MULTIUSER MIMO SYSTEMS AND INTERFERENCE AVOIDANCE

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ABSTRACT

We present application of interference avoidance methods to multiuser systems with multiple inputs and multiple outputs. A general signal space formulation is used which makes the approach applicable to any MIMO system model regardless of the choice of basis functions. Information is transmitted via multicode CDMA where symbols that comprise the data frame from a given user are "spread" over the available dimensions using a precoding matrix. Optimal precoding matrices that maximize signal-to-interference plus noise-ratio for all symbols/users are then obtained by application of distributed greedy interference avoidance methods. Numerical simulations have been performed and the signal-to-noise ratio distribution for receiver antennas and complementary cumulative distribution functions for sum capacity with optimal precoding matrices are also presented.

1. INTRODUCTION

Wireless communication systems with multiple inputs and multiple outputs (MIMO) in which many antennas are used for transmission and reception have received increased attention from the research community over the past years. Usually, multiple antennas are employed to provide spatial diversity and improve system performance by mitigating the effects of multipath fading [1–3]. New modulation schemes for multiple antenna systems have also been proposed and analyzed [4, 5] in an attempt to bring performance close to the theoretical limits [1, 2].

In this paper we present application of interference avoidance methods to multiuser MIMO systems such as those associated with the uplink of a wireless system in which users and the base station are equipped with multiple antennas. These methods provide distributed algorithms for codeword optimization in CDMA systems [6, 7] based on maximization of the signal-to-interference plus noise-ratio (SINR). Our approach is based on application of interference avoidance to general multiaccess vector channels [8, 9] for the particular case that corresponds to a multiuser MIMO system. We note that a vector channel representation is natural in the case of MIMO systems, and several models can be found in the literature [1–3, 5]. We also note that the approach is general and applicable to any MIMO system model.

Information is transmitted over the MIMO channel using multicode CDMA where a sequence of information symbols from a given user is "spread" over the available dimensions using a precoding matrix. Formulation of the MIMO channel problem in this CDMA context allows direct application of interference avoidance techniques [8, 9] to determine optimal precoding matrices which maximize the SINR for all symbols/users. The codeword ensemble formed by the optimal precoding matrices satisfies a simultaneous water filling solution which is an emergent property of interference avoidance algorithms [8, 9] and ensures also maximization of sum capacity [10]. However,interference avoidance is a codeword optimization procedure, and is in general different from water filling algorithms which optimize the signal covariance directly. Water filling algorithms have been proposed recently for multiuser multiple antenna systems [11].

2. THE MIMO SYSTEM MODEL

We consider the uplink of the multiuser system in Figure 1, in which L users communicate with the base station, and all are equipped with antenna arrays for transmission/reception.

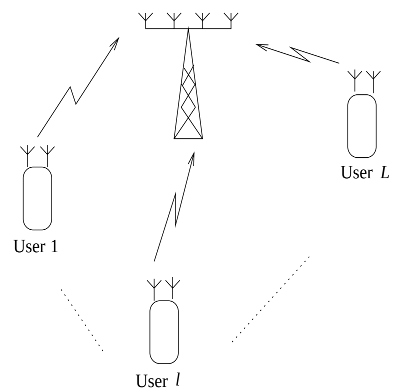

Fig. 1. Multiuser MIMO system in which users and the base station are equipped with antenna arrays for transmission/reception.

Let T_ℓ be the number of transmit antennas of user ℓ, $\ell = 1, \ldots, L$, and R be the number of receive antennas at the base station.

Using a general signal space formulation the multiuser MIMO system in Figure 1 is described by the multiaccess vector channel equation [1, 3, 5]

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n} \qquad (1)$$

with $\mathbf{x}_\ell$ being the N_ℓ-dimensional signal vector transmitted by user ℓ, $\mathbf{H}_\ell$ the $N \times N_\ell$ MIMO channel matrix corresponding to

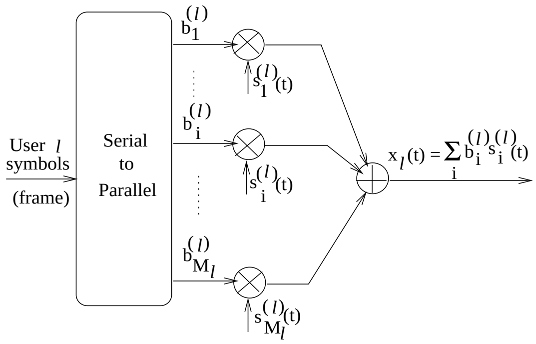

Fig. 2. Multicode CDMA approach for sending frames of information. Each symbol, $b_i^{(\ell)}$ in user ℓ's frame is assigned a distinct signature waveform $s_i^{(\ell)(t)}$ and the transmitted signal $x_\ell(t)$ is a superposition of all signatures scaled by their corresponding information symbols.

user ℓ, and $\mathbf{n}$ the noise vector at the receiver. We note that dimensions of the transmitter and receiver signal spaces depend on the number of transmit and receive antennas employed [1, 3, 5]. We assume that users transmit sequences of symbols consisting of zero-mean unit variance Gaussian random variables as frames using a multicode CDMA approach (Figure 2). For user ℓ the N_ℓ-dimensional transmitted vector $\mathbf{x}_\ell$ is obtained from the sequence of symbols to be sent $\mathbf{b}_\ell = [b_1^{(\ell)} \dots b_{M_\ell}^{(\ell)}]^\top$ through a spreading operation specified by the $N_\ell \times M_\ell$ precoding matrix $\mathbf{S}_\ell$, whose columns have unit norm and determine the "spreading" of corresponding symbols in the frame over the N_ℓ available dimensions. The transmitted vector sent by user ℓ becomes $\mathbf{x}_\ell = \mathbf{S}_\ell \mathbf{b}_\ell$, and our problem is to optimize precoder matrices $\mathbf{S}_\ell$, $\ell = 1, \dots, L$, such that the SINR corresponding to all codewords/users is maximized.

3. PRECODER OPTIMIZATION THROUGH INTERFERENCE AVOIDANCE

The fact that equation (1) is identical to that of a general multiaccess vector channel suggests that greedy interference avoidance can be used for precoder optimization [8, 9]. We note that numerous interference avoidance algorithms can be formulated based on repeated application of the greedy interference avoidance procedure, depending on the particular order in which codewords/users are selected for replacement. These are in general not water filling schemes although they yield a simultaneously water filling codeword ensemble as a consequence of the emergent water filling property of interference avoidance [8, 9].

However, this is different from finding the optimal transmit covariance matrices for all users such that the sum capacity of the multiple access vector channel defined by equation (1) is maximized. While the solution to both problems turns out to satisfy a simultaneous water filling condition [8–10], the latter problem implies an iterative water filling algorithm.

A straightforward way to implement a precoder optimization algorithm for MIMO systems based on interference avoidance is to sequentially update all codewords of a given user k until convergence and then iterate this procedure for all users. This procedure defines the eigen-algorithm for multiuser MIMO systems and is formally stated below:

The Eigen-Algorithm for Multiuser MIMO Systems

1. Start with a randomly chosen set of precoder matrices $\{\mathbf{S}_\ell\}_{\ell=1}^L$
2. For each user $k = 1 \dots L$
   (a) Compute the transformation that whitens the interference-plus-noise seen by user k
   (b) Change coordinates and compute transformed user k's MIMO channel matrix
   (c) Apply SVD and project the problem onto user k's signal space
   (d) Define the equivalent problem for user k as in [8, 9]
   (e) Adjust user k's transformed precoder matrix by replacing its columns sequentially using the greedy interference avoidance procedure
   (f) Iterate previous step until convergence
3. Repeat step 2 iteratively for each user until a fixed point is reached for which further modification of codewords will bring no additional improvement.

We note that steps 2(e)-(f) represent application of the basic eigen-algorithm [6, 7] and "water fill" user k's signal space while regarding the remaining users in the system as noise. Therefore, applied iteratively by each user, the eigen-algorithm for MIMO systems is an instance of iterative water filling and is thus guaranteed to converge to codeword ensembles which maximize sum capacity of the multiple access vector channel in equation (1).

However, we also note that the distributed and asynchronous nature of independent users and codeword updates might not admit such a simple tightly coordinated sequential approach. Fortunately, interference avoidance can still be applied under the assumption of asynchronous codeword updates since each update increases sum capacity [8, 9].

4. SIMULATION RESULTS

The MIMO channel model used for simulations is derived by using the same set of basis functions for the signal space as in [9, 12] consisting of real sinusoids (sine and cosine functions). Following [9, 12] we assume that the frame duration $\mathcal{T} \gg T_{ij}^{(\ell)}, \forall \ell,\ i,\ j$, which implies that sinusoids are eigenfunctions for all the channels in the multiple antenna link. We denote by N_c the number of frequencies used. The number of transmit antennas for user ℓ is T_ℓ and the number of receiver antennas is R. In this context the MIMO channel matrix $\mathbf{H}_\ell$ of user ℓ has dimension $2N_cR \times 2N_cT_\ell$ and is composed of $R \times T_\ell$ diagonal matrices of dimension $2N_c \times 2N_c$ containing gain matrices of all channels in the multiple antenna link between user ℓ and the base station. This MIMO channel model has been derived under the implicit assumption that the N_c periods of spanning sinusoids are large compared to the propagation delays between antenna elements so that the sine and cosine components are still approximately synchronized at the receiver even in the presence of multiple transmit and receive antennas. In addition, for simplicity, carrier synchronization for received signals has also been assumed.

4.1. Receiver SNR Distribution

With random precoding matrices and for a particular set of channels, the SNRs at different receive antennas have the bell-shaped distribution in Figure 3 (upper plot).Application of interference avoidance water fills the channels appropriately and results in a fixed set of SNRs at each receive antenna for given instances of the channel(s). As it can be seen from Figure 3 (lower plot), doubling the number of antenna elements in both transmitter and receiver results in about 3 dB improvement in the SNR. Also note that after interference avoidance the SNR is approximately the same for all receive antennas, even though no a priori assumption about equal SNRs at each antenna [1] has been made.

4.2. Fading Channels and Outage Capacity

In the case of fading environments, which is characteristic of wireless communications, the impulse responses of channels in the multiple antenna link change over time and it becomes difficult to apply interference avoidance to determine optimal precoding matrices corresponding to all channel realizations that occur during the duration of the transmission. In such cases interference avoidance is applied using average characteristics of the channels [13] to determine precoder matrices which are optimal for the average channel.

We have considered a frequency selective Rayleigh fading environment [13,14], and we first determine precoding matrices optimal for the average channel defined in terms of the average values of the Rayleigh random variables. Then we compute capacity values for distinct realizations of these Rayleigh random variables. The resulting CCDFs are presented in Figure 4 for a system with $L = 2$ users. In these plots we compare sum capacity in the case of only one transmit antenna per user and one receive antenna with the case of two transmit antennas per user and two receive antennas, and four transmit antennas per user and four receive antennas respectively.

5. CONCLUSIONS

Application of interference avoidance methods to multiuser MIMO systems has been presented and analyzed in the paper. Such systems are associated with the uplink of a wireless system in which users and the base station have multiple antennas.

Our approach is based on application of interference avoidance to general multiaccess vector channels in [8] for the particular case of multiuser MIMO systems. The approach is general and applicable to any MIMO system model. Information is sent in frames using multicode CDMA with spreading over the available dimensions implied by a precoding matrix. Optimal precoding matrices are obtained through application of greedy interference avoidance by all users in the system. For illustration an extension of the eigen-algorithm [6, 7, 9] is presented, which is an instance of iterative water filling [10].

Numerical results based on simulations were also presented in the paper. We note that these results are consistent with the well known results in the multiple antenna literature, namely that the use of multiple antennas in both the transmitter and the receiver is beneficial for system performance.

6. REFERENCES

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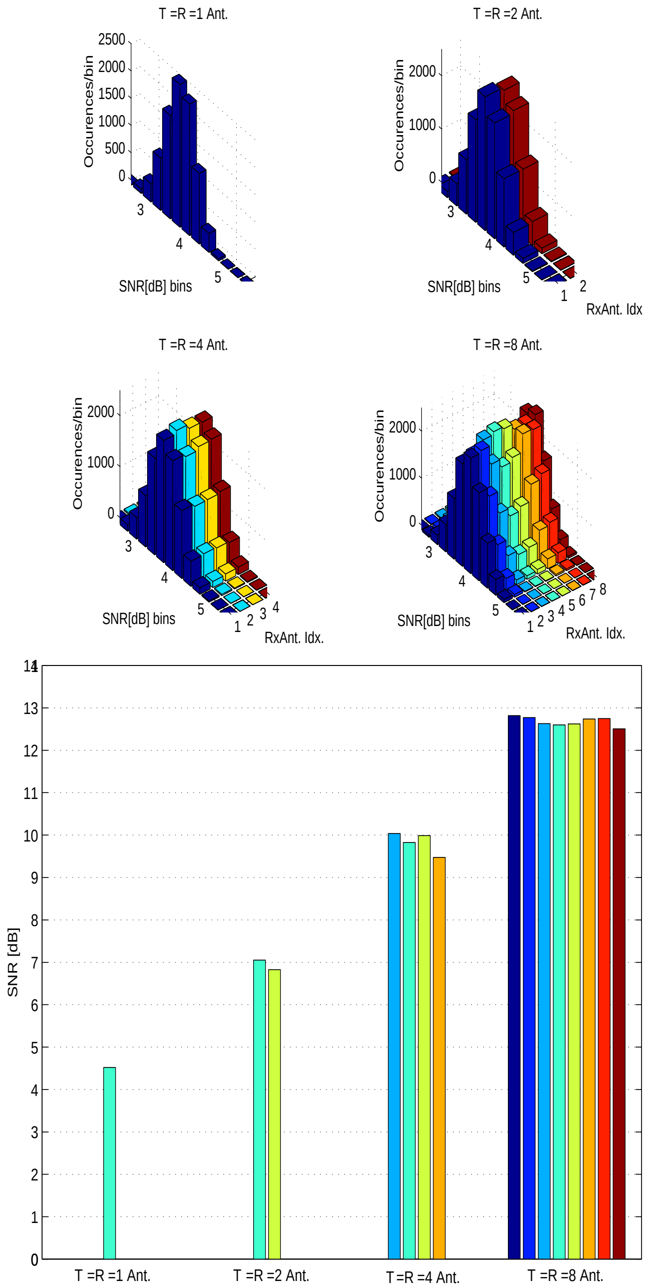

Fig. 3. SNR distributions at receiver antennas with random precoding matrices for a single user (upper plot), and SNRs with optimal precoding matrices yielded by interference avoidance(lower plot). Signal space has dimension $N = 10$, and the relative noise power at each receive antenna is $N_0 = 0.5$.

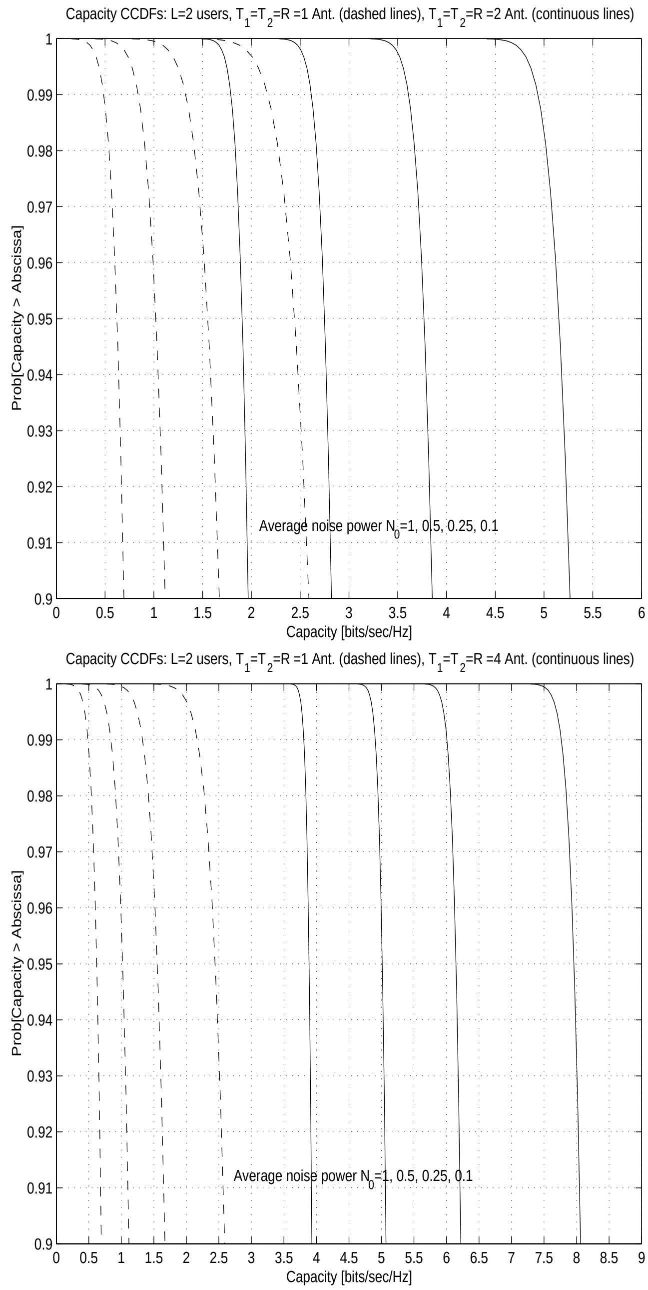

Fig. 4. Sum capacity CCDFs for a two-user MIMO system in a signal space of dimension $N = 10$. The $T_1 = T_2 = R = 1$ antenna case is compared with the $T_1 = T_2 = R = 2$ antenna case (upper plot) and with the $T_1 = T_2 = R = 4$ antenna case (lower plot) for various values of the relative noise power N_0.