BLIND MULTICHANNEL IMAGE RESTORATION USING SUBSPACE BASED METHOD

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ABSTRACT

We address the problem of image restoration in a multichannel system degraded by unknown blurs and additive noise. To identify the unknown blurs, modelled as FIR channels, we propose a subspace based method which exploits an orthogonality property between *signal* and *noise* subspaces. Finally to demonstrate the performance of the proposed method, we run simulations and compare the results with those of the *cross relation* method.

1. INTRODUCTION

In numerous applications, multiple blurred observed images of a single image are available whereas the original image and the blurs are unknown.

Spurred by promising results in 1-D signals, recently there has been a growing literature of so called *algebraic* methods applied to multichannel 2-D (image) restoration [2, 5, 7, 1, 4, 8]. In blind setting, where blurs and input image are both unknown, in general there are three possible approaches to restore the original image, (i) first the blurs are estimated and then one of the available multichannel restoration methods is employed to recover the original image [2, 5, 1], (ii) the restoration filters are directly estimated [7, 1], and (iii) the input image is directly estimated without estimating first the blurs or restoration filters [4, 8]. While the second and third approaches are conceptually more appealing than the first one, we argue that in most cases it is more convenient to restore the original image with known or estimated blurs, because we have more control on the characteristics of the desired image, i.e. restoration with regularization.

In this paper, we propose a subspace (SS) based method for 2-D blur identification in multichannel setting.

2. PROBLEM STATEMENT

Let K be the number of FIR channels $h_k(n_1, n_2), k = 1, \ldots, K$, each of size $(m_h \times n_h)$. Using single-input-multi-output (SIMO) model we have the following

$$\begin{aligned} \mathbf{y}(l_1, l_2) &= \sum_{m_1=0}^{m_h-1} \sum_{m_2=0}^{n_h-1} \mathbf{h}(m_1, m_2) x(l_1 - m_1, l_2 - m_2) \\ &\quad + \mathbf{n}(l_1, l_2) \end{aligned} \tag{1}$$

where $\mathbf{y}(l_1, l_2) = [y_1(l_1, l_2), \ldots, y_K(l_1, l_2)]^T$, $\mathbf{h}(l_1, l_2) = [h_1(l_1, l_2), \ldots, h_K(l_1, l_2)]^T$, $x(l_1, l_2)$ and $\mathbf{n}(l_1, l_2) = [n_1(l_1, l_2), \ldots, n_K(l_1, l_2)]^T$, denote the output images, blurs, input image, and independently distributed additive noise, respectively. In this paper we are concerned to the following problem: given (m_h, n_h) and $y_k(l_1, l_2), k = 1, \ldots, K$, each of size $((m_x - m_h + 1) \times (n_x - n_h + 1))$, where $(m_x \times n_x)$ denotes the size of $x(l_1, l_2)$, find $h_k(l_1, l_2), k = 1, \ldots, K$, in the absence of noise.

3. THE PROPOSED METHOD

The 2-D SS method we present here extends the idea of the corresponding 1-D method [3]. The originality of this method comes from the fact that it exploits the separation between the signal and noise subspaces, as well as the special structure of the multichannel convolution matrix. This leads to an advantage because by exploiting the particular generalized block Toeplitz structure of the unknown multichannel convolution matrix $\mathbf{H}$, the SS method does not require the channel input to be uncorrelated. In fact, as long as the estimated input covariance $\mathbf{R}_X$ (which will be defined below) is full rank, the SS method can identify the channel impulse responses $\mathbf{h}_k(l_1, l_2), k = 1, \ldots, K$, when $\mathbf{H}$ is full column rank.

Moreover unlike most of the single channel identification methods, e.g. Wiener filter, the input signal autocorrelation does not need to be known for SS method and the noise variance σ_N^2 may be unknown so long as the column dimension $d_{\mathbf{H}} = (m_w + m_h - 1)(n_w + n_h - 1)$ of the channel convolution matrix $\mathbf{H}$ is known or can be estimated.

Consider an $(m_w n_w \times 1)$ size sample vector obtained from vectorizing a $(m_w \times n_w)$ square area taken from the image output of the k-th channel

$$\mathbf{Y}_k = \mathbf{H}_k \mathbf{X} + \mathbf{N}_k \tag{2}$$

where

$$\begin{aligned}
\mathbf{Y}_k &= [\mathbf{y}_k^T(l_1, l_2), \ldots, \mathbf{y}_k^T(l_1 - m_w + 1, l_2 - n_w + 1)]^T \\
\mathbf{H}_k &= \begin{bmatrix} H_k^{(0)} & \ldots & H_k^{(n_h-1)} & & 0 \\ 0 & \ddots & & & \\ 0 & & H_k^{(0)} & \ldots & H_k^{(n_h-1)} \end{bmatrix} \\
\mathbf{X} &= [x(l_1, l_2), \ldots, x(l_1 - m_t + 1, l_2 - n_t + 1)]^T,
\end{aligned}$$

$\mathbf{N}_k$ has the same structure as $\mathbf{Y}_k$, and $H_k^{(j)}, j = 0, \ldots, n_h - 1$ denotes the 1-D multichannel convolution matrix corresponding to $h_k^{(j)} = [h_k(0, j), \ldots, h_k(m_h - 1, j)]$,

$$H_k^{(j)} = \begin{bmatrix} h_k^{(j)} & 0 & \ldots & 0 \\ 0 & h_k^{(j)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots \\ 0 & \ldots & 0 & h_k^{(j)} \end{bmatrix}, \tag{3}$$

$m_t = m_w + m_h - 1, n_t = n_w + n_h - 1$, $(m_w \times n_w)$ the observation window to construct data matrix from $\mathbf{y}_k$, and $(m_t \times n_t)$ the size of the corresponding area in the input image contributing to the observed $(m_w \times n_w)$ area in the output images.

Stacking all K-channel outputs gives us

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}. \tag{4}$$

where $\mathbf{Y} = [\mathbf{Y}_1 \ldots \mathbf{Y}_K]^T$, $\mathbf{H} = [\mathbf{H}_1 \ldots \mathbf{H}_K]^T$, and $\mathbf{N} = [\mathbf{N}_1 \ldots \mathbf{N}_K]^T$.

For the development of the SS method we use the following assumptions:

A 1 $Km_w n_w > (m_h + m_w - 1)(n_h + n_w - 1)$ *and* $\mathbf{H}$ *has the full column rank* $(m_h + m_w - 1)(n_h + n_w - 1)$.

A 2 $\mathbf{n}_k(l_1, l_2)$ *is white with zero mean and is uncorrelated with* $\mathbf{x}(l_1, l_2)$.

Based on the equ. (4), the output covariance matrix $\mathbf{R}_Y$ of size $Km_w n_w \times Km_w n_w$ can be written in the form

$$\mathbf{R}_Y = E\{\mathbf{Y}\mathbf{Y}^T\} = \mathbf{H}\mathbf{R}_X\mathbf{H}^T + \sigma_N^2 \mathbf{I} \tag{5}$$

where

$$\mathbf{R}_X = E\{\mathbf{X}\mathbf{X}^T\} \tag{6}$$

and

$$\mathbf{R}_N = E\{\mathbf{N}\mathbf{N}^T\} = \sigma_N^2 \mathbf{I} \tag{7}$$

denote the covariance matrices of input signal and channel noise, respectively and $E\{.\}$ is the expectation operator. The last term of (7) is the consequent of **A**2.

Additionally, we assume further that

A 3 $\boldsymbol{R}_X$ *has the full rank* $d_{\mathbf{H}} = (m_h + m_w - 1)(n_h + n_w - 1)$

Through eigenvalue decomposition, the covariance matrix $\mathbf{R}_Y$ can be diagonalized as

$$\begin{aligned}
\mathbf{U}^T \mathbf{R}_Y \mathbf{U} &= \begin{bmatrix} \Sigma_X^2 & 0 \\ 0 & 0 \end{bmatrix} + \sigma_N^2 \mathbf{I} \qquad (8) \\
&= \begin{bmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0 & \sigma_2^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_{Km_w n_w}^2 \end{bmatrix},
\end{aligned}$$

where

$$\begin{aligned}
\sigma_i^2 &> \sigma_N^2, \ \ i = 1, 2, \ldots, d_{\mathbf{H}}; \qquad (9) \\
\sigma_i^2 &= \sigma_N^2, \ \ i = d_{\mathbf{H}} + 1, \ldots, (Km_w n_w). \qquad (10)
\end{aligned}$$

We can then partition the eigen-vectors $\{U_i\}_{i=1}^{Km_w n_w}$ into a signal subspace $\mathbf{U}_X$ and a noise subspace $\mathbf{U}_N$ as

$$\mathbf{U} = [\underbrace{U_1 \ \ldots \ U_{d_{\mathbf{H}}}}_{\mathbf{U}_X} \ \underbrace{U_{d_{\mathbf{H}+1}} \ \ldots \ U_{Km_w n_w}}_{\mathbf{U}_N}]. \tag{11}$$

Using the identification condition that $\mathbf{H}$ has $(m_w + m_h - 1)(n_w + n_h - 1)$ independent columns, we obtain

$$\mathbf{H}\mathbf{R}_X\mathbf{H}^T = \mathbf{R}_Y - \sigma_N^2 \mathbf{I} = \mathbf{U}_X \Sigma_X^2 \mathbf{U}_X^T. \tag{12}$$

Accordingly, the unknown multichannel convolution matrix satisfies

$$\mathbf{H}\mathbf{R}_X^{1/2} = \mathbf{U}_X \Sigma_X \mathbf{V}, \tag{13}$$

where $\mathbf{V}$ is an unknown unitary matrix. Since we assume that $\mathbf{H}$ is full column rank and $\mathbf{R}_X$ is full rank, then Σ_X has full rank and

$$\mathbf{H} = \mathbf{U}_X \Sigma_X \mathbf{V} \mathbf{R}_X^{-1/2}, \tag{14}$$

It is clear then that $\mathbf{H}$ and $\mathbf{U}_X$ live in the same $d_{\mathbf{H}}$ dimensional space and are orthogonal to the noise subpace $\mathbf{U}_N$. Note that $\mathbf{U}_X$ spans the $d_{\mathbf{H}}$ dimensional signal subspace whereas $\mathbf{U}_N$ spans the noise subspace of dimension $Km_wn_w - d_{\mathbf{H}}$.

From the above observation, we have

$$\mathbf{U}_N^T\mathbf{U}_X = 0, \tag{15}$$

and consequently the following orthogonality relationship is satisfied

$$\mathbf{U}_N^T\mathbf{H} = 0. \tag{16}$$

Let $\mathbf{H}_{m_w,n_w} = \mathbf{H}$ and $\tilde{\mathbf{H}}_{m_w,n_w}$ be an alternative form of constructing the 2-D convolution matrix, by scanning first the rows then columns of $\mathbf{h}_k, k = 1,\ldots,K$. Then the uniqueness of $\mathbf{H}$ in (16) is establised by the following proposition.

Proposition 1 *[9] Assume:*

1. $K(m_w-1)n_w \geq (m_t-1)n_t$, $Km_w(n_w-1) \geq m_t(n_t-1)$, $m_w \geq m_h$, *and* $n_w \geq n_h$*;*
2. $\mathbf{H}_{m_w,n_w}$, $\mathbf{H}_{m_w,n_w-1}$, $\mathbf{H}_{m_w,n_w-1}$, $\tilde{\mathbf{H}}_{m_w-1,n_w}$, $\mathbf{H}_{m_w,n_h}$, *and* $\tilde{\mathbf{H}}_{m_h,n_w}$ *are full-column rank,*

then, the following conditions are equivalent:

- $\mathbf{H}'_{m_w,n_w}$ *is nonzero and* $\mathcal{R}(\mathbf{H}'_{m_w,n_w}) \subset \mathcal{R}(\mathbf{H}'_{m_w,n_w})$*;*
- $\mathbf{H}'_{m_w,n_w}$ *and* $\mathbf{H}_{m_w,n_w}$ *are proportional, i.e.* $\mathbf{H}'_{m_w,n_w} = \alpha\mathbf{H}_{m_w,n_w}$*, where* α *is a scalar factor,*

where $\mathcal{R}$ *denotes the range space.*

Generally the orthogonal relationship in equ. (16) is not sufficient for the identification of $\mathbf{H}$, if for not the special structure of $\mathbf{H}$ and if the conditions in Proposition 1 are not satisfied. As can be seen from the definition of $\mathbf{H}$, the number of unknown parameters is much smaller than the number of elements of $\mathbf{H}$. Making use of the special structure of $\mathbf{H}$, we can solve the equ. (16) by minimizing a quadratic cost function

$$\sum_{i=d_{\mathbf{H}}+1}^{Km_wn_w} ||U_i^T\mathbf{H}||^2. \tag{17}$$

To solve this minimization, we will use the special structure of $\mathbf{H}$. Remember that all K channel responses can be written in a super vector

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}_1(:) \\ \vdots \\ \mathbf{h}_K(:) \end{bmatrix} \tag{18}$$

Accordingly, each eigen-vector of $\mathbf{R}_X$ can also be partitioned into K smaller vectors, each of size m_wn_w

$$U_i = \begin{bmatrix} \mathbf{u}_1^{(i)}(:) \\ \vdots \\ \mathbf{u}_K^{(i)}(:) \end{bmatrix} \tag{19}$$

It is clear that the orthogonality condition can be reformulated as

$$U_i^T\mathbf{H} = \mathbf{h}^T\mathbb{U}_i = 0,\ i = d_{\mathbf{H}}+1,\ldots,Km_wn_w \tag{20}$$

where

$$\mathbb{U}_i = \begin{bmatrix} \mathbb{U}_1^{(i)} \\ \vdots \\ \mathbb{U}_K^{(i)} \end{bmatrix}, \tag{21}$$

and each $\mathbb{U}_k^{(i)}$ has the appropriate structure such that (20) is satisfied.

Hence from the known nullspace $\{\mathbb{U}_i\}_{i=d_{\mathbf{H}}+1}^{Km_wn_w}$, the unknown impulse responses $\mathbf{h}$ can be identified by minimizing

$$\begin{aligned} J(\mathbf{h}) &= \sum_{i=d_{\mathbf{H}}+1}^{Km_wn_w} ||U_i^T\mathbf{H}||^2 = \sum_{i=d_{\mathbf{H}}+1}^{Km_wn_w} ||\mathbb{U}_i^T\mathbf{h}||^2 \\ &= \mathbf{h}^T\left(\sum_{i=d_{\mathbf{H}}+1}^{Km_wn_w} \mathbb{U}_i\mathbb{U}_i^T\right)\mathbf{h} \end{aligned} \tag{22}$$

To avoid the trivial zero solution, one of several possible constraints can be used to take into account some information assumed or known about the channel, for example $||\mathbf{h}||^2 = 1$. Clearly, the solution translates to a minimum eigenvector problem.

4. SIMULATION

To test the performance of the proposed method we use simulation. An image of size of (100×100) is passed through $K = 4$ FIR blurs, each of size $(m_h \times n_h) = (5 \times 5)$, with coefficients drawn randomly from uniform distribution, and finally noise is added, with SNR = 30 dB. The original, blurred and noisy, and restored images are shown in Fig. 1. The restored image is obtained by using multichannel constrained least-squares method with regularization, based on the estimated blurs. Furthermore we compare also the mean squared error (MSE) performance of the SS method against the cross-relation (CR) based method [2] and the result is shown in Fig. 2. As we can observe, the SS method outperforms the CR method, which confirms the result for 1-D signals [6].

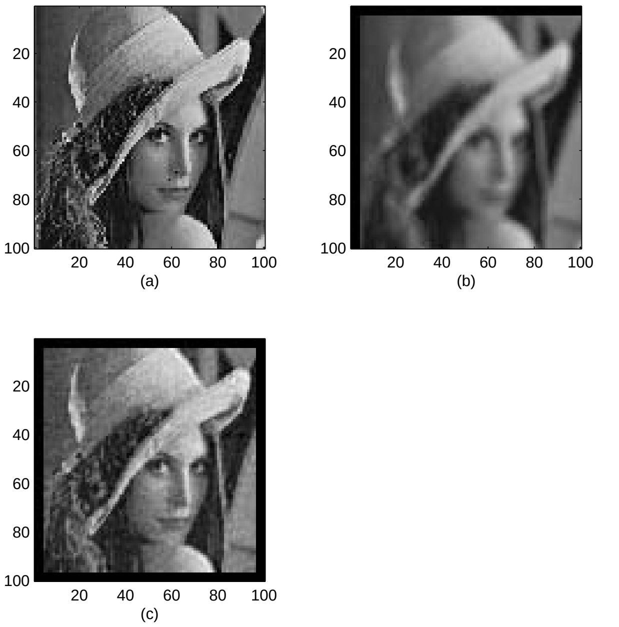

Fig. 1. Simulation. (a) Original image. (b) 1 of 4 blurred and noisy images; SNR=30 dB; $(m_h \times n_h) = (5 \times 5)$. (c) Restored image.

5. CONCLUSION

In this paper, we have proposed a SS based method to identify blurs in multichannel image restoration problem. The proposed method makes use of the orthogonality between the signal subspace and the noise subspace, as a result of the eigen decomposition of the estimated autocovariance matrix of the observed data. From the simulation we observed that the SS method performs better than the CR method, proposed in the literature [2].

6. REFERENCES

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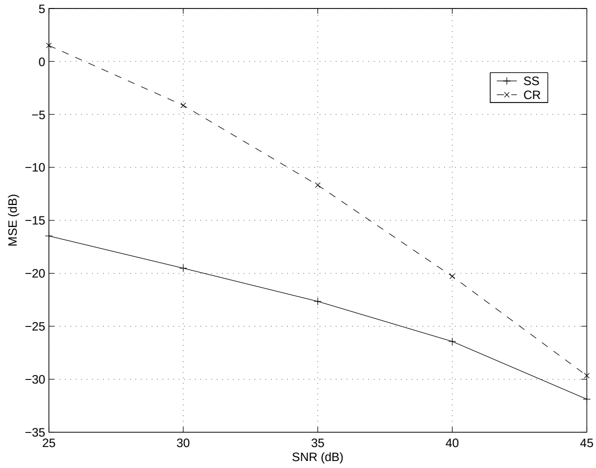

Fig. 2. Performance comparison of the SS method and the CR method. MSE versus SNR.

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