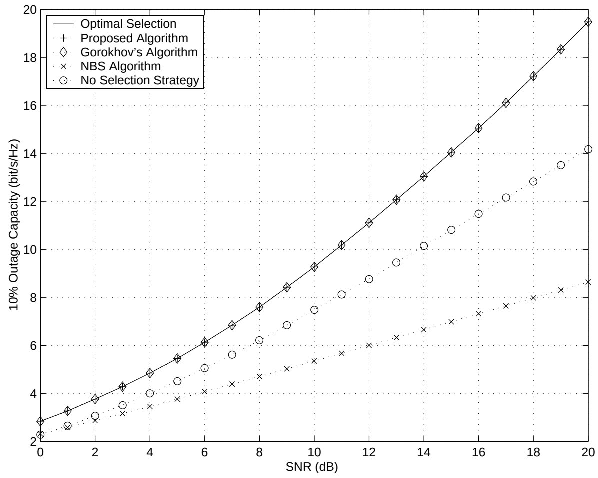

Fig. 2. 10% outage capacity versus the SNR. Second example.

to a correspondingly row-permuted matrix $\mathbf{G}$. However, the QR decomposition is performed in these algorithms by making use of the rows of $\mathbf{H}$ rather than $\mathbf{G}$.

An interesting observation following from the equivalence of the algorithms of Tables 1 and 2 to the QR decomposition of the matrix $\mathbf{H}^H$ at high SNRs is that after selecting N_s receive antennas, selecting any more antennas does not increase the capacity. Furthermore, if the rows of $\mathbf{H}$ are not independent (i.e., if $\mathbf{H}$ is rank deficient), increasing the number of antennas makes sense up to $L_r = \mathrm{rank}(\mathbf{H})$ antennas only.

5. SIMULATION RESULTS

To compare the performances of the proposed algorithm, the optimal antenna selection technique, the norm-based selection (NBS) method of [4], and the fast algorithm of [5], computer simulations have been carried out. The performance corresponding to the case where no advanced selection strategy is used is also shown (in this case, L_r receive antennas are always chosen *at random*). Through our simulations, $N_s = 4$, $N_r = 16$, and $L_r = 4$ are assumed. All results are averaged over 1000 channel realizations.

In the first example, we consider the Rayleigh channel case where the elements of $\mathbf{H}$ are independently drawn from a complex zero-mean Gaussian distribution with the unit variance.

Fig. 1 shows the 10% outage capacity versus the SNR. We see that the performances achieved by the proposed method and the algorithm of [5] are very close to that of the optimal selection procedure for a wide range of the SNR values, while the performance of the NBS algorithm is noticeably lower.

In the second example, we examine the performance of different antenna selection methods when the rows of $\mathbf{H}$ are linearly dependent. In this example, the elements of the first, fifth, nineth and thirteenth rows of $\mathbf{H}$ are independently drawn from a complex zero-mean Gaussian distribution with the unit variance, while there are four groups of identical rows: the rows $\#2, 3, 4$ are identical to the first row, the rows $\#6, 7, 8$ are identical to the fifth row, the rows $\#10, 11, 12$ are identical to the nineth row, and the rows $\#14, 15, 16$ are identical to the thirteenth row.

Fig. 2 shows the 10% outage capacity versus the SNR. This figure clearly demonstrates that the performance of the NBS method is very poor (and even much worse that the performance achieved when the antennas are selected at random), while the proposed method and the method of [5] perform nearly optimally. The reason for such poor performance of the NBS method is that in the second, third and fourth selection steps it choses the antennas that have the same fading coefficients as the antenna chosen in the first step. This does not increase the rank of $\mathbf{H}_n$ and, hence, results in a low capacity.

6. CONCLUSIONS

A new fast algorithm has been proposed for antenna selection in wireless MIMO systems. Our algorithm is shown to have a strong relationship to the QR decomposition of the channel matrix. This interpretation shows how the antenna subset selection procedure depends on the rank properties of the channel matrix.

7. REFERENCES

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