FREQUENCY OFFSET AND I/Q IMBALANCE COMPENSATION FOR OFDM DIRECT-CONVERSION RECEIVERS

Guanbin Xing, Manyuan Shen and Hui Liu

Department of Electrical Engineering, Box 352500
University of Washington, Seattle, WA 98195
{gxing,mshen,hliu}@ee.washington.edu

ABSTRACT

Two types of RF front-end imperfections in direct conversion receivers, namely frequency offset and I/Q imbalance, are addressed in this paper. The I/Q imbalance not only introduces an unwanted image interference but also degrades the accuracy of carrier estimation. In this paper, we propose a pilot-based scheme for both frequency offset and I/Q imbalance compensation. A Nonlinear Least Squares (NLS) frequency estimator robust to the I/Q imbalance is developed. Also derived is an I/Q compensation structure that consists of two stages: the frequency dependent imbalance is compensated with an FIR filter whereas the frequency independent part is corrected with an asymmetric phase compensator. The compensation coefficients are optimized by restoring the phase rotation embedded in the pilot symbols.

1. INTRODUCTION

The proliferation of wireless devices has been generating a renewed interest in RF direct-conversion that has inherent advantages in cost, package size, and power consumption. The tradeoff however, is a higher degree of RF imperfections including the I/Q imbalance. Abundant literatures exist on I/Q imbalance compensation; see [1][2] and references therein. Many of the existing approaches consider only frequency-independent I/Q imbalance, while in reality the I/Q imbalance is often frequency dependent, especially for wideband direct-conversion receivers. Furthermore, the presence of I/Q imbalance degrades the accuracy of carrier estimation, which may be problematic for carrier-sensitive systems such as OFDM. However, few existing algorithms address the frequency synchronization and I/Q imbalance jointly despite their tangled effects.

In this paper, we propose a pilot-based scheme for both frequency offset and I/Q imbalance compensation. In the presence of I/Q imbalance, the carrier estimation problem is nontrivial even with known training symbols since the received pilot not only contains the offset frequency but also its image component. Therefore, previously proposed frequency estimators, e.g. [3][4], are not directly applicable. Towards this end, we first derive a two-frequency model that formulates the I/Q imbalance effect on the frequency estimation. Based on this model, a Nonlinear Least Squares (NLS) method robust to the I/Q imbalance is developed. For I/Q imbalance compensation, an FIR filter is inserted into one of the I/Q branches to balance the frequency dependent imbalance, while the frequency independent error is corrected with an asymmetric phase compensator followed after. The compensation coefficients are optimized based on the phase rotation embedded between adjacent pilots, after the frequency offset has been accurately estimated using the method we propose.

2. I/Q IMBALANCE MODEL

Fig.1 shows the mathematical model of a direct-conversion receiver. The I/Q imbalance can be categorized into a frequency independent part and a frequency dependent part. An imbalanced mixer/LO usually generates a frequency independent I/Q imbalance which can be characterized by an amplitude mismatch α and a phase error ϕ. The subsequent I/Q amplification and filtering in general cause the frequency dependent I/Q imbalances, which are modelled by two mismatched LPFs ($H_I(f)$ and $H_Q(f)$). Since α can be treated as part of the mismatched LPF responses, the effect of α will be ignored in the ensuing discussion.

To understand the impact of I/Q imbalance on signal reception, we define the received signal as

$$\tilde{r}(t) = Re\{r(t) \cdot e^{j(\omega_c + \Delta\omega)t}\} \quad (1)$$

$$r(t) = r_I(t) + j \cdot r_Q(t) = s(t) \otimes c(t) \quad (2)$$

where $r(t)$, $s(t)$ and $c(t)$ are the baseband representations of the received signal, the transmitted signal, and the channel response, respectively. Following the derivation in [1] and taking into account the frequency offset ($\Delta\omega$), the down-converted signal $x(t)$ can be represented as

$$\begin{aligned} x(t) &= r(t)e^{j\Delta\omega t} \otimes g_S(t) + r^*(t)e^{-j\Delta\omega t} \otimes g_I(t) \\ &= e^{j\Delta\omega t}s(t) \otimes d(t) + e^{-j\Delta\omega t}s^*(t) \otimes v(t) \end{aligned} \quad (3)$$

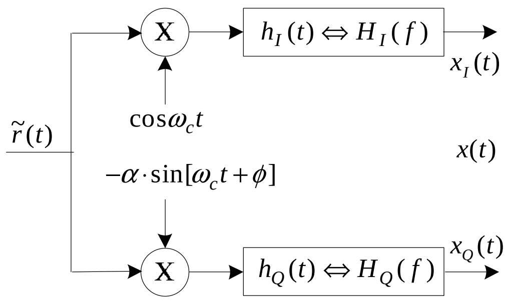

Fig. 1. Mathematical model of a direct-conversion receiver

where

$$g_S(t) = \mathcal{IDFT}\{[H_I(f) + e^{-j\phi} \cdot H_Q(f)]/2\}$$
$$g_I(t) = \mathcal{IDFT}\{[H_I(f) - e^{j\phi} \cdot H_Q(f)]/2\}$$
$$d(t) = c(t) \otimes g_S(t),\ v(t) = c^*(t) \otimes g_I(t) \tag{4}$$

Notice that different from an ideal down conversion, the desired signal is contaminated by its image interference (the second term in (3)). Such image interference, if not properly accounted for, will degrade the accuracy of carrier estimation and limit the reception performance.

3. ALGORITHM DEVELOPMENT

3.1. Frequency offset estimation with I/Q imbalance

Pilot-based synchronization methods have been well studied due to the needs for fast and accurate synchronization in burst transmissions. The pilot usually contains several identical symbols. Numerous carrier estimators have been proposed accordingly, e.g., [3][4]. However, as will be shown later, the accuracy of these estimators degrades in the presence of I/Q imbalance. Reference [5] analyzes the I/Q imbalance effect on carrier estimation but no effective solution is proposed. In this section, we propose a pilot structure that contains M identical symbols (each containing N samples) with all the even symbols rotated by $\pi/4$. A guard interval or cyclic prefix is inserted between symbols to avoid inter-symbol interference. The frequency estimation problem is reformulated based on the proposed pilot and an NLS based estimator robust to the I/Q imbalance is derived. For illustration, we first examine M identical pilot symbols without the additional phase rotation between symbols. The reason for the added rotation will become obvious later.

After the GI/CP removal, we stack the received pilot samples in a matrix as follows

$$\mathbf{X} = \begin{bmatrix} x(1,1) & x(1,2) & \cdots & x(1,N) \\ x(2,1) & x(2,2) & \cdots & x(2,N) \\ \vdots & \vdots & \ddots & \vdots \\ x(M,1) & x(M,2) & \cdots & x(M,N) \end{bmatrix} \tag{5}$$

where $x(m,n)$ stands for the nth sample of the mth received pilot symbol. Since the pilot contains M identical symbols, i.e., $s(m,n) = p(n)$ and $p(n)$ being the pilot symbol, each column of $\mathbf{X}$ in (5), denoted as $\mathbf{x}_n$, can be expressed as a superposition of two tones according to (3)

$$\mathbf{x}_n = \begin{bmatrix} e^{j\Omega} & e^{-j\Omega} \\ e^{j2\Omega} & e^{-j2\Omega} \\ \vdots & \vdots \\ e^{jM\Omega} & e^{-jM\Omega} \end{bmatrix} \begin{bmatrix} \alpha_n \\ \\ \beta_n \end{bmatrix} \stackrel{\text{def}}{=} \mathbf{\Omega} \begin{bmatrix} \alpha_n \\ \\ \beta_n \end{bmatrix} \tag{6}$$

where $\alpha_n = e^{j\Delta\omega n} p(n) \otimes d(n)$, $\beta_n = e^{-j\Delta\omega n} p^*(n) \otimes v(n)$ and $\Omega = \Delta\omega T$. The formulation above leads to a classic multi-line spectral estimation problem. The NLS method described in [6] becomes directly applicable. Particularly in our case, even though $\mathbf{x}_n$ consists of two frequencies, it actually only contains one variable with a different sign. As a result, only a low cost one-dimension search is required. The frequency estimation can thus be obtained by searching the maxima of the following

$$\hat{\Omega} = argmax_{\Omega} \left[tr\{\mathbf{\Omega}(\mathbf{\Omega}^H\mathbf{\Omega})^{-1}\mathbf{\Omega}^H\hat{\mathbf{R}}\} \right] \tag{7}$$

where $\hat{\mathbf{R}} = \mathbf{X}\mathbf{X}^H$ is the sample covariance matrix.

Note that the algorithm proposed in [3] can be considered as a special case of (7) in the absence of I/Q imbalance: when $\beta_n = 0$, (7) can be simplified as

$$\hat{\Omega} = argmax_{\Omega} \left[\mathbf{\Omega}^H\hat{\mathbf{R}}\mathbf{\Omega} \right] \tag{8}$$

with $\mathbf{\Omega} \stackrel{\text{def}}{=} \left[e^{j\Omega}\ e^{j2\Omega} \cdots e^{jM\Omega} \right]^T$.

Special attention should be given to $\mathbf{\Omega}$. In our application, $\mathbf{\Omega}$ becomes ill-conditioned when the initial offset is close to zero, which leads to poor estimation accuracy around zero frequency. To maintain a proper condition number regardless of the initial offset, an additional $\pi/4$ rotation can be introduced to all the even pilot symbols. The resulting $\mathbf{\Omega}$ thus becomes always well-conditioned

$$\mathbf{\Omega} = \begin{bmatrix} e^{j\Omega} & e^{-j\Omega} \\ e^{j2\Omega} \cdot e^{j\pi/4} & e^{-j2\Omega} \cdot e^{-j\pi/4} \\ \vdots & \vdots \\ e^{jM\Omega} \cdot e^{j\pi/4} & e^{-jM\Omega} \cdot e^{-j\pi/4} \end{bmatrix} \tag{9}$$

3.2. I/Q Imbalance Compensation

3.2.1. I/Q Compensation Structure

Fig.2 shows the proposed I/Q compensation structure. The FIR filter $w(n)$ in the I branch is meant for compensating the frequency-dependent imbalance. The coefficients of the filter should be optimized in such a way that its frequency response $W(f)$ is as close to $H_Q(f)/H_I(f)$ as possible.

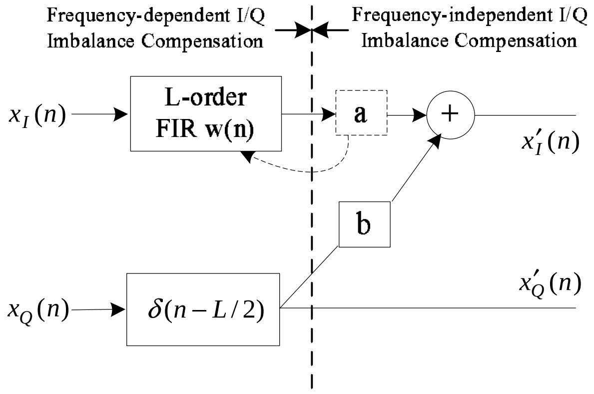

Fig. 2. Proposed compensation structure for I/Q imbalance

The $\delta(n-L/2)$ block in the Q branch is simply a delay unit for matching the delay of $w(n)$. The frequency-independent imbalance caused by the mixer/LO can be characterized by a matrix $\mathbf{\Phi}$ as follows

$$\begin{bmatrix} x_I \\ x_Q \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} r_I \\ r_Q \end{bmatrix} \stackrel{\text{def}}{=} \mathbf{\Phi} \begin{bmatrix} r_I \\ r_Q \end{bmatrix}$$

where $[r_I\ r_Q]^T$ and $[x_I\ x_Q]^T$ are the input and output of the mixer. The compensation is simply to left-multiply $\mathbf{\Phi}^{-1}$ on the down-converted vector $[x_I\ x_Q]^T$

$$\mathbf{\Phi}^{-1} = \underbrace{\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}}_{phase\ rotation} \begin{bmatrix} 1/\cos\phi & \tan\phi \\ 0 & 1 \end{bmatrix} \quad (10)$$

where the first part is just a phase rotation and will be absorbed in the channel equalization. The second part suggests an implementation as in the right half of Fig.2, where the gain factor a and b correspond to $1/\cos\phi$ and $\tan\phi$ in (10) respectively. Notice that the gain a in the I branch can be merged with $w(n)$ to further simplify the compensation. It is also worth pointing out that without frequency-dependent I/Q imbalance, $w(n)$ reduces to a scaler and the compensation structure reduces to the asymmetric compensator described in [2].

3.2.2. Optimization of the Compensation Coefficients

Having decided the compensation structure, the remaining task is to determine the optimum filter coefficients $w(n)$ and the gain factor b that best remove the I/Q imbalance.

Notice from (6) that without I/Q imbalance ($\beta_n = 0$), the two adjacent received symbols should only differ by a phase rotation, $e^{j\Omega}$, caused by the carrier offset. This phase rotation is known to the receiver after the frequency offset is accurately estimated. However, the presence of I/Q imbalance alters this relationship between the received pilot symbols. This observation suggests a procedure to determine $w(n)$ and b by restoring such phase rotation. In other words, the optimum $w(n)$ and b can be estimated by minimizing the following

$$[w(n), b]_{opt} = argmin_{w(n),b}\mathbf{J}(w, b)$$
$$\mathbf{J} = \sum_{m=1}^{M-1}\sum_{n=1}^{N} |x'(m+1,n) - C_m x'(m,n)|^2 \quad (11)$$

where

$$x'(m,n) = x'_I(m,n) + j \cdot x'_Q(m,n)$$
$$x'_I(m,n) = x_I(m,n) \otimes w(n) + b \cdot x_Q(m,n)$$
$$x'_Q(m,n) = x_Q(m,n)$$
$$C_m = e^{j\Omega_m} = \begin{cases} e^{j\Omega} \cdot e^{j\pi/4} & : \quad m = odd \\ e^{j\Omega} \cdot e^{-j\pi/4} & : \quad m = even \end{cases}$$

Note that the cost function in (11) has a quadratic form and therefore the minimization has a close form solution by using linear least squares method.

Since the compensation coefficients are determined by restoring the known phase rotation between adjacent pilot symbols, it is crucial that such phase rotation structure is robust with respect to the initial frequency offset. This reenforces the necessity of adding a $\pi/4$ rotation to all the even symbols, as mentioned in the previous subsection.

4. NUMERICAL RESULTS

Computer simulations are conducted to examine the performance of proposed scheme. Similar to the short SYNC symbols used in IEEE802.11a, a pilot containing 10 symbols (each with 16 samples) is generated. A three-ray channel with exponentially decaying power profile is used in all simulations. Two I/Q imbalance cases are considered: a) moderate I/Q mismatch scenario with only frequency independent imbalance ($\alpha = 1dB,\ \phi = 5^o,\ h_I(t) = h_Q(t) = \delta(t)$); and b) severe I/Q mismatch scenario with both frequency independent and frequency dependent imbalances ($\alpha = 1dB,\ \phi = 5^o,\ h_I(t) = [1\ 0.1],\ h_Q(t) = [0.1\ 1]$).

Performance of the proposed frequency offset estimator and the method developed by Li et al in [3] is given in Fig.3 and Fig.4. As shown in Fig.3, the performance of Li's approach fluctuates with the initial frequency offset, depending on how orthogonal the image interference is to the desired signal. In contrast, the proposed estimator works well across the entire frequency range. Also, it can be seen from Fig.4 that without considering I/Q imbalance, the estimation performance of Li's approach is floored by the image interference. On the other hand, by accounting for the interference structure, the accuracy of proposed estimator improves with the SNR and the estimation MSE is comparable to that without the I/Q imbalance.

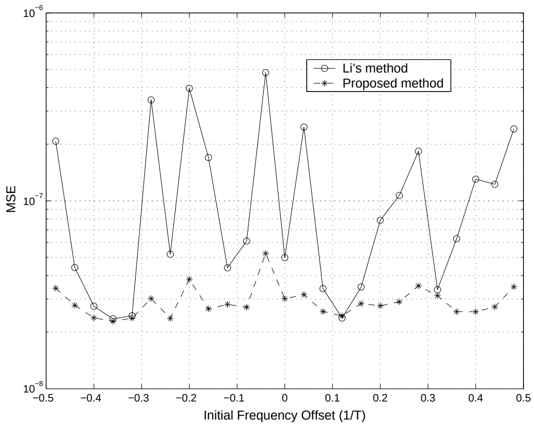

Fig. 3. Frequency estimation MSE vs. initial frequency offset. I/Q imbalance case A and SNR=25dB are assumed

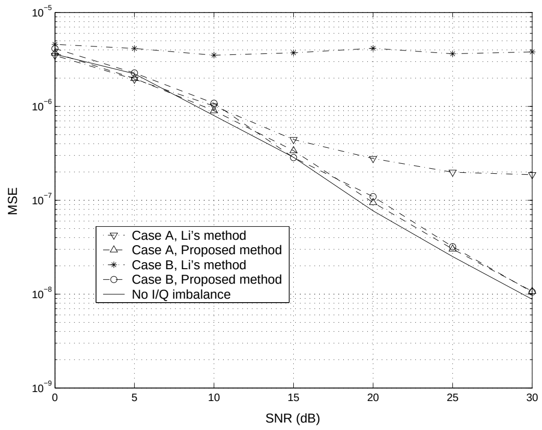

Fig. 4. Frequency estimation MSE vs. SNR under various I/Q imbalance cases. Initial frequency $\Delta\omega = 0.1/T$.

The average SNR of the demodulated OFDM signal is used to evaluate the impact of I/Q imbalance on the overall reception performance. An OFDM system with 512 subcarriers is simulated and QPSK/QAM with coherent demodulation is considered in the simulation. As shown in Fig.5, without I/Q compensation, the overall demodulation performance is again floored by the image interference. The reception may even fail completely when the I/Q imbalance is severe. After applying the proposed compensation with only a 5-order FIR filter, the demodulation SNR loss is negligible even with severe I/Q imbalance.

5. CONCLUSIONS

The I/Q imbalance of a RF front end not only introduces an unwanted image interference but also degrades the accuracy of carrier estimation, both of which are throttling the signal reception performance. This effect is even more severe in a direct-conversion OFDM receiver. In this paper we have proposed a pilot-based scheme for both carrier offset and I/Q imbalance compensation. An NLS based frequency estimator robust to I/Q imbalance has been developed. For the I/Q imbalance compensation, an FIR filter followed by an asymmetric phase compensator has been derived to correct both frequency dependent and independent I/Q imbalance. Computer simulations show that with a 5-order FIR filter, the SNR loss of demodulated signal is less than 1dB even with severe I/Q imbalance.

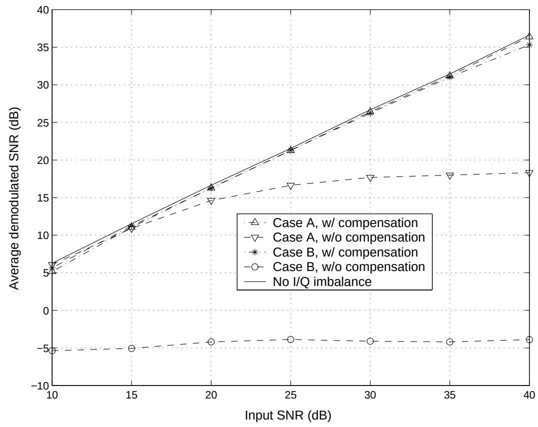

Fig. 5. Demodulated SNR of OFDM signal vs. input SNR under various I/Q imbalance cases. FIR tap $L = 5$.

6. REFERENCES

[1] M. Valkama, M. Renfors and V. Koivunen, "Compensation of frequency-selective I/Q imbalances in wideband receivers: models and algorithms," in *Proc. IEEE SPAWC'2001*, pp. 42-45, Taoyuan, Taiwan, Mar. 2001

[2] J. K. Cavers and M. W. Liao, "Adaptive compensation for imbalance and offset losses in direct conversion transceivers," *IEEE Trans. on Vehicular Tech.*, vol. 42, pp. 581-588, Nov. 1993

[3] J. Li, G. Liu and G. B. Giannakis, "Carrier frequency offset estimation for OFDM-based WLANs," *IEEE Trans. on Signal Processing*, vol.8, pp. 80-82, Mar. 2001

[4] R. V. Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House Publishers, 2000

[5] V.K.P. Ma and T. Ylamurto, "Analysis of IQ imbalance on initial frequency offset estimation in direct down-conversion receivers," in *Proc. IEEE SPAWC'2001*, pp. 158-161, Taoyuan, Taiwan, Mar. 2001

[6] P. Sotica and R. Moses, *Introduction to Spectral Analysis*, Prentice Hall, 1997