ROBUST EQUALIZATION BASED ON A NETWORK OF KALMAN FILTERS IN IMPULSIVE NOISE ENVIRONMENTS

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ABSTRACT

In this paper we investigate the problem of channel equalization in an *"$\epsilon-$contaminated"* impulsive noise environments. We show that the equalizer can be structured into a Network of Kalman Filters (NKF) operating in parallel. It is based on a state space description of the communication system, the approximation of the *a posteriori* pdf of the plant noise (related to the transmitted symbols in our case) by a Weighted Sum of Gaussian (WSG) density functions, and the knowledge of the pdf of the *"$\epsilon-$ contaminated"* noise which can be written as a sum of two Gaussians weighted by the probability of appearance of impulsive and Gaussian noises in the observations. The useful information can be extracted from the received sample at any time k even when impulses occur by exploiting the knowledge of the probability of appearence of the impulsive noise and its variance without using any clipping or localisation mechanism for impulses in observations. Simulations results show that the performance of the proposed algorithm is less vulnerable to the impulsive noise and is more robust than the conventional NKF algorithm based on the impulses clipping.

1. INTRODUCTION

High speed data transmission over communication channels is subject to intersymbol interference and noise. Equalization is the process which reconstructs the transmitted data from the received message combatting the distortion and interference of the communication link. The linear transversal filter is the simplest architecture in the class of symbol-by-symbol equalizers. The optimal solution, is the bayesian approach which is also the maximum *a posteriori* symbol-by-symbol decision equalizer [1]. In [2] an equalizer based on a Network of Kalman Filters (NKF) is proposed. These approaches of equalization processes are considered under Gaussian noise assumption. This often leads to analytically tractable solutions. Unfortunately, in many communication channels, such as, urban, indoor radio and underwater acoustic channels, the ambiant noise, through experimental measurements, exhibits Gaussian, as well as impulsive characteristics [3][4].

It is well known that non-Gaussian noise can cause significant performance degradation in conventional systems based on the Gaussian assumption [3], especially the filtering techniques based on a Kalman filter [5]. So, it is necessary to design an equalizer which can operate efficiently against impulsive noise. Therefore, various robust algorithms are worked out in order to increase the efficiency of Kalman approaches where the distribution of the state noise, used for the state estimation, deviates from the Gaussian distribution. The typical method used is the nonlinear receiver such as the hard-limiting receiver based on a Huber's min-max approach, [5][6].

In this paper, we deal with the *"$\epsilon-$contaminated"* noise model [5][6] (an additive combination of Gaussian and impulsive noises). We propose a new robust equalizer structured into a Network of Kalman Filters (NKF) operating in parallel which does not use any clipping or a localisation mechanism for impulses in the received signals. It is based on a state space description of the communication system, the approximation of the *a posteriori* probability density function (pdf) of the plant noise (related to the transmitted symbols of user in our case) by a Weighted Sum of Gaussian (WSG) density functions, and the knowledge of the pdf of the *"$\epsilon-$ contaminated"* noise which can be written as a sum of two Gaussians weighted by the probability of appearance of impulsive and Gaussian noises in the observations. The aim is to estimate the state vector (consisting of the last transmitted symbols) from the received message in a Minimum Mean Square Error (MMSE) sense. Each Kalman filter parameters are adjusted using one noise parameter (variance and contamination constant) and one Gaussian term in the *a posteriori* pdf approximation of the plant noise. So, we will have $2 \times Q$ Kalman filters working in parallel, where Q is the number of points in the signal constellation. The performance of the new algorithm are evaluated using simulations.

The outline of this paper is as follows. The next section presents the state-space description and the non Gaussian noise model. Section 3 describes our new robust equalizer based on a Network of Kalman filters. Section 4 gives some simulation results. Finally, section 5 draws our conclusion.

2. STATE SPACE DESCRIPTION AND NON GAUSSIAN NOISE MODEL

The baseband equivalent channel of interest is composed of the transmitter, the physical channel and the receiver. The transmitted sequence at the channel input $\{d(k)\}$ is composed of independent symbols belonging to a finite set $\Lambda = \{d_i, i = 1, \ldots, Q\}$ which is specific to the type of modulation used and having variance σ_d^2. In the case of a channel modeled by a finite memory linear filter, the received signal can be written as

$$y(k) = \mathbf{C}^T \mathbf{D}(k) + w(k) + b(k) \qquad (1)$$

where

$\diamond$ $\mathbf{C} = [c_0, c_1, \ldots, c_{M-1}]^T$ are the coefficients of the channel impulse response,

$\diamond$ $\mathbf{D}(k) = [d(k), \ldots, d(k-M+1)]^T$ is formed by the last M transmitted symbols,

$\Diamond$ $w(k)$ is an additive white Gaussian noise (AWGN) with zero mean and variance σ_w^2 and $b(k)$ is the impulsive noise.
In the following, the impulse noise is modeled as in [7] by:

$$b(k) = \gamma(k)g(k)$$

where $\{\gamma(k)\}$ stands for a Bernoulli process, a sequence of zeroes and ones with $p(\gamma = 1) = \epsilon$, where ϵ is the contamination constant, and $g(k)$ is a white Gaussian noise with zero mean and variance σ_b^2 such as $\sigma_w^2 \ll \sigma_b^2$.
Under this model, the probability density of the channel noise $v(k) = w(k) + b(k)$ can be expressed as

$$p(v(k)) = (1-\epsilon)\mathcal{N}(0, \sigma_w^2) + \epsilon\mathcal{N}(0, \sigma_b^2 + \sigma_w^2) \quad (2)$$

where $\mathcal{N}(m_x, \sigma_x^2)$ is the Gaussian density with mean m_x and variance σ_x^2. $\{v(k)\}$ is called a *"$\varepsilon-$ contaminated"* noise sequence [5][6]. The *"$\varepsilon-$ contaminated"* channel model is here expressed via both the observation equation (1) and the following state equation,

$$\mathbf{D}(k) = \mathbf{F}\mathbf{D}(k-1) + \mathbf{G}d(k) \quad (3)$$

where $\mathbf{F}$ is the $M \times M$ shift matrix, and $\mathbf{G}$ is the $M \times 1$ vector, given by,

$$\mathbf{F} = \begin{bmatrix} 0\,0\,0\ldots0 \\ 1\,0\,0\ldots0 \\ 0\,1\,0\ldots0 \\ \ldots\ldots\ldots \\ 0\ldots0\,1\,0 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

According to this formulation, the equalization is equivalent to estimating the state vector $\mathbf{D}(k)$ from the observation of the channel output $\mathbf{Y}^k = [y(k), y(k-1), ..., y(0)]$. We note that the estimation of $d(k)$ can be obtained at some delayed time $(k-r)$, where $0 \leq r \leq M-1$. This problem has been resolved by the Kalman observer. However, the filtering techniques, based on an ordinary Kalman filter, completely lose their optimal properties because of outliers in the observations to be processed [5]. Therefore, to cope with this problem, many kinds of "robust to impulsive noise" estimation algorithms have widely been studied, [5][6] and references within. The typical method for the removal of impulsive noise is employing the Huber's min-max approach. The performances of these algorithms depend on the threshold value which is generally unknown.
In the next section, we show how we can exploit the knowledge of impulsive noise parameters in order to have a robust equalization method.

3. ROBUST RECURSIVE STATE ESTIMATION BASED ON A NETWORK OF KALMAN FILTERS

The idea is to compute the *a posteriori* pdf of the state, $p(\mathbf{D}(k)|\mathbf{Y}^k)$, by approximating it by a Weighted Sum of Gaussian. The recursion on $p(\mathbf{D}(k)|\mathbf{Y}^k)$ is explicitely given by the following Bayes relation

$$p(\mathbf{D}(k)|\mathbf{Y}^k) = c_k p(\mathbf{D}(k)|\mathbf{Y}^{k-1}) p(y(k)|\mathbf{D}(k)) \quad (4)$$

with $\frac{1}{c_k} = p(y(k)|\mathbf{Y}^{k-1})$ where $\mathbf{Y}^{k-1} = [y(k-1), ..., y(0)]$
The likelihood of the observation $p(y(k)|\mathbf{D}(k))$ is easily determined if the noise pdf is known.

Using the approximation of the predicted pdf by a WSG, $p(\mathbf{D}(k)|\mathbf{Y}^{k-1})$, leads to the following expression [8],

$$p(\mathbf{D}(k)|\mathbf{Y}^{k-1}) = \sum_{i=1}^{\xi'(k)} \alpha_i'(k)\mathcal{N}(\mathbf{D}(k) - \mathbf{D}_i(k|k-1), \mathbf{P}_i(k|k-1)) \quad (5)$$

with $\{\mathbf{D}_i(k|k-1)\}_{i=1,\ldots,\xi'(k)}$ and $\{\mathbf{P}_i(k|k-1)\}_{i=1,\ldots,\xi'(k)}$ are vectors and matrices of dimension $M \times 1$ and $M \times M$, respectively, and, where the matrices $\mathbf{P}_i(k|k-1)$ approach the zero matrix.
From equation (2), the likelihood of the observation $p(y(k)|\mathbf{D}(k))$ can be written as a sum of two Gaussians,

$$p(y(k)|\mathbf{D}(k)) \simeq (1-\epsilon)\mathcal{N}(y(k) - \mathbf{C}^T\mathbf{D}(k), \sigma_w^2) + \quad (6)$$

$$\epsilon\mathcal{N}(y(k) - C^T\mathbf{D}(k), \sigma_b^2)$$

By replacing (6), (5) in (4) and by denoting $\lambda_1 = 1-\epsilon$, $\lambda_2 = \epsilon$, $\sigma_1^2 = \sigma_w^2$, and $\sigma_2^2 = \sigma_b^2$ we get,

$$p(\mathbf{D}(k)|\mathbf{Y}^k) = c_k \sum_{i=1}^{\xi'(k)} \sum_{j=1}^{2} \lambda_j \alpha_i'(k) A_i^j \quad (7)$$

where $A_i^j = \mathcal{N}(\mathbf{D}(k) - \mathbf{D}_i(k|k-1), \mathbf{P}_i(k|k-1)) \times \mathcal{N}(y(k) - \mathbf{C}^T\mathbf{D}(k), \sigma_j^2)$.

Based on the development done in [2], by rewriting

$$\mathbf{C}^T\mathbf{D}(k) = \mathbf{C}^T\mathbf{D}_i(k|k-1) + (\mathbf{D}(k) - \mathbf{D}_i(k|k-1))^T\mathbf{C} \quad (8)$$

and by defining

$$\mathbf{P}_{i,j}(k|k) = \left[\mathbf{P}_i(k|k-1)^{-1} + \frac{\mathbf{C}\mathbf{C}^T}{\sigma_j^2}\right]^{-1} \quad (9)$$

we obtain, using the inversion matrix lemma on (9),

$$\mathbf{P}_{i,j}(k|k) = \mathbf{P}_i(k|k-1) - \mathbf{K}_{i,j}(k)\mathbf{C}^T\mathbf{P}_i(k|k-1) \quad (10)$$

$$\mathbf{K}_{i,j}(k) = \mathbf{P}_i(k|k-1)\mathbf{C}\left[\sigma_j^2 + \mathbf{C}^T\mathbf{P}_i(k|k-1)\mathbf{C}\right]^{-1} \quad (11)$$

By introducing,
$\mathbf{D}_{i,j}(k|k) = \mathbf{D}_i(k|k-1) + \mathbf{K}_{i,j}(k)\left[y(k) - \mathbf{C}^T\mathbf{D}_i(k|k-1)\right]$
and by doing some rearrangements in equations, we can show that $p(\mathbf{D}(k)|\mathbf{Y}^k)$ can be written as a WSG :

$$p(\mathbf{D}(k)|\mathbf{Y}^k) = \sum_{\substack{l=(i,j) \\ l=(1,1)}}^{2\xi'(k)=\xi(k)} \alpha_l(k)\mathcal{N}(\mathbf{D}(k) - \mathbf{D}_l(k|k), \mathbf{P}_l(k|k)) \quad (12)$$

with

$$\mathbf{D}_{i,j}(k|k) = \mathbf{D}_i(k|k-1) + \mathbf{K}_{i,j}(k)\left[y(k) - \mathbf{C}^T\mathbf{D}_i(k|k-1)\right] \tag{13}$$

$$\begin{aligned}
\alpha_{l=(i,j)}(k) &= \frac{\lambda_j \alpha_i^{'}(k)\beta_{i,j}(k)}{\sum_{l=1}^{2\xi^{'}(k)} \lambda_j \alpha_i^{'}(k)\beta_{i,j}(k)} \\
\beta_{i,j}(k) &= \mathcal{N}(y(k) - \mathbf{C}^T\mathbf{D}_i(k|k-1), \sigma_j^2 + \mathbf{C}^T\mathbf{P}_i(k|k-1)\mathbf{C}) \\
\mathbf{K}_{i,j}(k) &= \mathbf{P}_i(k|k-1)\mathbf{C}\left[\sigma_j^2 + \mathbf{C}^T\mathbf{P}_i(k|k-1)\mathbf{C}\right]^{-1} \\
\mathbf{P}_{i,j}(k|k) &= \mathbf{P}_i(k|k-1) - \mathbf{K}_{i,j}(k)\mathbf{C}^T\mathbf{P}_i(k|k-1)
\end{aligned} \tag{14}$$

We have then established the way of computing the density function $p(\mathbf{D}(k)|\mathbf{Y}^k)$ as a sum of Gaussian density functions when the density function $p(\mathbf{D}(k)|\mathbf{Y}^{k-1})$ is a sum of Gaussian density functions.

The predicted pdf $p(\mathbf{D}(k+1)|\mathbf{Y}^k)$ for the next iteration is determined according to this Bayesian relation :
$p(\mathbf{D}(k+1)|\mathbf{Y}^k) = \int p(\mathbf{D}(k)|\mathbf{Y}^k)\, p(\mathbf{D}(k+1)|\mathbf{D}(k)) d\mathbf{D}(k)$
with $p(\mathbf{D}(k+1)|\mathbf{D}(k)) = p(\mathbf{G}d(k+1))$
Using the assumption of i.i.d symbols, we can write,

$$p(\mathbf{G}d(k+1)) = \sum_{q=1}^{Q} p_q \mathcal{N}(\mathbf{G}d(k+1) - \mathbf{G}d_q, \mathbf{Q}_q)$$

where $p_q = \frac{1}{Q}$ and $\mathbf{Q}_q = \epsilon_0 I_M$ ($\epsilon_0 \ll 1$).

By denoting $\mathbf{D}_{i,j,q}(k+1|k) = \mathbf{F}\mathbf{D}_{i,j}(k|k) + \mathbf{G}d_q$ and $\mathbf{P}_{i,j,q}(k+1|k) = \mathbf{F}\mathbf{P}_{i,j}(k|k)\mathbf{F}^T + \mathbf{Q}_q$,
we can show that $p(\mathbf{D}(k+1)|\mathbf{Y}^k)$ can be obtained as,

$$p(\mathbf{D}(k+1)|\mathbf{Y}^k) = \sum_{m=1}^{\xi^{'}(k+1)} \alpha_m^{'}(k+1) \times \tag{15}$$

$$\mathcal{N}(\mathbf{D}(k+1)\mathbf{D}_{l,q}(k+1|k), \mathbf{P}_{l,q}(k+1|k))$$

with $m = (l,q) = (i,j,q)$, $\xi'(k+1) = \xi(k) \times Q$, $\alpha_m^{'}(k+1) = p_q \alpha_l(k)$
Finally, we can resume the algorithm as follows,

Prediction

$$\boxed{\begin{aligned}
\xi'(k) &= \xi(k-1) \times Q \\
\alpha_m^{'}(k) &= p_q \alpha_l(k-1) \text{ avec } m = (l,q) = (i,j,q) \\
\mathbf{D}_m(k|k-1) &= \mathbf{F}\mathbf{D}_{i,j}(k-1|k-1) + \mathbf{G}d_q \\
\mathbf{P}_m(k|k-1) &= \mathbf{P}\mathbf{F}_{i,j}(k-1|k-1)\mathbf{F}^T + \mathbf{Q}_q
\end{aligned}}$$

Estimation

$$\boxed{\begin{aligned}
\xi(k) &= 2\xi^{'}(k) \\
\mathbf{D}_{i,j,q}(k|k) &= \mathbf{D}_{i,j,q}(k|k-1) + \\
&\mathbf{K}_{i,j,q}(k)\left[y(k) - \mathbf{C}^T\mathbf{D}_{i,j,q}(k|k-1)\right] \\
\mathbf{P}_{i,j,q}(k|k) &= \mathbf{P}_{i,j,q}(k|k-1) - \mathbf{K}_{i,j,q}(k)\mathbf{C}^T\mathbf{P}_{i,j,q}(k|k-1) \\
\mathbf{K}_{i,j,q}(k) &= \frac{\mathbf{P}_{i,j,q}(k|k-1)\mathbf{C}}{\sigma_j^2 + \mathbf{C}^T\mathbf{P}_{i,j,q}(k|k-1)\mathbf{C}} \\
\alpha_{i,j,q}(k) &= \frac{\lambda_j \alpha_{i,j,q}^{'}(k)\beta_{i,j,q}(k)}{\sum_{i,j,q}^{2\xi(k-1)Q} \lambda_j \alpha_{i,j,q}^{'}(k)\beta_{i,j,q}(k)} \\
\beta_{i,j,q}(k) &= \mathcal{N}(y(k) - \mathbf{C}^T\mathbf{D}_{i,j,q}(k|k-1), \\
&\sigma_j^2 + \mathbf{C}^T\mathbf{P}_{i,j,q}(k|k-1)C)
\end{aligned}}$$

Then, the MMSE estimation of the state vector, $E(\mathbf{D}(k)|\mathbf{Y}^k)$, is

$$\widehat{\mathbf{D}}_{MMSE}(k|k) = \sum_{i,j,q}^{\xi(k)} \alpha_{i,j,q}(k)\mathbf{D}_{i,j,q}(k|k) \tag{16}$$

We can see that each predicted state, $\mathbf{D}_{i,j,q}(k|k)$, at the output of the Kalman filter indexed by (i,j,q) is weighted by a coefficient $\alpha_{i,j,q}$ that depends on the probability of appearance of the Gaussian and impulsive noises ($1-\epsilon$ and ϵ resp.) and the error covariance of each Kalman filter ($\beta_{i,j,q}$). When an impulse occurs in the received sample $y(k)$, the $\beta_{i,1,q}$ terms tend to zero, otherwise, the $\beta_{i,2,q}$ terms tend to zero. So, the algorithm extracts the information symbol even when the impulses are present without need of any threshold. In that way, the impulses do not affect the good behavior of the algorithm.
Remark : If $\epsilon = 0$, we obtain the original NKF-based equalizer published in [2]
In a binary transmission, the equalizer output with a delay r, $\hat{d}_{MMSE}(k-r)$, is given by the sign of the $(r+1)^{th}$ component of the estimated state. The equation (16) proves that the estimated state, in an MMSE sense, is a convex combination of the outputs of $\xi(k)$ parallel Kalman filters. The covariance matrix of the estimation error is

$$\overline{\mathbf{P}}(k) = \sum_{i,j,q}^{\xi(k)} \alpha_{i,j,q}(\mathbf{P}_{i,j,q}(k|k) + \tag{17}$$

$$(\mathbf{D}_{i,j,q}(k|k) - \mathbf{D}(k|k))(\mathbf{D}_{i,j,q}(k|k) - \mathbf{D}(k|k))^T)$$

As established above, the complexity of the algorithm, evaluated by $\xi(k)$, grows exponentially through iterations. To make it of practical use for on-line processing, the sum giving $p(\mathbf{D}(k)|Y^k)$ in (12) will be constrained to contain only one term after the filtering step, let $\xi(k) = 1$. This is done by setting $\mathbf{D}_i(k|k)$ to the value of the estimated state $\hat{\mathbf{D}}_{MMSE}(k)$, $\mathbf{P}_i(k|k)$ to $\overline{\mathbf{P}}(k)$ and $\alpha_{i,j,q}(k)$ to 1 for the next prediction step.
This approximation is rational because the *a posteriori* state pdf is assumed to be localized around the MMSE estimation if the equalizer performs correctly. It reduces the number of parallel Kalman filters from $\xi(k)$ to $2Q$.

4. SIMULATION RESULTS

In this section, we give some simulation results for the proposed algorithm based on a network of Kalman filters for a BPSK modulation alphabet ($Q = 2$). In figure 1, we include a channel $\mathbf{C}$, which is a realization of the typical channel Model A specified by Hiperlan2 standard. The delay estimation r is fixed to 8 symbols. We plot the BER obtained from three methods. In the first, we choose $\epsilon = 0$, so, we have the influence of the Gaussian noise only. In the second and third curves, the contamination parameter or the probability of impulsive noise is chosen $\epsilon = 8 \times 10^{-3}$ with $\frac{\sigma_b^2}{\sigma_w^2} = 500$. We employ our algorithm described in section 3 and the classical NKF [2] with a fixed threshold, as is done in [6], in order to prevent the divergence of the algorithm when impulses occur.

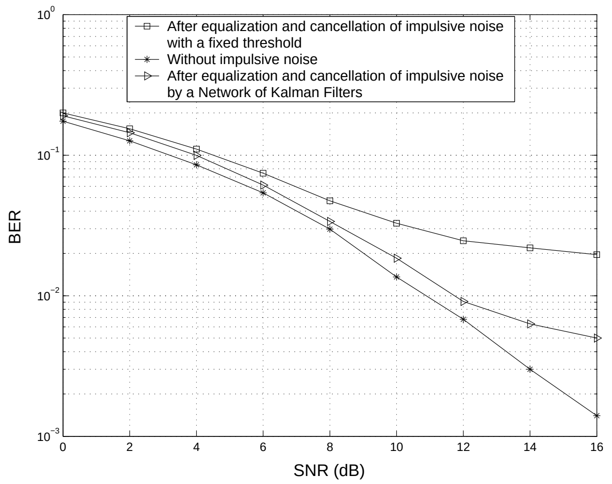

Fig. 1. BER performance when we consider a channel $\mathbf{C}$

We can notice, the good behavior of our method under these circumstances, since the curve after equalization and cancellation of impulsive noise is only marginally different from the curve obtained with Gaussian noise only. We can also remark that the robust NKF outperforms the NKF+threshold method. It is clear, that its performance depend on the value of this threshold.

In figure 2, we show the efficiency of impulse noise cancellation in the proposed structure. We employ a channel $\mathbf{C} = 1$ in order to cancel the effect of the channel and highlight the impact of impulsive noise on observations. We plot $E_s/MSE\ (dB)$ versus $SNR = E_s/\sigma_w^2\ (dB)$ where E_s is the symbol energy. The MSE is computed between the estimated symbols, $\hat{d}$, obtained from the observations corrupted by both Gaussian and impulsive noises ($\epsilon \neq 0$), and the received signal in the case of Gaussian noise only, $d + b$. So, MSE can be written as $MSE = E\{||d + b - \hat{d}||^2\}$. Therefore, if the algorithm cancels the effect of the impulsive noise, we will have a reliable estimation of the transitted data (i.e. $\hat{d} = d$). So, the value of the MSE will tend to the Gaussian noise variance σ_w^2. From figure 2, we notice that the robust NKF equalizer is not vulnerable to the impulsive noise and cancels its effect since its curve closely follows the curve containing the Gaussian noise only.

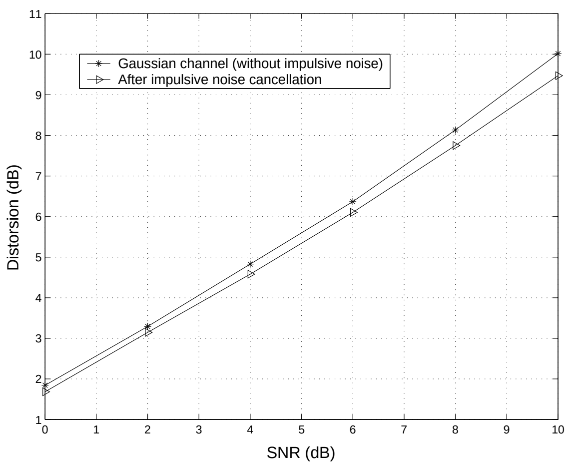

Fig. 2. Efficiency of impulsive noise cancellation in the proposed structure

5. CONCLUSION

In this paper, we have described a new equalizer based on a NKF given us the optimal MMSE estimation of the state vector formed by the last M transmitted symbols. It is based on a state-space description of the communication system, the approximation of the density functions of the data signals by a WSG density functions and the knowledge of the pdf of the measurment noise which can be written as two Gaussian terms weigthed by the probability of appearance of Gaussian and impulsive noises. In this approach, neither clipping nor impulses correction mechanism are employed. Simulation results show that the proposed algorithm is less vulnerable to the impulsive noise and presents a better behavior compared to the NKF+threshold scheme. Many extensions are under consideration, in order to increase the practical usefulness of this approach.

6. REFERENCES

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