POWER ALLOCATION AND BIT LOADING FOR SPATIAL MULTIPLEXING IN MIMO SYSTEMS

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ABSTRACT

The power assignment problem is important for Multiple-Input-Multiple-Output (MIMO) systems to achieve high capacity. Although this problem is solved by well-known water filling algorithms, this does not provide an optimal solution if the system is constrained to a fixed raw bit error rate threshold and to discrete modulation orders. In this work an approximate approach, called QoS based WF, is proposed to solve the power assignment problem with such constrains. It is shown to outperform quantization of the conventional water filling solution and a well known bit loading algorithm (Chow's algorithm) used in the Digital Subscriber Lines (DSL).

1. INTRODUCTION

MIMO systems with multiple antennas at both the transmitter and the receiver sides can significantly increase the capacity of band-limited wireless channels, provided a sufficiently rich multi-path scattering environment. In this respect, it is ideal for future high data rate wireless communications [1] [2].

The capacity of a MIMO system is maximized by using simple singular value decomposition (SVD) weights combined with optimal power distribution over the transmit antennas which is known as water filling (WF) [3]. This WF solution requires infinite-length codebook and continuous modulation order and continuous power level. Thus it is not possible to use the result directly in practice.

A group of iterative techniques were proposed to approach the WF solution with the constrains of discrete modulation order for the Digital Subscriber Line (DSL) case. These bit loading algorithms also distribute the available energy among a set of parallel AWGN channels as to maximize the overall bit rate for discrete loading problem in practice: the Hughes-Hartogs greedy algorithm, the algorithm of Chow [4], and etc. However, these bit loading algorithms use the famous constant gap concept to approximate the SNR threshold difference between the Shannon bound and a practical system without channel coding both under certain bit error rate (BER) constraints [5]. The gap is assumed to be constant once the BER threshold and the modulation type is fixed, but this gap is constant *only* for very low BER conditions, typically 10^{-6} for uncoded bits (like the practical realizations of DSL). However, it is not realistic for a wireless MIMO link to have such a low uncoded BER and typically the raw BER is around 10^{-3} to 10^{-2} instead.

Furthermore, the WF algorithm itself gives a considerable gain in capacity compared to the simpler V-BLAST structure only in low SNR region in a MIMO system [6], where the constant gap assumption does not hold. For low BER and high SNR conditions the WF solution is not practical. Since the gap approximation is poor in the operating regions of wireless systems, the use of DSL bit loading methods for wireless MIMO systems is limited.

The complexity of bit loading algorithms in DSL applications is often critical. A large number of sub-channels (typically 256) are used and several modulation schemes may be utilized leading to a highly complex optimization problem. Some trade off between complexity and performance must be made for practical DSL bit loading algorithms. Therefore, although some DSL bit loading methods do not require a constant gap assumption, they are still sub-optimal in the throughput sense because of this trade off. In wireless MIMO systems however, the number of spatial sub-channel and modulation orders is typically much smaller. Therefore, sub-optimal methods approaching optimal performance may be considered.

The MIMO power assignment problem under quantized modulation order has been studied for the V-BLAST case. However, the performance of these approaches is limited by the V-BLAST structure itself [7] [8].

In the sequel we will present a new method, called QoS based WF, to distribute power for wireless MIMO system under the constrains of BER threshold and discrete modulation order. This method distributes the transmit power more efficiently than rounding off the conventional WF solution and Chow's DSL bit loading algorithm.

Section 2 describes the system model that will be assumed and Section 3 formulates the problem of QoS constrained discrete water filling. The ideal solution to the QoS constrained discrete water filling problem is presented in Section 4. Two approximate solutions: rounding off conventional WF and QoS based WF are described in detail in Sections 5 and 6. Section 7 presents simulation results and Section 8 concludes the paper.

2. SYSTEM MODEL

We will consider a simple narrow band MIMO model with M_t transmit antennas and M_r receiver antennas operating co-channel:

$$\mathbf{r} = \mathbf{HWs} + \mathbf{v} \tag{1}$$

where $\mathbf{r} \in \mathbb{C}^{M_r}$ is the base band received symbol vector. The elements of the channel matrix, $\mathbf{H} \in M_{M_r \times M_t}$, contain the gain and phase between each transmitter and receiver. It is assumed that each element of $\mathbf{H}$ is iid complex Gaussian random variable with unit variance. The transmitted symbol vector at one time slot is $\mathbf{s} \in \mathbb{C}^{M_s}$ and M_s is the number of concurrent symbol steams. The average symbol energy is normalized to unity. Each element of the noise vector $\mathbf{v} \in \mathbb{C}^{M_r}$ is assumed to be iid zero-mean complex Gaussian with variance σ^2. In order to keep the average transmit power equal to one, the transmit weight matrix $\mathbf{W} \in M_{M_t \times M_s}$ is normalized as:

$$trace(\mathbf{WW}^*) = 1$$

Modulation is adapted to the channel conditions both in time and also in the spatial dimension. A raw bit error rate will be used as target and no channel coding is considered.

3. QOS CONSTRAINED DISCRETE WATER FILLING

When we assume the input of MIMO channel, $\mathbf{Ws}$, is independent and circularly symmetric Gaussian, the capacity of this MIMO system is [2]:

$$C = \sum_{i=1}^{M} \log_2 \left(1 + \frac{\lambda_{W,i}^2 \lambda_{H,i}^2}{\sigma^2} \right) \tag{2}$$

where $M = \min(M_t, M_r)$, $\lambda_{W,i}$ and $\lambda_{H,i}$ are the i^{th} singular values of $\mathbf{W}$ and $\mathbf{H}$ respectively.

We denote the SVD of the channel matrix $\mathbf{H}$ and transmit weight matrix $\mathbf{W}$ as: $\mathbf{U}_H \mathbf{\Sigma}_H \mathbf{V}_H^*$ and $\mathbf{U}_W \mathbf{\Sigma}_W \mathbf{V}_W^*$. The capacity of (2) is maximized by the well known WF solution:

$$\lambda_{W,i}^2 = \left(\xi - \frac{\sigma^2}{\lambda_{H,i}^2} \right)_+, \quad i = 1, 2, \cdots, M \tag{3}$$

$$\mathbf{U}_W = \mathbf{V}_H \tag{4}$$

where $(x)_+$ is $\max(0, x)$ and ξ is chosen so that the following power restrain is satisfied:

$$trace(\mathbf{\Sigma}_W^2) = 1$$

The matrix $\mathbf{V}_W$ does not affect the capacity, so we can choose any unitary matrix with correct size. The easiest way is to select $\mathbf{V}_W$ as an identity and a simple receiver weight $\mathbf{U}_H^*$ as suggested in [3], so the decision statistics will be

$$\tilde{\mathbf{s}} = \mathbf{U}_H^* \mathbf{r} = \mathbf{\Sigma}_H \mathbf{\Sigma}_W \mathbf{s} + \mathbf{U}_H^* \mathbf{v} \tag{5}$$

Because $\mathbf{U}_H$ is unitary, the noise part in the equation above is still white Gaussian. Thus, WF decouples the channel into orthogonal eigenmode sub-channels.

We denote M_i, ρ_i and β_i as the modulation order, SNR per symbol, and the BER for the i^{th} orthogonal sub-channel respectively:

$$\rho_i = \frac{\lambda_{W,i}^2 \lambda_{H,i}^2}{\sigma^2} \tag{6}$$

For a certain modulation type, we can easily find a map of BER to SNR and modulation order as found in [9]. The unique relation can be expressed as:

$$\beta_i = \mathcal{F}(\rho_i, M_i) \tag{7}$$

Many such $\mathcal{F}$ functions can only be solved numerically, but a number of them have a closed analytic form.

Our problem of QoS constrained discrete water filling can be posed as:

$$\begin{aligned} \text{Maximize :} \quad & B = \sum_{i=1}^{M_s} (1 - \beta_i) \log_2 M_i \\ \text{Subject to :} \quad & \beta_i = \mathcal{F}(\rho_i, M_i) \le \Psi \\ & M_i \in 2^n, \;\; n = 1, 2, \cdots \\ & \sum_{i=1}^{M_s} \lambda_{W,i}^2 = 1 \end{aligned}$$

where Ψ is the raw BER threshold.

4. IDEAL OPTIMAL SOLUTION

It follows directly from the WF algorithm in (3) that the ideal form of solution for the above maximization problem with constraints should apply:

$$\lambda_{W,i}^2 \left(1 + \frac{1}{\Gamma_i} \right) = \xi, \;\; i = 1, 2, \cdots, M \tag{8}$$

where

$$\Psi = \mathcal{F}(\Gamma_i, M_i) \tag{9}$$

However, this is only an ideal *form*, not a feasible solution to calculate because of the non-linearity. It is also impossible to get close form expressions of ρ_i or $\lambda_{W,i}$ because of the $\mathcal{F}$ function. Two approaches of finding approximate solutions to this QoS constrained discrete WF problem are now presented below: rounding off WF and QoS based WF.

5. ROUNDING OFF WF SOLUTION

The most direct solution of the QoS constrained discrete water filling problem is to simply round off the original non-constrained WF solution. The rounding off WF solution is:

Step 1 Calculate the initial $\lambda_{W,i}$ based on WF.

Step 2 Find the maximum usable M_i from QoS constrain:

$$\rho_i = \frac{\lambda_{W,i}^2 \lambda_{H,i}^2}{\sigma^2}$$
$$\beta_i = \mathcal{F}(\rho_i, M_i) \leq \Psi \quad \Rightarrow M_i$$

This means that we first calculate the power assignment according to WF solution without any constraints, and then we decide the corresponding discrete modulation order from the QoS requirement. This algorithm is easy to implement based upon the existing WF algorithm, if a dictionary of $\mathcal{F}$ function is pre-stored.

Some sub-channels marked as active by the WF solution may have such poor SNR that even the BER threshold for BPSK cannot be satisfied. These poor sub-channels will then be closed by the rounding off WF method to keep a consistent QoS requirement.

6. QOS BASED WF SOLUTION

The basic idea is to use power efficiently considering that a fixed discrete modulation order is used over a certain SNR region if there is some QoS constraint. If the modulation and the BER threshold are fixed, there is no need to increase unnecessary power in each sub-channel, because extra power can only improve the SNR but not the modulation order and the total bit rate. We can keep the power to the lowest level which just meets the QoS need and therefore obtain some residual power. The residual power can then be re-allocated to other sub-channels to improve their possible modulation orders.

We call this algorithm as QoS based WF and it is described with the concept of a *re-distribution routine*, which means only using the minimum power to achieve the BER threshold in each sub-channel. The re-distribution routine with a control parameter α is defined as:

1 Calculate the current residual power:

$$P_{res} = \alpha \left(1 - \sum_{i=1}^{M_s} \lambda_{W,i}^2\right)$$

2 Find the maximum usable M_i from the QoS constraint:

$$\rho_i = \frac{\lambda_{H,i}^2}{\sigma^2}(\lambda_{W,i}^2 + P_{res})$$
$$\beta_i = \mathcal{F}(\rho_i, M_i) \leq \Psi \quad \Rightarrow M_i$$

3 Find the necessary SNR for M_i:

$$\Psi = \mathcal{F}(\Gamma_i, M_i) \quad \Rightarrow \Gamma_i$$

4 Calculate the necessary power level for this sub-channel:

$$\lambda_{W,i}^2 = \Gamma_i \frac{\sigma^2}{\lambda_{H,i}^2}$$

The QoS based WF method is a two-step tight rounding off and tries to apply the residual power to each active sub-channel, in the order from the strongest sub-channel to the weakest one, to check if a higher order modulation can be used. If a higher order modulation can be reached by applying the residual power, change the modulation order to that higher order and re-calculate the residual power.

The QoS based WF algorithm is listed below:

Step 1 Calculate the initial $\lambda_{W,i}$ based on WF.

Step 2 $\alpha = 0$. Run re-distribution routine for $i = 1 : M_s$.

Step 3 $\alpha = 1$. Run re-distribution routine for $i = 1 : M_s$.

Step 4 Re-scale the power assignment for all the active sub-channels such that no final residual power is left.

$$\text{New} \quad \lambda_{W,i}^2 = \frac{\lambda_{W,i}^2}{\sum_{i=1}^{M_s} \lambda_{W,i}^2}$$

The QoS based WF considers the QoS constraint and comes up with a trade off between the BER and the modulation order. The total bit rate is increased at the cost of a lower BER just satisfying the QoS constraint. Thus we can expect a throughput gain compared to the simple rounding off WF algorithm.

The complexity of this QoS based WF algorithm is the same as the conventional continuous WF. It is not simpler than the WF algorithm because it contains the WF algorithm as the first step to calculate the initial solution.

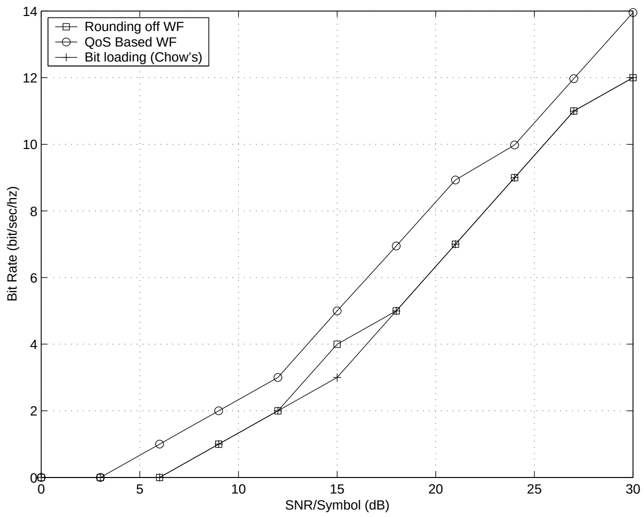

Fig. 1. Total bit rate versus SNR per symbol. Comparison of rounding off WF, QoS based WF, and Chow's bit loading algorithm for a 4×4 uncoded MIMO system, BER threshold $\Psi = 10^{-3}$.

7. SIMULATION RESULTS

Simulations are performed for a 4×4 MIMO system with Gray-coded MPSK. No channel coding is added. The SNR per symbol is defined as $1/\sigma^2$. Each SNR point is calculated by $50,000$ channel realizations. The total bit rate is defined with an outage probability of 1% for all the channel realizations. The QoS requirement is BER below $\Psi = 10^{-3}$. We use Chow's algorithm as a typical DSL bit loading solution for comparison.

Figure 1 shows the resulting total bit rate of three methods: the simple rounding off WF solution, the QoS based WF and the Chow's algorithm. It is easily seen that the simple rounding off WF is sub-optimal. Chow's algorithm gives almost the same total bit rate as the rounding off WF. The QoS based WF can provide higher throughput than the other two schemes: it increases the throughput over the rounding off WF solution by 50% to 100% in the low SNR region and 10% to 30% in the high SNR region.

8. CONCLUSIONS

In this work, we have defined the power assignment problem of wireless MIMO system with constraints of QoS requirement and discrete modulation orders. An approximate approach, called QoS based WF, is proposed to solve this problem. It is shown to outperform a simple round off of the water filling solution and the DSL bit loading algorithm (Chow's) both in terms of the total bit rate throughput.

The QoS based WF is of the same complexity as the water filling algorithm and the Chow's algorithm. For wireless MIMO systems, usually only a small number of transmitter and receiver antenna elements are used, therefore a limited number of spatial channels will typically be employed and QoS based WF algorithms for power allocation should be of great interest.

We only take into account the discrete modulation order in this work, but the power level is not quantized. Future work, therefore, could also examine the performance under quantized power levels to further optimize the power and bit allocation problem for wireless MIMO systems. In addition, joint optimization of channel coding and power assignment can be studied to further improve the system throughput.

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