PRAGMATIC MULTI-USER SPATIAL MULTIPLEXING WITH ROBUSTNESS TO CHANNEL ESTIMATION ERRORS

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ABSTRACT

Using ideas from power control and downlink beamforming, we suggest a pragmatic approach for spatial multiplexing in systems with several access points, each serving one or more mobile terminals. We assume that all channels are known globally, but in the case of fast fading, it is sufficient to know the second order statistics of the channels. Robustness to channel estimation errors is easily incorporated in the algorithm. Since global channel knowledge is required, the algorithm is mainly intended for system simulations where it provides a benchmark for the performance of any spatial multiplexing scheme with equal rate on the spatial channels.

1. INTRODUCTION

The concepts of MIMO (Multiple Input Multiple Output) processing for wireless communication has attracted large attention during the last years. Both information theoretic considerations and practical algorithms show that the performance, in terms of data rate and quality, can be improved significantly using multiple antennas both the receivers and transmitters [1–3].

However, most results that have appeared so far concentrate on the performance of each single link, not on the system level performance. Using more spatial degrees of freedom for each individual link increases the overall interference level in the system and may even reduce the total system throughput. There is no complete information theory for a general multi-user system, but a few studies have been published where the Shannon channel capacities of each link are added together, viewing the interference from other users as spatially colored noise [4, 5]. The interesting conclusion is that it often is better, in terms of system capacity, not to use all available spatial degrees of freedom for each link. Other attempts to characterize or reach the capacity region of a system are described in for example [6, 7].

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This paper considers so-called *spatial multiplexing* systems, where the receiver exploits channel knowledge to form parallel spatial channels. In particular, it is assumed that the knowledge of all channels is available on a system level. On the link level, it is well-known [3] that optimal channel capacity can be reached from the singular value decomposition (SVD) of the channel matrix, using the left and right singular vectors at the receiver and transmitter, respectively, combined with water filling to determine the data rate of each of the resulting orthogonal spatial channels. In a practical implementation it may be more convenient to use the same rate (or a small selection of different rates) for all the spatial channels. This observation is used in [8, 9] to find alternative spatial multiplexing schemes. For a single link using spatial multiplexing combined with OFDM over a frequency selective channel, the optimal solutions corresponding to a number of different optimality conditions is found using a unified approach in [10].

Here, we follow the same general idea but try to find a solution optimized at the system level, not at the link level, using the following pragmatic approach.

- The signal to interference plus noise ratio (SINR) of the received signal is used as the quality of service (QoS) criterion.
- Some controlled self interference between the spatial channels of each user is permitted.
- Given a desired QoS level for each user, the overall interference level in the system is minimized.

This approach has been presented for the case of full channel knowledge in [11] and similar ideas have been proposed in [12, 13] using MSE and an approximate BER expression, respectively, as the QoS criterion. Here, we extend the ideas to a situation where only the second order statistics of the channels are known. This formulation applies to several scenarios. In situations with rapid fading, it is clearly infeasible to track the instantaneous channels, but it may still be realistic to collect channel information averaged over the small scale fading. Also, it is easy to add robustness to channel uncertainty.

2. SYSTEM MODEL

Assume a system with one or more transmitters each communicating with one or several receivers. Both the transmitters and receivers are equipped with array antennas with N and M antenna elements each, respectively. For simplicity, we will assume a narrowband system and only study users that share the same carrier frequency. The discrete time equivalent complex valued baseband signal at receiver r is given by the $M \times 1$ vector

$$\mathbf{y}_r(n) = \sum_t \mathbf{H}_{t,r}\mathbf{x}_t(n) + \mathbf{n}_r(n) \tag{1}$$

where the $M \times N$ matrix $\mathbf{H}_{t,r}$ denotes the channel from transmitter t to receiver r and $\mathbf{x}_t(n)$ denotes the $N \times 1$ vector of signals emitted from the antennas of transmitter t. The additive noise $\mathbf{n}_r(n)$ is assumed to be spatially and temporally white, $\mathrm{E}\{\mathbf{n}_r(n_1)\mathbf{n}_r^H(n_2)\} = \sigma_r^2\mathbf{I}\delta_{n_1,n_2}$. The channel state information is given in terms of the correlation matrices

$$\mathbf{R}_{t,r} = \mathrm{E}\left\{\mathrm{vec}[\mathbf{H}_{t,r}]\,\mathrm{vec}^H[\mathbf{H}_{t,r}]\right\} \tag{2}$$

The special case of complete channel knowledge corresponds to $\mathbf{R}_{t,r}$ of rank one.

The communication between each transmitter receiver pair is done by multiplexing the data over one or more spatial channels, formed by linear transformations at the receiver and transmitter. The lth spatial channel for receiver r is determined by a transmit beamformer $\mathbf{v}_{r,l}$ and a receive beamformer $\mathbf{u}_{r,l}$. To emphasize that self interference between spatial channels is treated exactly the same as multiuser interference and in order to keep the notation as simple as possible, a single index i is used below instead of the pair r, l to identify each spatial channel. The receiver using spatial channel i is denoted by $\rho(i)$ and the corresponding transmitter is denoted by $\tau(i)$. The signal transmitted at transmitter t is given by

$$\mathbf{x}_t(n) = \sum_{i;\tau(i)=t} \mathbf{v}_i s_i(n) \tag{3}$$

where s_i denotes the data stream transmitted over the spatial channel i and $\mathbf{v}_i$ is the corresponding transmit beamforming vector. At the corresponding receiver, the data stream s_i is estimated using the receive beamforming vector $\mathbf{u}_i$,

$$\hat{s}_i(n) = \mathbf{u}_i^H \mathbf{y}_{\rho(i)}(n)\ . \tag{4}$$

Thus, the resulting spatial channel i including transmit and receive beamformers, is a scalar channel with total gain $\mathbf{u}_i^H\mathbf{H}_{\tau(i),\rho(i)}\mathbf{v}_i$. Using properties of the vec operator and Kronecker product, the average power gain is given by

$$\begin{aligned}\mathrm{E}\{|\mathbf{u}^H\mathbf{H}\mathbf{v}|^2\} &= (\mathbf{v}^T \otimes \mathbf{u}^H)\mathbf{R}(\mathbf{v}^* \otimes \mathbf{u}) \\ &= \mathbf{u}^H(\mathbf{v}^T \otimes \mathbf{I})\mathbf{R}(\mathbf{v}^* \otimes \mathbf{I})\mathbf{u} \\ &= \mathbf{v}^H(\mathbf{I} \otimes \mathbf{u}^T)\mathbf{R}^T(\mathbf{I} \otimes \mathbf{u}^*)\mathbf{v}\ .\end{aligned} \tag{5}$$

3. ALGORITHM

The strategy proposed in this paper is a straightforward generalization of optimal downlink beamforming [14–16], minimizing the total transmitted power under the constraint that the received signal of each spatial channel has a sufficient average signal to interference plus noise ratio (SINR). Note that minimizing the total transmit power will also help reducing the overall interference level in the system.

Assume that the signals multiplexed over the spatial channels are mutually uncorrelated and have equal power normalized to one. If γ_i denotes the SINR threshold for spatial channel i, the resulting optimization problem is

$$\begin{aligned} &\min \sum_j \|\mathbf{v}_j\|^2 \\ &\text{s.t.}\ \frac{\mathrm{E}\{|\mathbf{u}_i^*\mathbf{H}_{\tau(i),\rho(i)}\mathbf{v}_i|^2\}}{\sum_{j\neq i}\mathrm{E}\{|\mathbf{u}_i^*\mathbf{H}_{\tau(j),\rho(i)}\mathbf{v}_j|^2\} + \sigma_{\rho(i)}^2\|\mathbf{u}_i\|^2} \geq \gamma_i \\ &\quad \|\mathbf{u}_i\|^2 = 1,\ \ i = 1, \ldots, I \end{aligned} \tag{6}$$

This is a non-linear and non-convex optimization problem in the variables $\mathbf{v}_i$ and $\mathbf{u}_i$ which makes it difficult to find the global optimum. However, using (5) it is easy to show that for fixed transmit beamformers, the optimum receive vectors are given by the standard maximum SINR solution and for fixed receive beamformers, the problem reduces to the problem of jointly optimal transmit beamforming which can be solved using the algorithms in [14–17]. Iterating between these receive and transmit solutions will decrease the cost function in each step and is thus guaranteed to converge. In [18], the two steps are combined into a single loop (that algorithm is formulated for situations with completely known channel matrices but can easily be extended to the case where only $\mathbf{R}_{t,r}$ are known. However, numerical experiments have shown that the algorithm often diverges even if a feasible starting point is given. Inspired by [16], a two-stage approach was used, first optimizing a common factor in all γ_i to find a feasible solution, if possible, and then using this feasible solution to minimize (6). Both stages are performed iterating between transmitter and receiver optimization. In both stages, each optimal SINR receiver beamformer is found as the solution of a generalized eigenvalue problem. In the first stage, the transmitter beamformers are determined using the algorithm in [16, Table I]. In the second stage, the transmit power is minimized using any of the downlink beamforming algorithms in [14–17].

4. ROBUSTNESS TO CHANNEL MISMATCH

The design strategy formulated above may easily be extended to include robustness to channel uncertainties. Assume that lower and upper bounds on the true channel cor-

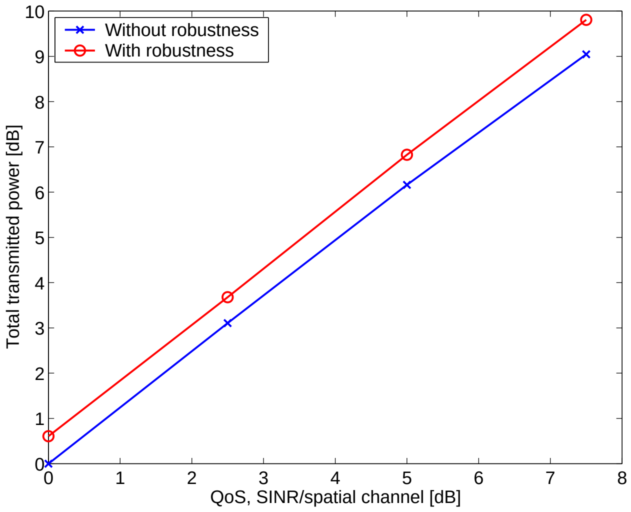

Fig. 1. Comparison of robust and non-robust spatial multiplexing.

relation matrices $\mathbf{R}_{t,r}$ are available in the form $\underset{\smile}{\mathbf{R}}_{t,r} \preceq \mathbf{R}_{t,r} \preceq \overset{\frown}{\mathbf{R}}_{t,r}$, where $\mathbf{A} \preceq \mathbf{B}$ denotes that $\mathbf{B} - \mathbf{A}$ is positive semidefinite. The simplest example, used in the numerical evaluations below, is to use $\underset{\smile}{\mathbf{R}}_{t,r} = \hat{\mathbf{R}} - \epsilon_{t,r}\mathbf{I}$ and $\overset{\frown}{\mathbf{R}}_{t,r} = \hat{\mathbf{R}} + \epsilon_{t,r}\mathbf{I}$. This corresponds to constraint of the form $\|\hat{\mathbf{R}}_{t,r} - \mathbf{R}_{t,r}\|_2 \leq \epsilon_{t,r}$, where, $\|\cdot\|_2$ denotes the spectral norm, i.e., the largest eigenvalue.

To define the optimal robust $\mathbf{u}_i$ and $\mathbf{v}_i$, (6) is modified such that the constraint should hold for all values of the $\mathbf{R}_{t,r}$ within these bounds. As shown in [15], this results in a problem of the same form as (6), where $\mathbf{R}_{t,r}$ is replaced by $\underset{\smile}{\mathbf{R}}_{t,r}$ in the numerators of the constraints and by $\overset{\frown}{\mathbf{R}}_{t,r}$ in the denominators of the constraints.

5. NUMERICAL EXAMPLES

5.1. Robustness to Channel Mismatch

To illustrate the robustness to channel estimation errors, a simple scenario has been simulated with a single transmitter with 6 antenna elements communicating with two receivers, each equipped with 2 antenna elements. For simplicity, the true channels were i.i.d Gaussian fading and the estimation errors were also Gaussian i.i.d with standard deviation 1% of the standard deviation of each element.

A slowly varying fading process was assumed, where the transmitter has access to noisy estimates of the true channel, which means that $\hat{\mathbf{R}}$ is rank one. The robust formulation of Section 4 was used with $\underset{\smile}{\mathbf{R}}_{t,r} = \hat{\mathbf{R}} - \epsilon_{t,r}\mathbf{I}$ and $\overset{\frown}{\mathbf{R}}_{t,r} = \hat{\mathbf{R}} + \epsilon_{t,r}\mathbf{I}$, where ϵ was chosen such that the bound holds with an outage probability of about 10%.

Figure 1 shows the total transmit power necessary to transmit two data streams to each user, using the robust and the non-robust formulations.

5.2. Rapid Fading

In many MIMO system deployments, it is reasonable to let the receiver track the instantaneous channel, based on known pilot signals in the data, but it is typically not feasible to have global knowledge of all instantaneous channels. However, it may still be practically possible to collect the second order statistics of all the channels, averaged over the small-scale fading and use this information to design the transmit vectors. In these situations, the linear precoding at the transmitters should ideally be designed assuming that the receivers use the optimal MMSE receiver for the instantaneous channel. However, the resulting optimization problem is almost impossible to solve and the solution of (6) may be used to find good transmitter weights, even though they are designed for a scenario where also the receiver only knows the second order statistics of the channels. Then, the use of a better receiver should add a diversity gain, which could be used to lower the fading margin needed in the SINR target in (6). For Rayleigh fading channels, using fixed transmit and receive weights, it can be shown, similar to [19], that the instantaneous SINR, γ_{inst}, has an outage probability bounded by

$$\Pr[\gamma_{\text{inst}} < \gamma_{\text{target}}] \leq 1 - e^{-\gamma_{\text{target}}/\gamma_{\text{aver}}}, \tag{7}$$

where γ_{aver} denotes the average signal power divided by the average interference plus noise power. This means, for example, that an fading margin of 10dB is necessary to obtain an outage level around 10%, i.e. that γ in (6) which corresponds to γ_{aver} should be chosen 10dB larger than the desired target SINR value. The spatial multiplexing scheme proposed in this paper was evaluated in a simulated scenario with four base stations and four randomly positioned mobiles, each equipped with a 4-element antenna array. The propagation channels were modeled by 1-5 randomly placed clusters of scatterers around each mobile, each with a small angular spread and log-normal distributed shadow fading. The transmit weights were determined from (6) based on the second order statistics of the channel. Receiver weights were determined both from (6) and using knowledge about the instantaneous channels. The results are best summarized by the following two conclusions,

- Trying to use more than a single spatial subchannel per link will mostly not lead to any feasible solution, since using more spatial degrees of freedom for each link will increase the overall interference level in the system. The conclusion is that ordinary beamforming at the transmitters and receivers often is best in terms of system performance. Similar conclusions have also been reported e.g. in [5, 11].

- The average diversity gain of determining the receiver weights from the instantaneous channel instead of the covariance of the statistics is above 15dB, which more than compensates for the fading margin implied by (7). Note, however, that the design criterion (6) does not necessarily favor a solution with high diversity gain. Note also that this diversity gain will be smaller if more spatial degrees of freedom are used for each link.

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